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A TEXT BOOK OF

INTERMEDIATE PHYSICS

By

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PREFACE

Physics is considered by average college student to be an interesting but a difficult subject. This difficulty often arises from the fact that student is not able to grasp well the basic ideas and principles of the subject. An attempt has been made in this book to present the subject in a manner that may possibly lead to a better understanding and grasp. The author has taught Physics for more than twenty-five years to all classes from the Intermediate to M.Sc. and has done original work and investigations in the subject. The book is based on the long experience gained during these years in Physics classes and the Physics Laboratory.

The author shall feel very grateful if the fellow lecturers of the subject in colleges, shall communicate to him their own views and opinions about any topic dealt in the book.

P. U. College,
Hoshiarpur
May, 1953

H. R. SARNA
Professor of Physics.

Intermediate Physics

CHAPTER I

SCIENCE AND SCIENTIFIC METHOD

1. Science—A Study of Nature.

In the world around us various natural phenomena are constantly occurring. The sun rises and sets causing the succession of day and night. The seasons follow in regular cycles. The weather undergoes constant changes. The stars, lightning, rainbows and many other things we notice have been the cause of wonder and curiosity to man.

Gifted with an inborn intelligence, man has constantly tried to know the cause of the natural phenomena and the natural changes which he observes in the world. This inquiry carried on through generations, has led to a considerable amount of knowledge which we call Science. It is a Latin word which means to "Know". In a general way, therefore, knowledge of all kinds is included in science, but in ordinarily accepted sense science is that branch of knowledge which studies nature, its working, and its laws. Knowledge connected with living organisms, plants and animals, concerns the Biological sciences; while that dealing with non-living bodies or inert masses, which go to make up the world is called as Physical sciences, and includes such subjects as Physics, Chemistry, Astronomy, Geology etc.

2. The scientist is a seeker of truth.

He makes careful observations of the facts and phenomena of nature, often involving laborious measurements and experimentation. As a result of these observations he makes rational conclusions about the principles and laws governing working of nature. These laws are then tested by their ability to explain new knowledge as it is acquired. If they can successfully explain all the known facts, the laws are accepted as true and dependable knowledge; otherwise they are discarded and investigation continues in search of truth.

Physics concerns itself with an investigation of matter and of energy. In the world around us various transformations are taking place in which matter and energy are undergoing changes. In fact the activity of nature is a consequence of changes in matter and energy. Investigation of matter and energy, therefore, has formed the chief pursuit of the physicists.

The growth of science has been very rapid in the last three hundred years. In this period the experimental study of nature has been pursued and it has proved extremely fruitful in revealing new knowledge, and as a means of deciding between different theories. Galileo (1564-1642) may be called the father of experimental sciences. His famous experiments on falling bodies done at the leaning tower of Pisa were a straight and a simple way of proving that falling bodies dropped from the same height take the same time to reach the earth. People at that time believed that the heavier bodies fall at a faster rate than the lighter ones, a view, which was taught by the Greek philosopher Aristotle and accepted for more than a thousand years. Galileo proved that such light bodies as papers and feathers take longer time to fall because of air resistance, but if the air resistance is reduced or eliminated all bodies fall to the earth from same height in equal time. Galileo was prosecuted and imprisoned for defying the established thoughts of his age. In fact, most of the pioneers of truth have met the same fate. Yet Galileo introduced a new outlook, the experimental outlook in the field of scientific investigation. Since then the experimental method has been followed with great success. The present times called the age of science owes this name to the great experimental work and researches of a large number of scientists.

3. Measurement an Essential Part of Scientific Investigations.

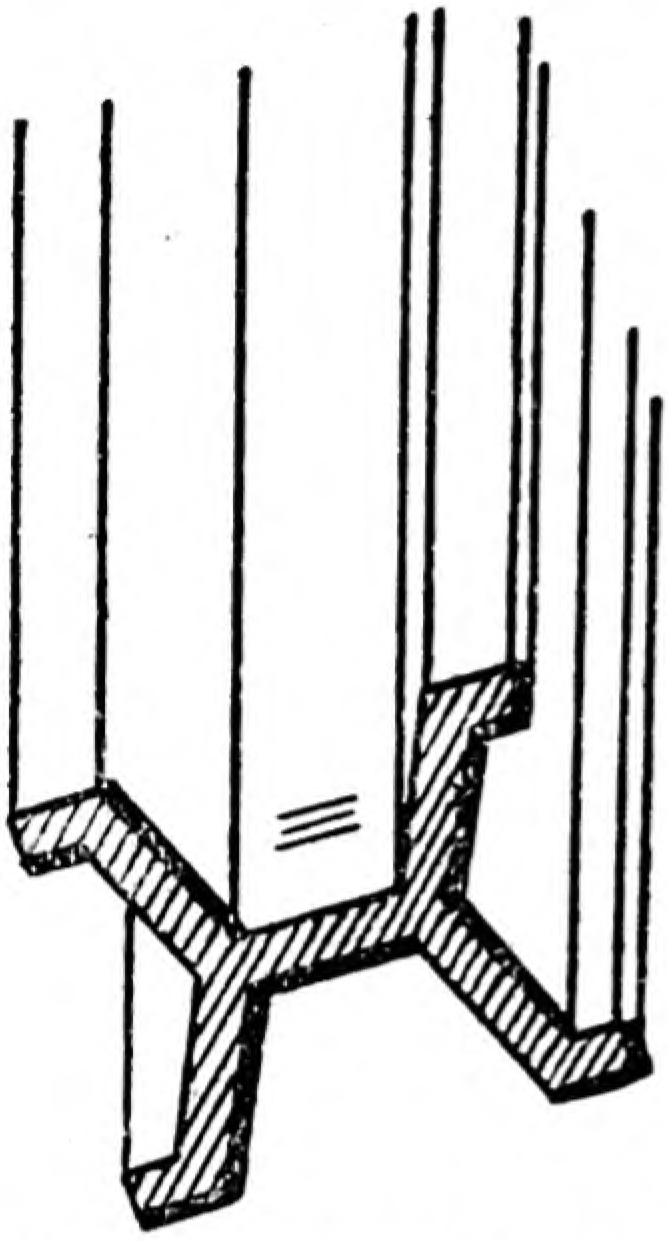
The scientist in his inquiry of nature is not content with a qualitative knowledge but wishes the knowledge to be made as exact as possible. For this purpose refined measurement of various quantities is required and measuring appliances of many designs have been introduced to aid the scientists in their studies. Measuring scales in diverse shapes and forms are used for determining values of many a quantity. Physics more than other sciences has laid the greatest importance on exact measurement and on this account the laws of Physics are more definite, and subject to mathematical expression. Physics deals with basic problems of nature and on this account has been called the most fundamental science.

4. Necessity of Units.

To measure any quantity one must select a unit in terms of which the measurement is to be made. Thus when a draper measures a length of cloth, he uses the yard as the unit of measurement or when a grocer sells a commodity, he weighs it in terms of a suitable unit, say a pound. The scientist has to measure a great many quantities as velocities, forces, work, temperature, electric currents and so on and he obviously requires a unit for each one of these quantities. The student will come across in the study of Physics many units which are used for measuring the various physical quantities, but it is found that a good many of units are obtainable from certain simpler units of more fundamental physical quantities. The three fundamental quantities recognized by the physicist are length, mass and time, and the units adopted by law for measuring these fundamental quantities are called fundamental units. Other units which are derivable from those of length, mass and time are called derived units.

5. The C.G.S. or Metric System of units.

There are two commonly used systems of fundamental units. One of them is called the *metric system* or the



A section of the standard meter
Fig. 1.1

C. G. S. system. The centimeter, gram, and second are the units of length, mass and time on this system. It originated in France and is most widely used in scientific work. The unit of length on this system is a centimeter which is defined as one hundredth part of the meter. Meter is defined as the distance between two marks on a bar made from an alloy of platinum and iridium, when it is at the temperature of melting ice (O°C). This bar which is called the *standard meter* or International meter is kept at the International Bureau of Weights & Measures at Sevres, near Paris. Copies of this original meter are issued by the International Bureau to the Bureaus of Standards of different countries. A section of the standard meter showing the marks at one end is illustrated in Fig. 1.1. The alloy of platinum and iridium is used, because it is very resistant to corrosion etc. Beside the centimeter and the meter other multiples of these are used. Thus a decimeter is one-tenth of a meter. A kilometer equals 1000 meters. The various multiples or sub multiples are obtained by multiplying or dividing by ten or powers of ten and, therefore, enable easy conversion among themselves. On this system the following prefixes are very frequently used for denoting multiples.

Kilo	...	Thousand-fold
Mega	...	Million-fold.

And for denoting sub multiples :

Deci	...	Tenth
Centi	...	Hundredth
Milli	...	Thousandth
Micro	...	Millionth

It was originally intended by the designers of the meter that its length should be one ten millionth of the distance from the pole of the earth to the equator measured along a meridian of longitude. But in practice this ideal could not be realised, and the standard meter is about 2 millimeters shorter than what was originally desired.

The unit of mass on C. G. S. system is the *gram*, which is defined as one thousandth part of the Kilogram. The standard kilogram is the mass of a cylinderical block made of the same platinum and iridium alloy as is used for the standard meter, carefully preserved by the International Bureau. This unit was originally chosen to make its mass equal to the mass of 1 litre (one thousand cubic centimeters) of pure water at 4°C, although this equality could not be exactly realized.

The unit of time is the *mean solar second.* The mean solar day is the average value calculated from all days of the year for the time taken by the earth between two successive transits of the sun across the meridian at any place. The rotation of the earth around its axis causes the apparent movement of the sun during the day across the sky. If we determine the time when the sun is exactly overhead on one day to the same position on the next day i.e., the time from noon to noon on two successive days, then the time interval between these two moments is called the solar day. The solar day is not a constant time interval, but varies somewhat throughout the year. If the measurement of the solar day is made for all days in the year and its mean value found out, it is called the mean solar day. It is sub divided into 24 hours with a further sub division of each hour into 60 minutes and each minute into 60 seconds. Therefore, $\frac{1}{24 \times 60 \times 60} = \frac{1}{86400}$ part of the mean solar day is the mean solar second. This is the standard of time indicated on clocks or watches.

If the length of the day is determined by a reference to some star instead of the sun then the time elapsing when the star is exactly at zenith on one night to the same position on the successive night is called a sidereal day. Because of the fact that the earth is moving in its orbit around the sun while at the same time rotating on its axis, it comes about that the solar day is longer by about 4 minutes than the sidereal day. Sidereal time is used by astronomers who have to watch the positions and the movements of the stars.

6. The British System of Units.

On this system which originated in England the units of length, mass and time defined by law are a foot, a pound and a second. It is, therefore, commonly referred to as Foot-Pound-Second system or F. P. S. system. It is widely used in engineering and in commerce.

A foot is $\frac{1}{3}$ part of the British or Imperial Standard yard, which has its legal definition as the distance between two lines on a bronze bar kept at the office of the Exchequer in London, when its temperature is 62° F.

The standard of mass on the British System is the Pound Avoirdupois, which is defined as the mass of a piece of platinum preserved at the office of Exchequer of London.

The unit of time on this system also is the mean solar second.

Following relationship exists between some of the C. G. S. and F. P. S. units.

1 Meter = 39·37 Inches.
1 Inch = 2.54 Centimeters.
1 Kilogram = 2.205 Pounds.
1 Pound = 453·6 Grams.
1 Oz = 28.35 Grams.
1 Gram = 15·432 grains.

A third system based on the meter as the unit of length, the kilogram as the unit of mass and the second as the unit of time is called m.k.s, system. By using this system the derived units particularly in electricity are of convenient size.

7. Derived Units.

It is easily seen that simple physical quantities as area, volume, density, velocity, acceleration do not require any special units of their own, but can be measured in terms of units depending upon the fundamental units of length, mass and time. When we say that the area of the floor of a room is 80 square feet, we are using a square foot as the unit for measuring areas. The square foot is the area of a square surface, each side of which has a length of 1 foot. Similarly when we say that the volume capacity of a bottle is 200 cubic centimeters, we are using a cubic centimeter as the unit of volume *i.e.*, the volume of a cube each side of which is 1 centimeter in length. Thus the units of length *i.e.*, the foot and the centimeter also serve for expressing the units of area or the units of volume. The latter are called the derived units.

Again, we express the density of mercury as 13.6 grams per cubic centimeter. The physical quantity called density is being measured in terms of a unit obtainable from the fundamental units of mass and length.

Similarly when we express the speed of an aeroplane as 300 miles per hour, we are using the multiples of the fundamental units of length and time to obtain a unit of speed.

We have only discussed a few simple physical quantities and seen that their units are derivable from the fundamental units. It will be found in the study of the subject that various other physical quantities also have units expressible in terms of the fundamental units. The student will, therefore, realize the great importance of fixing upon exact and precise units for measuring the fundamental quantities as he has learnt in the present chapter.

8. Dimensions of Physical Quantities.

We have seen above that an area is measurable in terms of the square of length. This is expressed by saying that area has the dimensions of $(\text{Length})^2$ or L^2. Similarly the dimensions of volume are written as $(\text{Length})^3$ or L^3. On the same principle since density is $\frac{\text{mass}}{\text{volume}}$ we may write the dimensions of density as $\frac{\text{mass}}{\text{Length}^3}$ or $\frac{M}{L^3}=M.L^{-3}$ and the dimensions of velocity which equals $\frac{\text{distance}}{\text{time}}$ as $\frac{L}{T}=L.T^{-1}$. We see that the physical quantities: area, volume, density, velocity, are expressed as having dimensions of the fundamental quantities of length, mass and time. This process can be applied to other more complicated quantities and is some times very useful in understanding the nature of those quantities and the relationship between them.

Summary

Physics is that branch of science which deals with the study of matter and energy.

The three fundamental quantities to be measured in Physics are length, mass and time. The units, defined by law, for measuring these quantities are called fundamental units. Larger or smaller multiples of the fundamental units are called derived units. The units of area, volume, speed and other physical quantities which can be expressed in terms of the fundamental units, are also called derived units.

There are two systems of fundamental units, the C.G.S. system and the F. P. S. system. In the C.G.S. system centimeter (cm.) is the unit of length, gram (gm) is the unit of mass and second (sec.) is the unit of time. In the F.P.S. system, foot (ft.), pound (℔) and second (sec.) are the units of length, mass and time respectively.

The dimensions of a physical quantity show how it is expressed in terms of the three fundamental quantities of length, mass and time. Thus the dimensions of speed are LT^{-1} and of density are ML^{-3}.

Relation between C.G.S. and F.P.S. units.

1 meter	=39·37 inches
1 inch	=2·54 cms.
1 Kilogram	=2.205 ℔s.
1 ℔.	=453.6 grams

1 oz =28.35 grams
1 gram =15.432 grains

Exercises.

1. How many Kilometers are there in one mile? (1.61 K. m.)

2. The diameter of the earth is 8000 miles. Express it in Kilometers. (12880 K. m.)

3. Find the number of grams in one ounce. (28.35 gm.)

CHAPTER II

MOTION-VELOCITY-ACCELERATION

1. Rest and Motion.

If a body does not change its position with reference to its surroundings it is said to be at rest relative to the surroundings. When a change in the position of a body takes place, it is said to suffer displacement. Displacement is produced by the motion of a body.

Mechanics is that branch of Physics which treats of the motion of bodies and of forces acting upon them. The study of motion shows that it occurs according to definite laws. Rest and motion are, however, relative terms. Thus a person sitting in the carriage of a train may be considered at rest with respect to other objects in the carriage while the train itself is speeding at a high velocity. In the same way we ordinarily consider a building as at rest, while we know that along with the earth it is rotating and also moving around the sun at a high speed of nearly 18 miles a second.

Average speed.

If a body is changing its position, it is said to be in motion. The speed of a body in motion is the rate at which it is being displaced. For example if a cyclist covers a distance of 10 miles in one hour, his average speed is 10 miles per hour. In this case the speed must have changed during the period considered but if we divide the total distance travelled by total time taken, we get an average value of speed. Similarly if an automobile travels over a distance of say 200 miles in 5 hours, its average speed is $200/5=40$ miles per hour. It must have sometimes slowed and sometimes increased its speed. It may even have had to stop at intervals, but the average speed is 40 miles per hour. A knowledge of average speed is useful in such cases as of a train or a motor car, or aeroplane as it gives an idea of the time taken to cover long distances.

3. Uniform Speed.

The speed of a moving body is said to be uniform if it traverses equal distances in equal intervals of time "HOWEVER SMALL." If a railway train travelling with a speed of 60 miles per hour is found to move one mile every minute, and 88 feet every second, we would say its speed is uniform, if a second is counted as the smallest interval. If, however, the train moves over 8.8 feet in every tenth of a second its speed will be considered to be more uniform. This illustrates the necessity of the words "HOWEVER SMALL" when defining uniform speed.

Suppose v represents the uniform speed of a body, then the distance travelled in a time t is $s=\underline{v}\times t$. It is necessary u
to express the values of v and t in appropriate units.

Example. An aeroplane is flying with a uniform speed of 300 miles per hour. Find the distance travelled per minute. $s=300\times 1/60=5$ miles.

Example. A railway train is travelling with a speed of 60 miles per hour. Find the distance travelled per second. $s=60\times 1/60\times 1/60=1/60$ miles per second or $5280/60=88$ feet per second.

The student is advised to memorise that a speed of 60 miles per hour is equivalent to 88 feet per second. The relation helps to convert any other speed given in miles per hour into feet per second quite easily. Thus 15 miles per hour is at once seen to be 22 feet per second, and 6 miles per hour as 8. 8 feet per second.

4. Velocity.

When considering the motion of a body we quite often use the word *velocity* instead of speed. The word velocity also specifies the direction in which the body is moving instead of merely stating its rate of motion.

Without specifying the direction of a moving body, we will not know its position after any particular interval of time. Suppose that a ship sails at the rate of 30 miles per hour, then if it leaves a port at some time, we will not know its position after say one hour, unless we also state in what direction it is sailing. The speed of the ship is 30 miles per hour, but to express its velocity we must say 30 miles per hour due east or due north as the case may be depending on the direction in which it is sailing.

5. Vector and Scalar Quantities.

Velocity is a typical example of physical quantities for the statement of which, their magnitude and also their direction must be stated in order to convey the meaning fully and completely. Other cases of such quantities are force, acceleration etc. in which a statement of their direction is very necessary and important. These quantities are called vector quantities or sometimes only vectors. On the other hand such physical quantities as mass of a body, its volume or density, are fully expressed if we simply state just the value or magnitude. Such quantities are called scalars or scalar quantities. If we say that the volume of some water is 2 litres, the statement is quite complete, whereas if we write that the velocity of an aeroplane is 5 miles per minute, the expression is not complete, for we cannot know the position of the aeroplane, unless the direction in which it is flying is also stated.

Scalar quantities can be added and subtracted by simple arithmetic methods. Thus combining two litres of water with another 2 litres will make 4 litres in all, but combining two velocities or two forces does not necessarily give the total velocity or the total force equal to the arithmetic sum of the individual velocities or forces. For consider the case of a boat the velocity of which in still water is say 4 miles per hour. Now if the velocity of flow of water in a river is 3 miles per hour, then the velocity of the boat in the river going up-stream, is only 1 mile per hour, and going down-stream it is 7 miles per hour while going across the stream the velocity will have somewhat different value. We clearly see that the addition of velocities is a very different process from the addition of volumes. In the general case we will learn that the addition of vectors is done by a process of quite different type from that for the addition of scalars. The vector addition process is given more fully in the next chapter.

6. Acceleration.

We have seen that a moving body is said to possess a constant or a uniform velocity, if it covers equal distances in equal time intervals, even when the time intervals are very small. There are other cases of moving bodies in which the speed is variable. A railway train when it starts from a station picks up speed at first gradually and then faster, till it acquires the maximum speed. A stone dropped from the roof of a building increases its speed as it falls. We say that the velocity of the train in the above case and the falling stone is being accelerated.

Similarly when a train approaches a station, it slows down its velocity till it comes to stop. A stone thrown upward decreases in velocity as it ascends till at the highest point it reaches, it momentarily comes to rest before beginning to fall again. In these cases the velocity decreases with time and is said to be retarded or decelerated.

The change of velocity per unit time is called acceleration, or in other words, acceleration is the time rate of change of velocity.

If the change of velocity occurs by equal amounts in equal time intervals however small, the acceleration is called uniform. Suppose v_1 represents the original velocity of a moving body. Now if this velocity is being accelerated, then suppose that in an interval of time t the velocity of the body becomes v_2 which is its final velocity. Then the acceleration or the time rate of change of velocity is

$$\frac{\text{Final velocity} - \text{Original Velocity}}{\text{Time Interval}}$$

$$a = \frac{v_2 - v_1}{t} \qquad \text{......(1)}$$

In the case of a train starting from a station the initial velocity is zero. Suppose that it acquires a velocity of 10 miles per hour in two minutes, then according to the above formula, its acceleration is

$$\frac{\text{10 miles per hour}-0}{\text{2 minutes}}=\text{5 miles per hour per minute.}$$

Next counting from the second minute if the velocity of the train becomes 30 miles per hour after another two minutes then the acceleration of the train in this time interval is :

$$\frac{\text{30 miles per hour}-\text{10 miles per hour}}{\text{2 minutes}}=\frac{\text{20 miles per hour}}{\text{2 minutes}}$$

$$=\text{10 miles per hour per minute.}$$

If both the velocities and time in the above case are expressed in feet and seconds then

$$\text{Acceleration}=\frac{\text{44 feet per second}-\text{14.66 feet per second}}{\text{120 seconds}}$$

$$=\text{0.24 feet per second per second.}$$

Since acceleration is change of $\frac{\text{velocity.}}{\text{time}}$,the unit of time in it occurs twice and unless we state it like that the expression for acceleration is not complete.

Equation (i) may be written as

$$v_2-v_1=a.t.$$

When the acceleration is uniform, velocity is changing at a constant and definite rate, and thus the mean velocity during the interval t is

$\frac{v_1+v_2}{2}=\bar{v}$, When v_1 and v_2 are the initial and final velocities.

Further, since the distance travelled is given by the mean velocity multiplied by time we have

$$s=\bar{v}.t$$

$$=\frac{v_1\text{[illegible]}}{2}\times t \qquad \text{or} \quad v_2+v_1=\frac{2s.}{t} \qquad \text{......(2)}$$

Multiplying the equation (2) with equation (1) we obtain

$$v_2^2-v_1^2=2\,a\,s \qquad \text{......(3)}$$

Again from equation (1) $v_2=v_1+a.\,t$.

Substituting this value of v_2 in equation (2) we get

$$s=\left(\frac{v_1}{2}+\frac{v_1+at}{2}\right)t$$

$$=\frac{v_1{\cdot}t}{2}+\frac{v_1{\cdot}t}{2}+\tfrac{1}{2}\,at^2$$

$$=v_1{\cdot}t+\tfrac{1}{2}\,at^2 \qquad \ldots\ldots(4)$$

The above four equations are all used in problems concerning bodies moving with uniformally accelerated motion. These equations are rewritten below :—

$$a=\frac{v_2-v_1}{t} \quad \text{or} \quad v_2=v_1+at \qquad \ldots\ldots(1)$$

S= a t -

$$s=\frac{v_2+v_1}{2}.\,t \qquad \ldots\ldots(2)$$

$$v_2^2-v_1^2=2as \qquad \ldots\ldots(3)$$

$$s=v_1.t+\tfrac{1}{2}\,at^2 \qquad \ldots\ldots(4)$$

Five quantities v_2, v_1, a, t, and s are involved in these equations, but in any one of them four out of these five are being used. Most often in problems dealing with uniformally accelerated bodies, three of the five quantities enumerated above are given and the other one or two are to be determined by calculation. The student has to choose which one of the four equations is to be applied to a particular problem and carry out the calculations. All equations must, therefore, be memorised.

Example : A train is moving with a velocity of 30 miles per hour, when brakes are applied and it comes to rest in 30 seconds. What is its average negative acceleration or retardation.

Quantities given are :

$v_1=30$ miles per hour$=44$ feet per second.

$v_2=0$

$t=30$ seconds.

Therefore, $a=\dfrac{v_2-v_1}{t}$

$$= \frac{0-44}{30} = \frac{-4.4}{30} \text{ feet per second per second.}$$

$$= -1.466 \text{ feet per second per second.}$$

Example : An automobile is moving at some particular place and time with a velocity of 40 miles per hour and an acceleration of 2 miles per hour per minute. If the acceleration remains constant, how far will the car travel in 15 minutes ?

Quantities given are :

$v_1 = 40$ miles per hour

$a = 2$ miles per hour per minute

$t = 15$ minutes $= \frac{1}{4}$ hour

Equation to be used $= s = v_1 . t + \frac{1}{2} at^2$

Since acceleration is 2 miles per hour per minute, the gain of velocity in 15 minutes is 30 miles per hour. Therefore, the final velocity $v_2 = 70$ miles per hour.

And by using equation (2)

$$s = \frac{v_1 + v_2}{2} t$$

$$= \frac{40+70}{2} \cdot \frac{1}{4} = \frac{110}{2} \cdot \frac{1}{4} = \frac{55}{4} = 13.75 \text{ miles.}$$

If we use equation (4) all units must be converted into miles and hours.

$v_1 = 40$ miles per hour.

$a = 2$ miles per hour per minute $= 2 \times 60$

$= 120$ miles per hour

$t = 15$ minutes $= \frac{15}{60} = \frac{1}{4}$ hour.

$$s = v_1 . t + \frac{1}{2} at^2$$

$$= 40 \times \frac{1}{4} + \frac{1}{2} \times 120 \times (\tfrac{1}{4})^2$$

$$= 10 + \frac{1}{2} \times 120 \times \frac{1}{16} = 13.75 \text{ miles.}$$

7. Velocity-time curves.

A good understanding of the problem of moving bodies is acquired by drawing the velocity time curve of the body. On this curve the velocity of the moving body is represented on the Y axis and the intervals of time on the X axis.

A body moving with a uniform velocity will be indicated on this curve by a straight line parallel to the axis of X as in Figure 2.1.

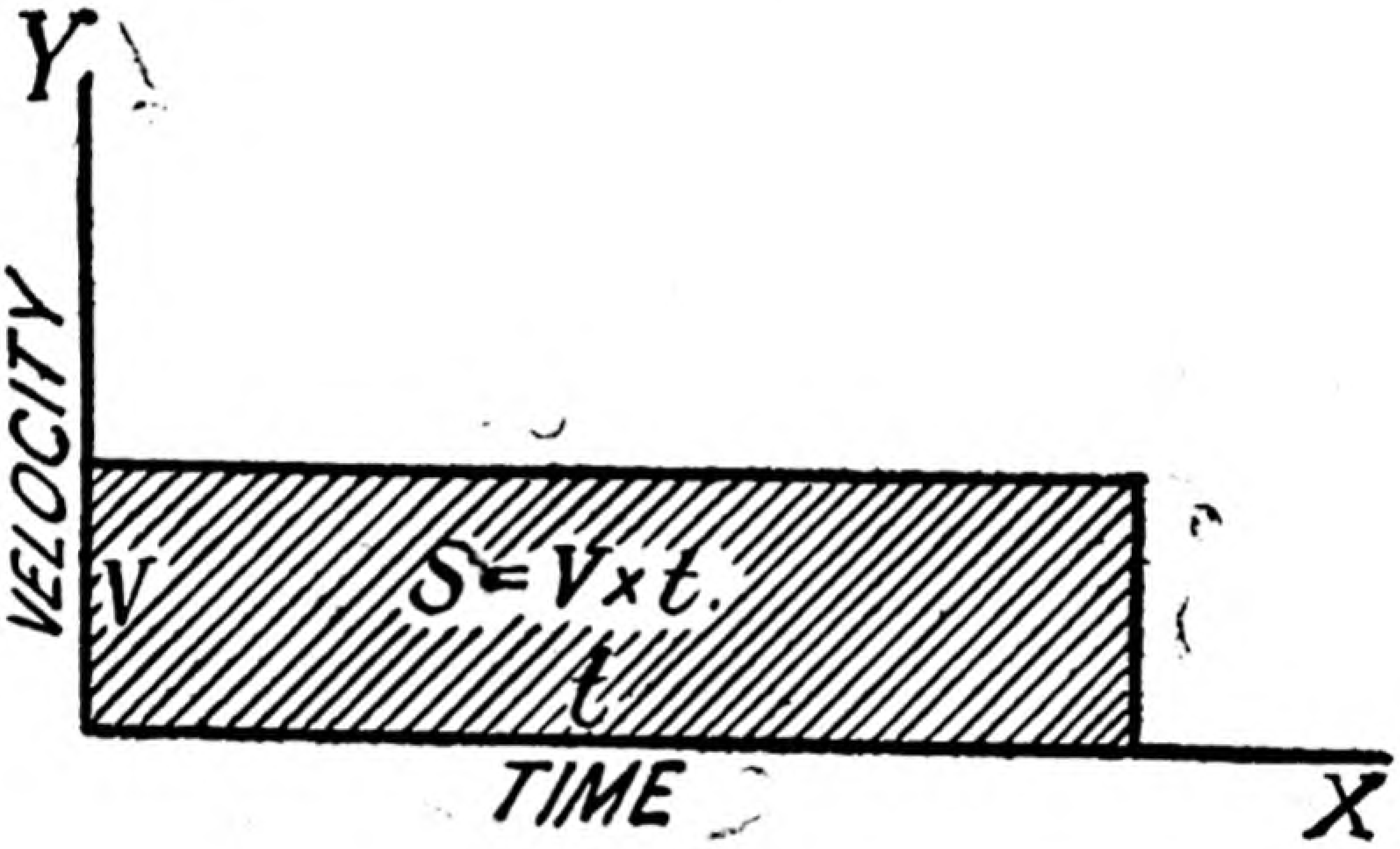


Fig. 2.1

The distance of every point on this line from the axis of X is constant which implies that the velocity remains constant as time passes. The distance covered in a time *t* equals *v*. *t* and on the Fig. 2.1., it is the area of the rectangle of which the base is equal to *t*, and the height equal to *v*.

In general the velocity of the body may be complicated, but it can always be expressed by a graph although the shape of this graph may not be simple. Suppose the curved line AB in figure 2.2 represents

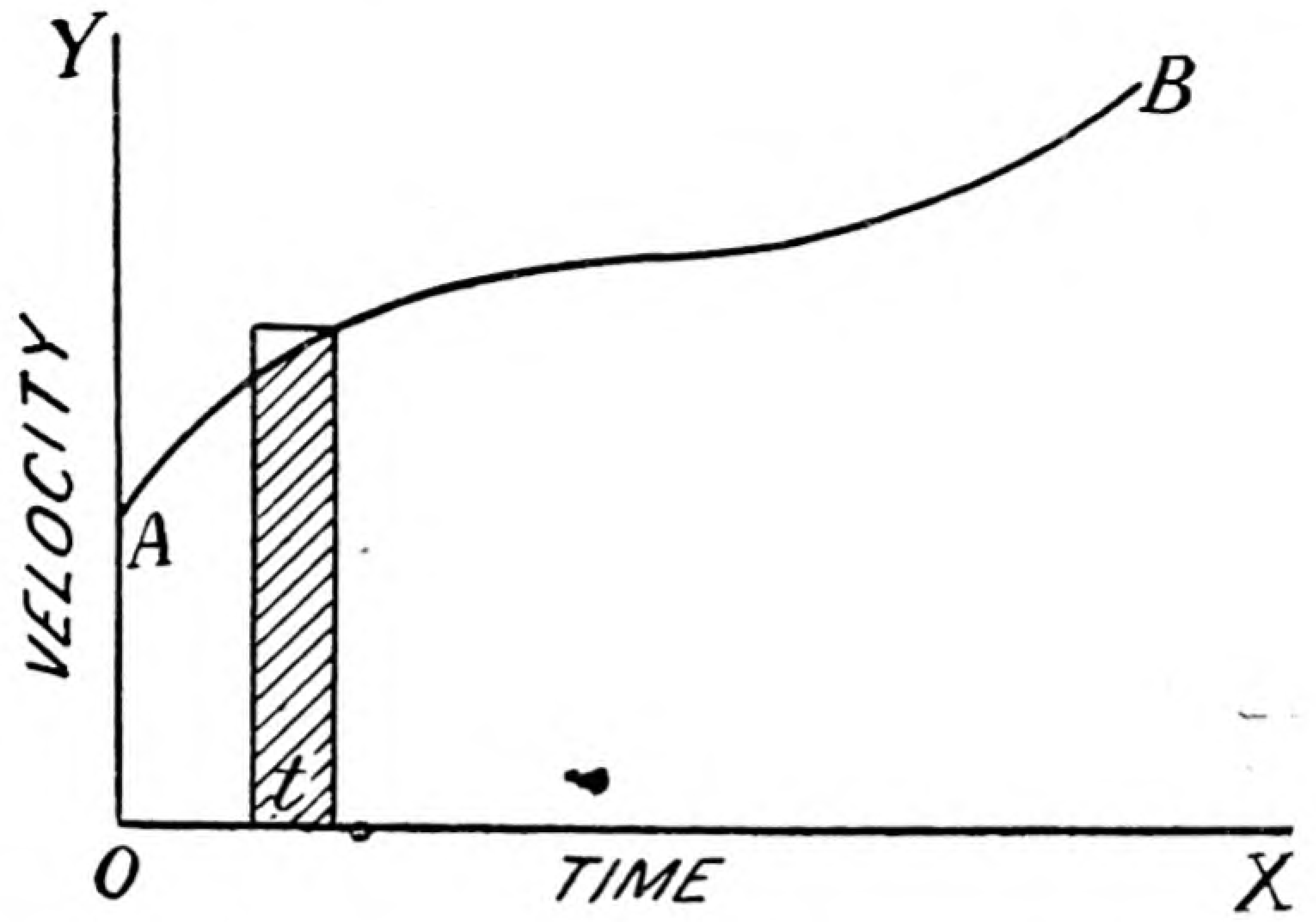


Fig. 2.2

the velocity curve of the moving body; although the curve is not simple, yet a very small segment of the curve can be taken to be a straight line and the distance moved by the body in a very small interval of time t during which the velocity of the body is represented by a small straight segment is obtained by drawing ordinates at the beginning and at the end of the small interval of time. The area between these ordinates, the time axis and the small segment of the velocity curve represents very closely the distance moved by the body in this interval. Any bigger interval can be considered as made up of a large number of small intervals and for each small interval the distance moved would be given as above. Therefore, the distance moved in a definite interval of time is given by the area included between the time axis, the ordinates at the beginning and at the end of that interval and the velocity curve, whatever may be the shape of this curve.

Let us now consider the velocity time graph of a body moving with uniformly accelerated motion. Suppose at some moment the initial velocity of the body is v_1 and it increases by an amount a in every second. Its velocity becomes v_1+a after one second, v_1+2a after 2 seconds, v_1+3a in three seconds etc. The velocity time graph is obtained by marking points having heights of v_1, v_1+a, v_1+2a, etc, as shown in the Figure 2.3.

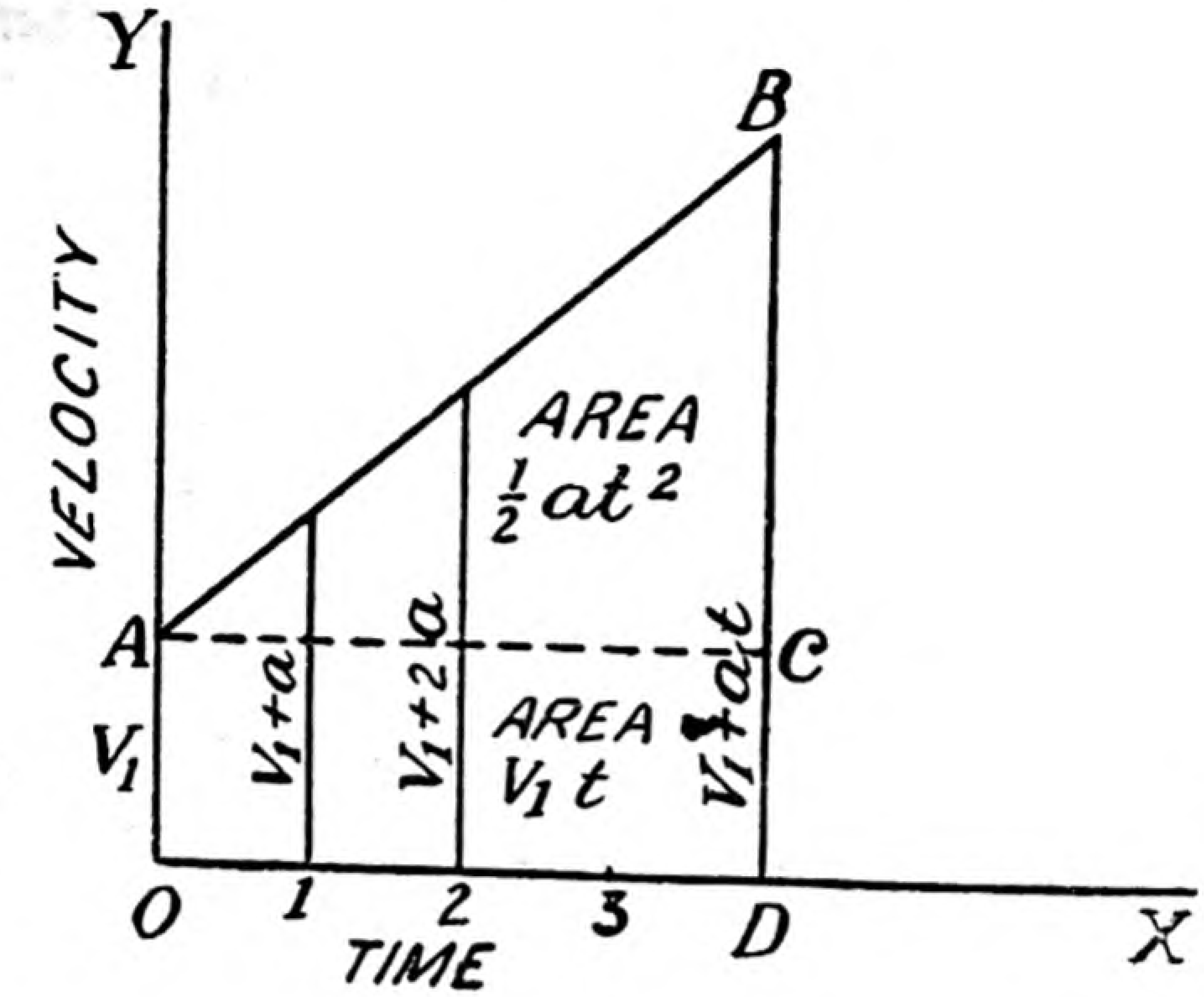


Fig. 2.3

The graph obtained by joining these points is a straight line inclined upwards. The distance travelled in a certain interval of time t, is obtained by drawing ordinates at the beginning and at the end of this interval so that they cut the velocity-time graph. The area included between the axis of time, the two ordinates and the velocity graph represents the distance travelled. In this case the area consists of a rectangle OACD of base t and height v_1 and a triangle of base t and height $BC=at$. The area of this triangle is $\frac{1}{2}t \,.\, at=\frac{1}{2}at^2$. The total area which represents the distance travelled by the uniformally accelerated body in time t and having the initial velocity v_1 and acceleration a is $v_1.t+\frac{1}{2}at^2$.

This is the same formula as we derived earlier on general considerations without velocity time graph.

8. Acceleration due to Gravity.

A freely falling body is a common example of uniformally accelerated motion. If a stone is dropped it falls towards the earth with a constantly increasing velocity having uniform acceleration provided the resistance of air is negligible.

*Newton showed that if a light body as a feather, and a heavier body as a metallic coin, are allowed to drop through a glass tube from which air has been exhausted by a vacuum pump, they fall at the same rate, and therefore with an equal acceleration. If they are dropped from the upper end of the tube at the same instant, they reach the lower end at the same time. If however the tube contains air, then the feather drops more slowly due to the greater air resistance.

The acceleration due to gravity is denoted generally by 'g'. Its value at sea level and 45° latitude is 32.17 feet per second per second or 980.6 cm per sec^2. For most purposes involving calculations it is sufficiently accurate to take the value of g as 32 feet per sec^2 or 980 cm/sec^2.

If a ball is thrown upwards, it decreases in its velocity at the rate of 32 feet per sec^2 until at the highest point it reaches, its velocity becomes zero and then it begins to fall down with an increase in velocity of 32 $feet/sec^2$.

The equations of motion of uniformally accelerated bodies as developed in the last pages will exactly apply to the case of freely falling bodies. For a, the acceleration of the body, we may write g and for s, the distance moved we may write h, the height through which the body falls. The equations thus become :

---

*Sir Isaac Newton (1642-1727). Newton was born in the same year in which Galileo died. He is regarded as one of the greatest scientific and mathematical genius. Was educated at Grantham and Trinity College, Cambridge. He discovered the law of universal gravitation and was the founder of the differential calculus. He found out the composite nature of light as shown by the spectrum. He also constructed the first reflecting telescope. He was professor of Natural Philosophy at Cambridge and was President of the Royal Society.

$$v_2=v_1+gt$$

$$h=\frac{(v_1+v_2)}{2}t.$$

$$h=v_1\times t+\tfrac{1}{2}gt^2$$

$$v_2^2-v_1^2=2gh.$$

Time t sec	Falling Body	Velocity v ft./sec	Distance h ft.
0	●	0	0
1	●	32	16
2	●	64	64
3	●	96	144
4	●	128	256

Fig. 2.4

Position and velocity of a falling body at different times.

9. Effect of Resistance on falling bodies.

Whenever a solid body moves through a fluid the latter offers a resistance to the motion. This resistance is like an opposing force to the motion of the body. The resistance of air upon a falling body depends upon the speed. For the first few moments after the body begins to fall the frictional resistance of the air is small but as the velocity of the falling body increases, the resistance also increases, being proportional to the square of the velocity for large velocities. This resistance reduces the net downward force acting on the body, and its rate of fall begins to decrease until for a certain speed of the falling body, called the 'terminal speed', the air resistance exactly equals the down-ward force of the earth on the body. From this moment onward the body falls not with an accelerated motion, but with a uniform velocity or terminal speed.

In the case of a man jumping from an aeroplane, before he opens the parachute, the terminal velocity is about 120 miles per hour, and after opening the parachute it is about 14 miles per hour. Thus a parachute trooper reaches the earth dropping with a constant velocity of 14 miles per hour. This is about the velocity that a man acquires when jumping from a height of 7 feet, and it does not cause great shock on striking against the ground.

Rain drops falling through air also acquire a constant terminal velocity which depends upon the size of the drop. Extremely small size drops of a radius of 1/100 millimeter diameter, as are formed in a mist, fall only with a velocity of 1½ centimeter per second. Larger drops with radius of 1/10 of a centimeter fall with a velocity of about 800 centimeter per second.

Example :

If the resistance due to air friction is neglected what would be the velocity with which a hail stone falling from a height of 8000 feet will reach the earth.

Quantities given :

$v_1 = 0$

$g = 32$ per second per second

$h = 8000$ feet.

Quantity required v_2

Formula to be used : $v_2^2 - v_1^2 = 2gh$.

Since v_1 is zero.

Therefore, $v_2^2 = 2 \times 32 \times 8000$

$= 512000$

$= 51.2 \times 10^4$

and $v_2 = 715.6$ feet per second.

Example :— A stone is dropped into a well. If it reaches the water in the well in 3.5 seconds, what is the depth of the water below the surface of the ground ?

Quantities given $v_1 = 0$

$t = 3.5$ Seconds.

$g = 32$ feet per second per second.

Quantity required h

Equation to be used : $h = v_1.t + \frac{1}{2}gt^2$

Since $v_1 = 0$

$$\begin{aligned} h &= \tfrac{1}{2}gt^2 \\ &= \tfrac{1}{2} \times 32 \times (3.5)^2 \\ &= 16 \times 12{\cdot}25 \\ &= 196.00 \\ &= 196 \text{ feet.} \end{aligned}$$

Summary

Rest and Motion.

A body is said to be at rest when its position does not change with respect to its surroundings. When its position undergoes a displacement relative to its surroundings, it is said to be in motion. Rest and motion are relative terms.

Speed is the rate of change of position of a body. Velocity is the speed of a body in some given direction.

Uniform velocity. A body is said to have a uniform velocity if it moves through equal distances in equal intervals of time, however small, without change of direction.

Acceleration. The rate of change of velocity of a body is called its acceleration. If the velocity of a body changes by a feet per second after every second, its acceleration is a feet per second per second (a ft./sec.^{2}). A body has a uniform acceleration if its velocity changes by equal amounts in equal intervals of time, however small.

Those physical quantities which have magnitude only are called scalars or scalar quantities. Mass, volume, area and time are scalars. Quantities which have magnitude as well as direction are called vectors or vector quantities. Velocity, acceleration and force are examples of vector quantities.

$$\text{Average velocity of a body} = \overline{v} = \frac{\text{Distance covered}}{\text{time taken}}$$

Average velocity of a body moving with uniform acceleration $= \overline{v} = \dfrac{\text{Initial velocity} + \text{Final velocity}}{\text{time}} = \dfrac{v_1 + v_2}{t}$.

Formulas for uniformly accelerated motion

$$v_2 = v_1 + at \text{ or } a = \frac{v_2 - v_1}{t} \qquad \dots \quad \text{(i)}$$

$$s = \frac{(v_1 + v_2)}{2} t \qquad \dots \quad \text{(ii)}$$

$$s = v_1 t + \tfrac{1}{2} a t^2 \qquad \dots \quad \text{(iii)}$$

$$v_2^2 - v_1^2 = 2\,as \qquad \ldots \quad \text{(iv)}$$

where

v_1=initial velocity

v_2=final velocity.

a=acceleration. If acceleration is opposed to v_1, negative sign is given to it.

s=distance covered.

t=time.

Acceleration due to gravity $= g = 32$ ft./sec² $= 980$ cm/sec².

All bodies, whatever their mass, fall to the earth with the same uniform acceleration due to gravity, if resistance due to air is absent.

For bodies falling under the action of gravity, the above formulas are written as

$$v_2 = v_1 + gt \text{ or } g = \frac{v_2 - v_1}{t} \qquad \ldots \quad \text{(v)}$$

$$s = \frac{(v_1 + v_2) \times t}{2} \qquad \ldots \quad \text{(vi)}$$

$$s = v_1 t + \tfrac{1}{2} g t^2 \qquad \ldots \quad \text{(vii)}$$

$$v_2^2 - v_1^2 = 2\,gs \qquad \ldots \quad \text{(viii)}$$

g is given negative sign if it is opposed to v_1.

Exercises

1. A train starts from a station and increases its velocity at a uniform rate. If its velocity after a time of 5 minutes is 45 miles per hour, determine its acceleration and the distance travelled in this time.

Solution :

45 Miles per hour = 66 feet per second.

$$a = \frac{v_2 - v_1}{t}$$

$$= \frac{\overset{11}{\cancel{66}} - 0}{5 \times \underset{10}{\cancel{60}}} = \frac{11}{50} \text{ feet per second per second.}$$

$$s=\frac{v_1+v_2}{2}\,t$$

$$=\frac{66\times5\times60}{2}=9900 \text{ feet.}$$

2. A train is travelling at a speed of 60 miles per hour, when brakes are applied, and it slows down at a constant rate coming to rest 1 minute after the application of the brakes. How much distance does it move during this period ?

$$s=\frac{v_1+v_2}{2}\,t$$

$$\frac{60+0}{2}\cdot\frac{1}{60}=\tfrac{1}{2}\text{ mile.}$$

3. Find distance travelled in the n^{th} second by a uniformally accelerated body.

Suppose a is the acceleration and v_1 the initial velocity of the body.

The distance travelled in n seconds is

$S_n=v_1\cdot n+\frac{1}{2}an^2$

And the distance travelled in $n-1$ seconds is

$S_{(n-1)}=v_1\cdot(n-1)+\frac{1}{2}a(n-1)^2$

Therefore, the distance travelled in the n^{th} second is

$$S_n-S_{(n-1)}=\{v_1\cdot n+\tfrac{1}{2}an^2\}-\{v_1(n-1)+\tfrac{1}{2}a(n-1)^2\}$$

$$=v_1+\frac{2n-1}{2}\,a.$$

4. A stone is dropped from a rising balloon at a height of 200 ft. above the ground and it reaches the ground in 6 seconds. What was the velocity of the balloon just at the moment when the stone was dropped ? (*C. U.*)

(62.7 ft./Sec)

5. A shot fired horizontally from the top of a tower, with a velocity of 600 feet per second, hits the ground in 2 seconds. What is the height of the tower, and the

distance from the foot of the tower, where the shot strikes the ground ? (height 64 ft.; distance 1200 ft.)

(Punjab University)

6. A body starts from rest, and is found to traverse 60 cms. in the 6th second of its motion. Calculate the acceleration if it is uniform. (10.9 cm/Sec^2).

7. Explain clearly why time is stated twice in stating an acceleration. A body describes 24 feet in the 2nd second and 100 feet in the 4th second of its motion. If the motion is uniformly accelerated, how far will it go in the 5th second. 138 ft. *(P. U. 1952)*

8. A bullet is fired vertically upward with a velocity of 800 ft. per second. Determine (*i*) the highest point to which the bullet rises, (*ii*) time to reach the highest point, (*iii*) total time of flight of the bullet and (*iv*) the velocity of the bullet when it strikes the ground.

(*i*) $v_1 = 800$ ft./Sec.

$v_2 = 0$ at the highest point.

$\therefore \quad v_2^2 - v_1^2 = -2\,gh$ (acceleration is negative)

$\therefore \quad 0 - 800 \times 800 = -2 \times 32 \times h$ or $h = 10{,}000$ ft.

(*ii*) $v_2 - v_1 = -gt.$

$0 - 800 = -32\,t$

Time to reach highest point $= \dfrac{800}{32} = 25$ Sec.

(*iii*) Time to come to earth from the highest point is given by the formula $h = \frac{1}{2}\,gt^2$ since the velocity of bullet at the highest point is zero.

$10000 = \frac{1}{2} \times 32 \times t^2$ or $t = 25$ seconds.

$\therefore$ total time of flight = time for upward journey + time for downward journey.

$= 25 + 25 = 50$ sec.

(*iv*) Velocity with which it strikes the ground = The velocity it acquires when it has fallen for 25 seconds from the highest point.

$= 25 \times 32 = 800$ ft./Sec.

Parts (*iii*) and (*iv*) can be solved alternatively in the following way. Since the bullet comes back to the ground

resultant distance covered $=0$

$s=v_1t-\frac{1}{2}gt^2$ (g is opposed to v_1)

$$0=800t-\frac{32}{2}t^2$$

or $t=50$ seconds $=$ total time of flight.

Again $v_2^2-v_1^2=2as$ and $s=0$

$\therefore v_2^2=v_1^2$

or $v_2=v_1=800$ ft/sec $=$ velocity when it strikes ground.

9. A ball starting from rest rolls down an inclined plane with uniformly accelerated motion. If its acceleration is 20 feet/sec^2, find

(*a*) the distance it travels in 10 seconds ;

(*b*) the distance it rolls in the eighth second;

and (*c*) its velocity at the end of the tenth second.

(*P. U. 1945*).

(*a*) 1000 ft.; (*b*) 150 ft.
(*c*) 200 ft./sec.

10. A ball thrown up vertically, returns to the thrower after 6 seconds. How high did it go ? 144 ft. (*P. U.*)

11. A stone is thrown vertically upward with a velocity of 200ft/sec. When will the stone be at the height of 336 feet and what will be its velocity at that time ?

If the stone reaches the height of 336 ft. in t seconds, then

$$336=200t-\frac{1}{2}\times 32t^2$$

or $2t^2-25t+42=0$

$(t-2)(2t-21)=0$

$t=2$ seconds and 10.5 seconds.

velocity after 2 seconds $=200-2\times 32=136$ ft/sec going up.

velocity after $10\frac{1}{2}$ seconds$=200-\frac{21}{2}\times 32=136$ ft. /sec. coming down.

12. The speed of a train changes from 15 miles/hr. to 30 miles/hr. in 10 seconds. Express the acceleration of the train in (*i*) miles per hour per minute, (*ii*) feet per minute per minute. (*iii*) feet per second per second.

(*i*) 90 miles per hour per minute.
(*ii*) 7920 feet per minute per minute.
(*iii*) 2.2 feet per second per second.

13. A stone is thrown vertically up with a velocity of 200 ft./sec. Two seconds later, a second stone is thrown in the same direction with a velocity of 192 ft./sec. When and where will the two stones meet in the air ?

(*i*) 6 seconds after the second stone is thrown; (*ii*) 576 ft. above the ground.

CHAPTER III.

VECTOR QUANTITIES

1. Definition.

When a body undergoes motion a displacement of its position takes place. Displacement is the simplest example of a quantity the direction of which must also be stated along with its magnitude in order to convey full meaning. Suppose a man walks a distance of 3 miles in a straight path, then he has been displaced from his initial position by a length of 3 miles but we cannot determine the final position of the man unless we state in what direction from his starting point he walks over the distance. In fact, in the above case if we do not know the direction, all we can say is that he may be anywhere on the circumference of a circle the radius of which is 3 miles, but to know his exact position we must give the direction in addition to the distance. So that, if we express that he walks a distance of 3 miles due East, from the starting point, we know his real position after he has traversed the above distance.

Other physical quantities such as velocity, acceleration and force are similar in their nature to the simple quantity, displacement, discussed above. In their case also, one must state both their direction and magnitude to impart full meaning to the statement. All such quantities which require their direction to be stated are called '*vectors*' or vectorial quantities.

2. Straight line representation of vectors.

A vector quantity is represented in physics by means of a straight line. The length of the line denotes the magnitude of the vector quantity on a suitable scale chosen for the purpose. The direction in which the line is drawn represents the direction of the particular quantity and an arrow head on the line gives the sense in which the direction is to be considered. The particular case of a man walking 3 miles due East would thus be represented by a straight line drawn in the East West direction and of a length 3 cms. or 3 inches according as we choose a scale of 1 cm, or 1 inch equivalent to 1 mile. An arrow head pointing towards East will show that the displacement under consideration is due East. The line AB (Fig. 3.1) represents the case. Similar procedure is used in representing any other vector.

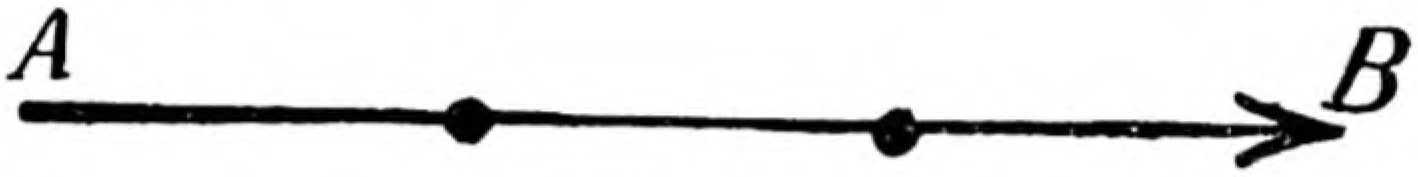


Fig. 3.1

3. Addition of vectors.

Suppose that the man after walking a distance of 3 miles on a straight path due East, changes his direction and walks a further distance of 4 miles in a straight path due North. His two displacements of 3 miles due East and then 4 miles due North are represented by straight lines AB and BC in the Fig. 3.2.

The total or added effect of these two displacements is to cause the resultant displacement of the man from the initial positon A to the final position C. If he had moved from A to C along the straight path AC, his effective displacement would have been the same, as by the combined displacements AB and BC.

The student will easily realise that the sum of a displacement of 3 miles, and another displacement of 4 miles is not just 3 plus 4 but it depends on the directions of the two dis-

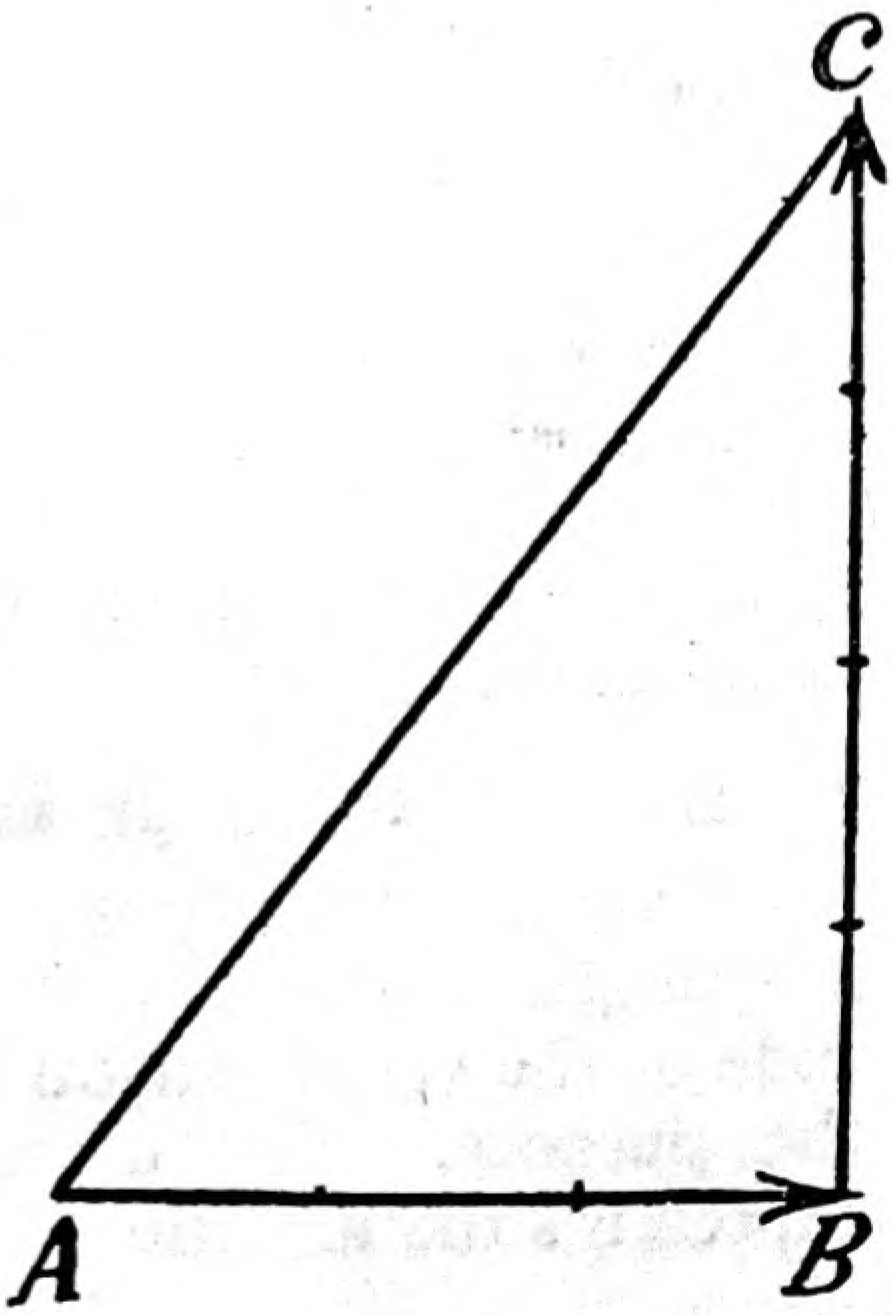


Fig. 3.2

placements 3 and 4. In the case considered above when the two are perpendicular to each other, the resultant or the effective displacement of the man is $\sqrt{3^2+4^2}=5$. This result is obtained from the geometry of the right angled triangle. We say that the vectors AB and BC have been added geometrically to produce a sum equal to the vector AC. It is easily seen that the sum of the vectors 3 and 4 equals 5 only when the two vectors are at right angles to each other. If they are inclined at other angles their sum will be different. When they are in the same direction the combined effect or sum is just their arithmetic sum. The displacement of 3 miles due East plus a further displacement of 4 miles also due East produces a final displacement of 7 miles due East. When the two vectors act along opposite directions their combined effect is the difference of their magnitudes. A displacement of 3 miles due East and a displacement of 4 miles due West produces a resultant displacement of 1 mile due West. We, therefore, see that the resultant of two vectors can have any value ranging from their arithmetic sum to their arithmetic difference according to the relative directions of the two vectors, because, addition of vectors is a geometric process, while the addition of scalars is an arithmetic process.

The procedure for adding two vectors is to draw a straight line to represent the first vector on an appropriate scale and to place an arrow head at the end of this line to indicate the direction of the vector. Then from the tip of the arrow head draw a second straight line on the same

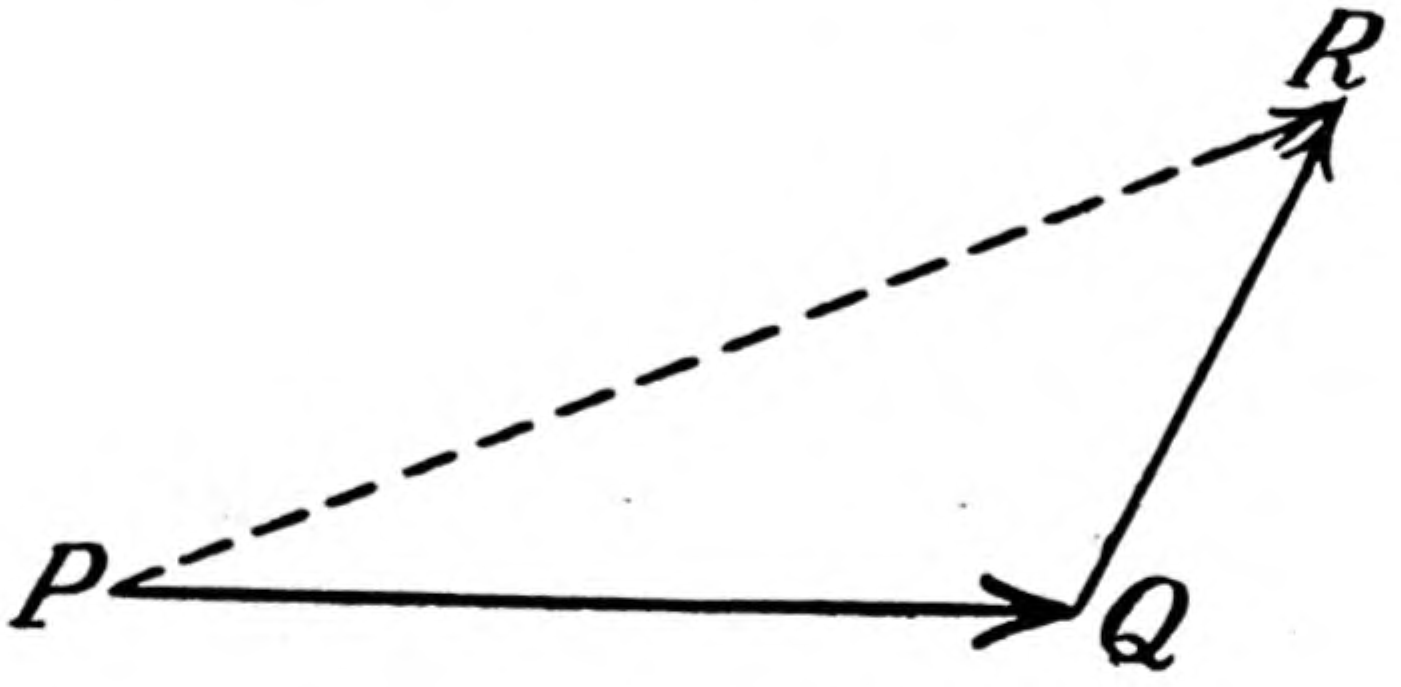


Fig. 3.3

scale as that chosen for the first line and in the direction of the second vector again put an arrow head at its end. In the Fig. 3·3, the two vectors are shown by PQ and QR. Their sum equals the vector PR. The actual quantity which we have called a vector may be a displacement or velocity, acceleration or force or any other quantity requiring a direction for its statement.

4. Law of Parallelogram of Vectors.

This law is more commonly called the law of parallelogram of velocities or the law of parallelogram of forces, according as velocities or forces are being treated, but in the general form it is called the law of parallelogram of vectors. In order to make the treatment more concrete, let us consider the case of velocities. If two velocities v_1 and v_2 are at the same time acting upon a body, then their resultant velocity is obtained by the use of this law. Two straight lines AB and AC are drawn from a common point to represent on a suitable scale the magnitude and the direction of the two velocities acting on the body. The angle θ between these lines gives the angle of inclination between the directions of the two velocities, (fig. 3·4)

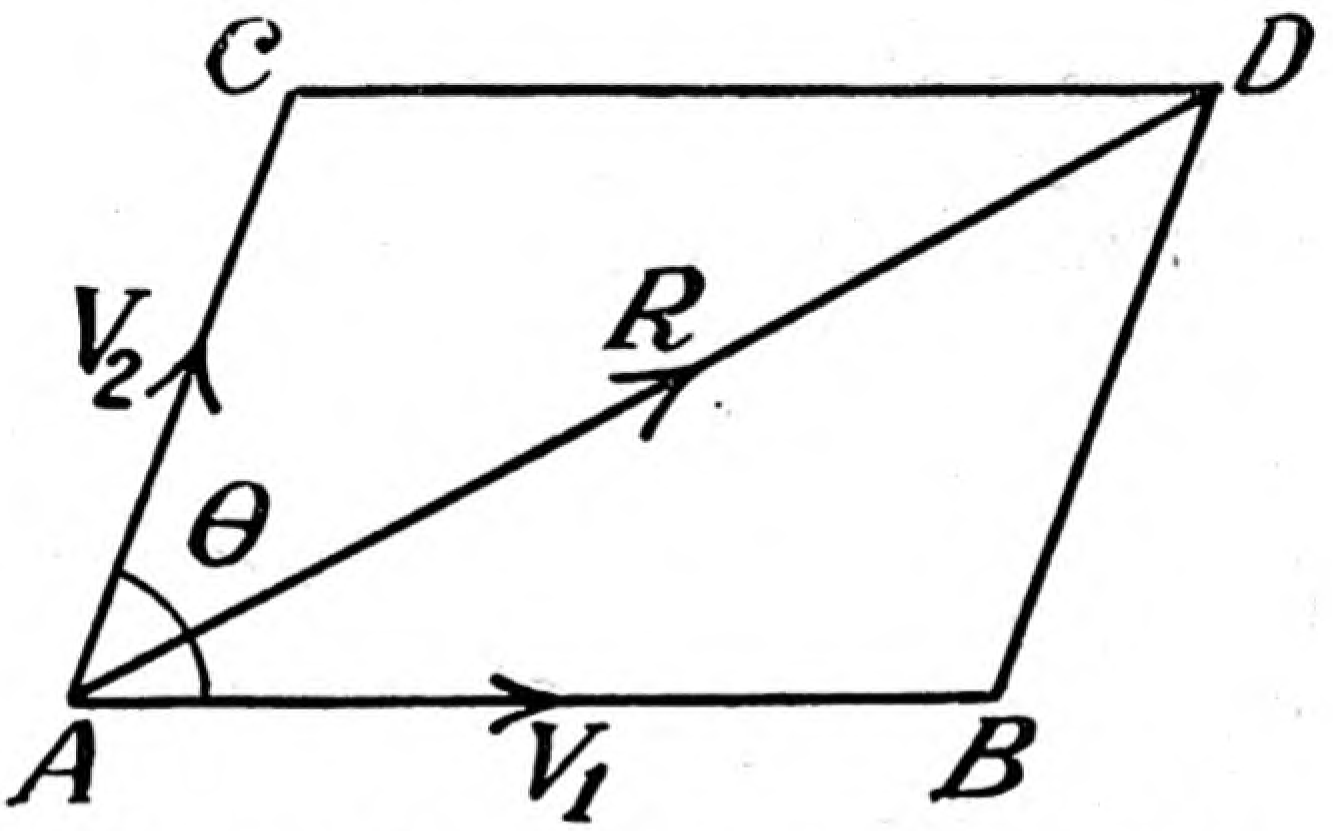


Fig. 3.4 Parallelogram of velocities

To find the resultant complete the parallelogram ABCD by drawing lines BD and CD respectively parallel to AC and AB and intersecting at the point D. Join A to D, to give AD the diagonal of the parallelogram. Then this diagonal represents in magnitude and direction the resultant of the velocities v_1 and v_2 denoted by the straight lines, AB and AC.

So far it is a graphical construction, and the value of the resultant is determined by measuring the length of the diagonal AD. This measurement will possess greater accuracy if the length to be measured is not too small. The accuracy of the graphical method is increased by choosing such a scale, that as large a parallelogram as is possible with the size of the paper is drawn.

The magnitude and direction of the resultant can also be found out by a calculation instead of making the diagram and greater accuracy is obtained by this method. To calculate R, the resultant velocity, produce AB (Fig. 3.5), and from the point D drop the perpendicular DE on AB produced. Then angle $DBE=\theta=$ the angle CAB. Also $\frac{DE}{DB}=\sin\theta$, and $\frac{BE}{BD}=\cos\theta$. Now, $DE=v_2\sin\theta$. Since the side BD of the parallelogram is equal and parallel to AC, which represents the velocity v_2 and $BE=v_2\cos\theta$. Further $AE=AB+BE=v_1+v_2\cos\theta$.

AD is the hypotenuse of the right angled triangle AED, therefore, $AD^2=(AE)^2+(ED)^2$ or

$$R^2=(v_1+v_2\cos\theta)^2+(v_2\sin\theta)^2=v_1^2+v_2^2\cos^2\theta+ 2v_1 v_2\cos\theta+v_2^2\sin^2\theta$$

$$=v_1^2+v_2^2(\sin^2\theta+\cos^2\theta)+2v_1 v_2\cos\theta.$$

Since $\sin^2\theta+\cos^2\theta=1$

We have $R^2=v_1^2+v_2^2+2v_1 v_2\cos\theta.$

i. e. $R=\sqrt{v_1^2+v_2^2+2v_1 v_2\cos\theta}$

By this formula we can calculate the magnitude of the resultant velocity knowing the individual velocities v_1, and v_2, and the angle of their inclination θ, when these velocities are acting together on the body.

The direction of the resultant R can also be calculated. Let α be the angle, the resultant R makes with a velocity v_1, *i.e.* the angle DOE of the Figure (3.5).

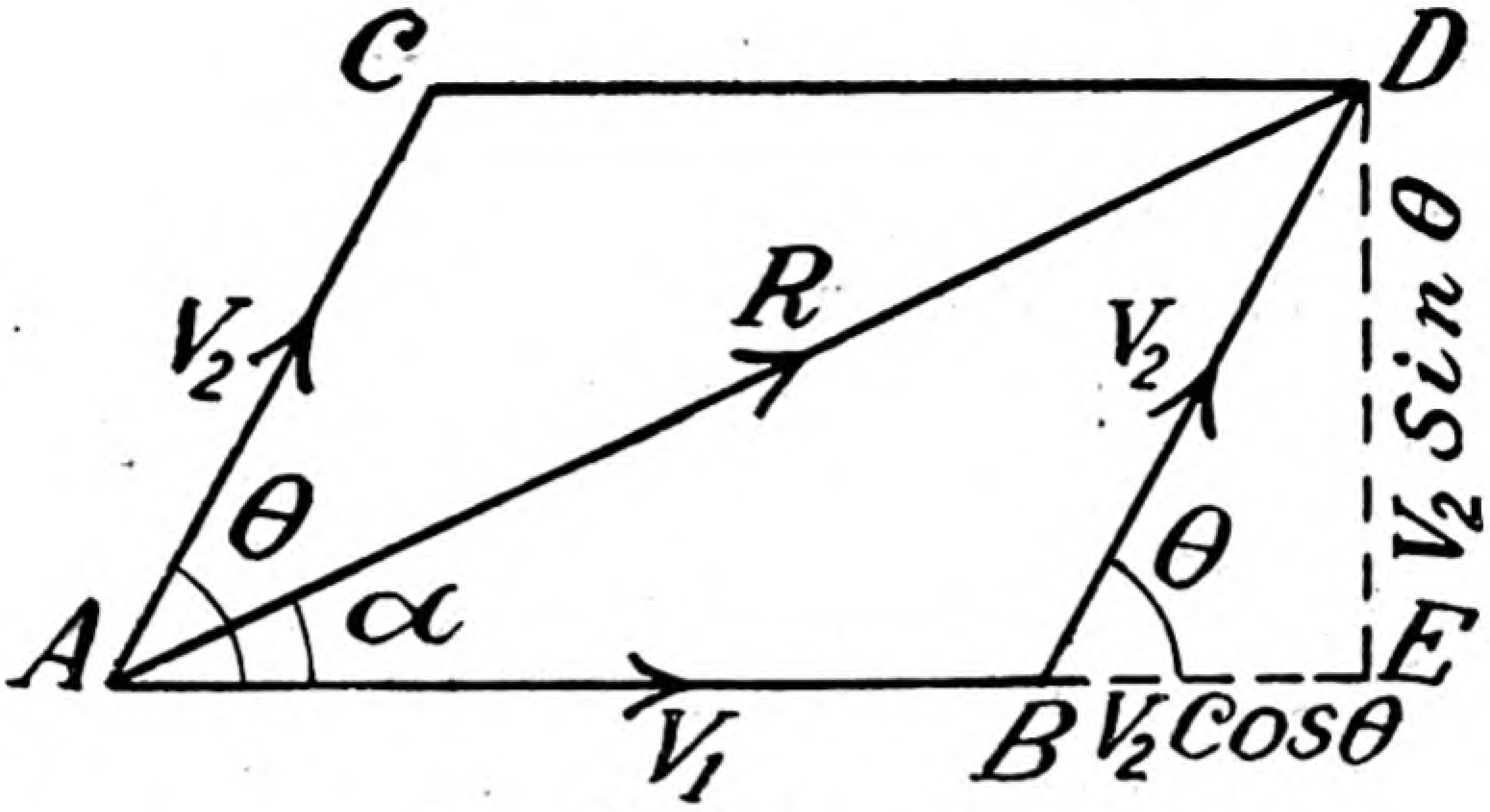


Fig. 3·5

$$\text{Then } \tan\alpha = \frac{DE}{AE} = \frac{DE}{AB+BE} = \frac{v_2 \sin\theta}{v_1 + v_2 \cos\theta}$$

Thus knowing v_1, v_2, and θ, the angle α can be calculated.

Also in the case of forces, if we call F_1 and F_2 as the two forces acting at the same time on a body, and inclined at an angle θ to each other, the joint effect of these two forces on the body would be the same as that of a single force F called their resultant, and given by the following formula which is quite similar to that for velocities.

$F = \sqrt{F_1^2 + F_2^2 + 2\,F_1\,F_2 \cos\theta}$ and the inclination α of the resultant F to the force F_1 by the formula $\tan\alpha = \dfrac{F_2 \sin\theta}{F_1 + F_2 \cos\theta}$

We may now consider a few special cases.

I. If the two velocities are perpendicular to each other $\theta = 90°$ and $\sin 90° = 1$, while $\cos 90° = 0$,

Therefore, $R = \sqrt{v_1^2 + v_2^2}$ and $\tan\alpha = \dfrac{v_2}{v_1}$. The parallelogram in this case becomes a rectangle.

II. If the two velocities are acting in the same direction $\theta = 0$, $\sin\theta = 0$, $\cos\theta = 1$, and $R = \sqrt{v_1^2 + v_2^2 + 2\,v_1\,v_2} = v_1 + v_2$ and $\alpha = 0$.

III. If the direction of the two velocities is opposite to each other $\theta=180°$, $\sin\theta=0$, and $\cos\theta=-1$,

Therefore, $R=\sqrt{v_1^2+v_2^2-2v_1v_2}=v_1-v_2$

Example :—In still water a man can row a boat with a speed of 5 miles per hour. If he is rowing the boat at right angles to a stream flowing at the rate of 5 miles per hour, what will be the actual velocity of the boat ?

In this case $\theta=90°$

The Resultant velocity $R=\sqrt{5^2+5^2}$

$=\sqrt{50}$

$=7.09$ M. P. H.

$\tan\alpha=1$

Therefore, $\alpha=45°$

The velocity of the boat is, therefore, 7.09 miles per hour inclined at an angle of 45° to the stream.

5. Triangle of Vectors :

In the last article we have used the method of parallelograms for determining the resultant of two vector quantities. The graphical construction can be somewhat simplified in the following way :

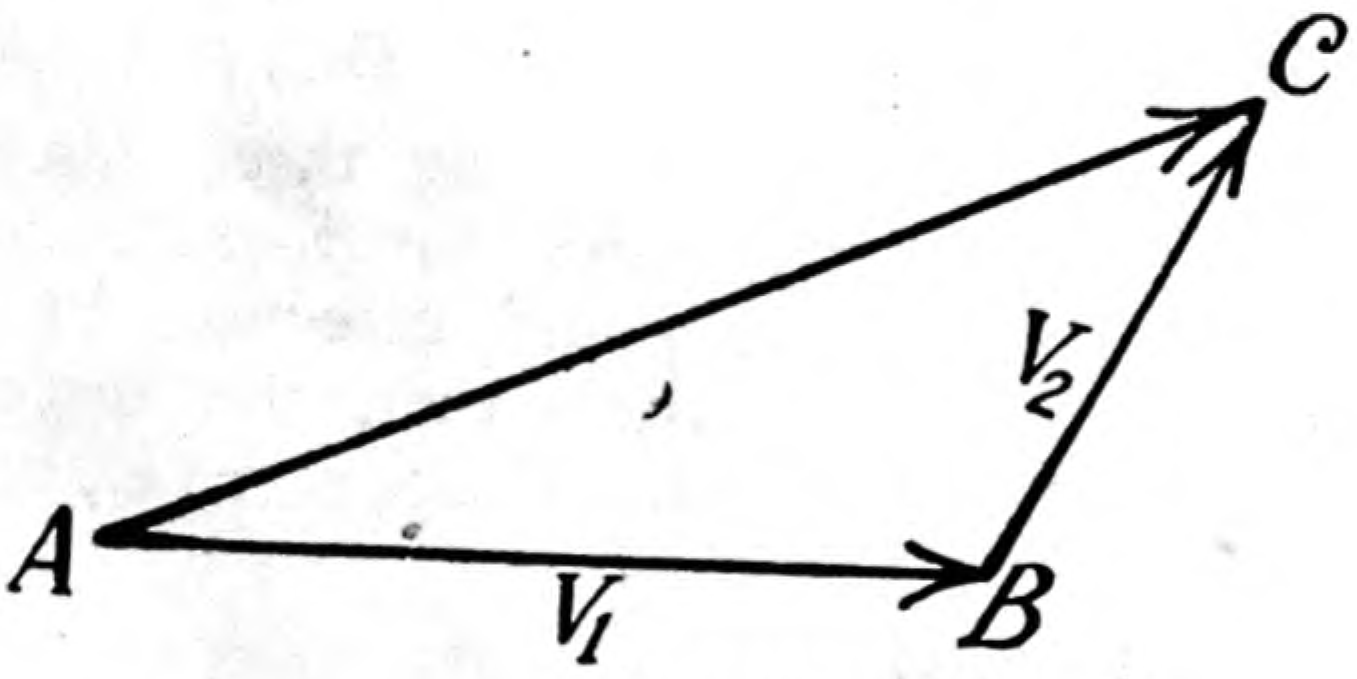


Fig 3.6

Suppose the two velocities acting together are v_1 and v_2. Draw a straight line AB (Fig 3.6) to represent on a suitable scale the magnitude of v_1, and drawn parallel to the direction v_1. Put an arrow head at B to indicate that the velocity considered is in the direction A to B. From the point B draw another straight line BC to represent the velocity v_2; its length on the same scale as used for v_1, representing its magnitude, and the direction parallel to v_2, is represented by an arrow. The tail of the arrow for v_2, coincides with the head of the arrow corresponding to v_1. Join AC. Then the straight line AC gives the resultant of the velocities v_1 and v_2. An arrow head placed on AC at the point C indicates the direction of the resultant.

An exactly similar construction would be used for finding the resultant of two forces. This method simplifies the construction and is called the method of triangle of Vectors. It will be observed that the resultant is given by the same calculation as considered in the method of parallelogram of Vectors. We see that if a third velocity equal to CA, (Fig. 3.6), were to act on the body on which two velocities v_1 and v_2 represented by AB, and BC are already acting, the three velocities will cancel each other's effect, and the body will remain at rest. This is, because, the resultant of AB and BC is AC, and the third velocity CA is equal and opposite to AC, and therefore, cancels its effect. We can generalise that if three velocities acting simultaneously on a body can be represented by the three sides of a triangle taken in order, then the three velocities are in Equilibrium, and the body remains at rest. Mark the words "in order," which means that the direction of the three velocities represented completely by the triangle ABC is AB, BC, and CA.

A similar proposition concerning three forces acting at the same point on a body, states that if these forces can be represented in magnitude and direction by the three sides of a triangle taken in order, then the forces are in equilibrium, and their resultant disappears i.e., has a value equal to zero.

6. Polygon of Vectors :

If the resultant of more than two velocities, jointly acting on a body is to be determined, then the law of Polygon is used. Suppose a number of velocities v_1, v_2, v_3 and v_4 acting in different directions are imposed, at the same time, on a body. To determine a single velocity, which will produce the same motion in the body, as the combined effect of the various velocities, i.e., to find their resultant, we make the following construction :—

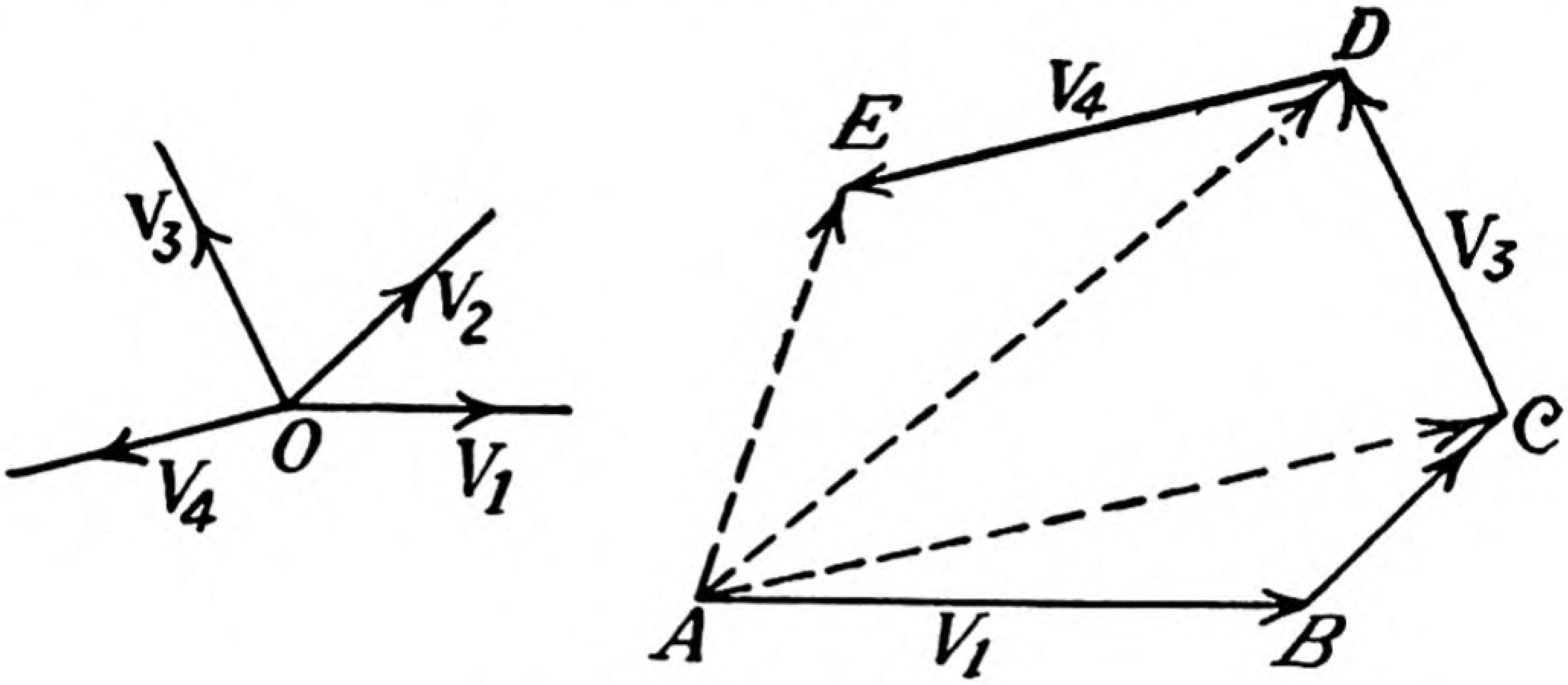


Fig. 3·7

Draw a straight line AB to represent v_1 in magnitude and direction (Fig. 3·7). From the point B draw a straight line BC to represent the velocity v_2 in the same way. From C draw the straight line CD for the velocity v_3, and from D, the line DE representing the velocity v_4. Show arrow heads at B, C, D and E to specify the direction. If now we join A with E, then the straight line AE represents the resultant of the four velocities v_1, v_2, v_3, and v_4.

This is easily proved to be so, for AC is the resultant of AB and BC by law of triangles as seen above. Further, two velocities represented by AC and CD will have a resultant AD, i.e., the resultant of AB, BC, and CD is AD, and similarly the resultant of AD and DE equals AE. Thus velocity represented by AE is the resultant of the four velocities. The method may be used to combine any number of velocities, or forces acting together on a particle.

A conclusion which easily follows from the above treatment is that if a number of forces or velocities are acting together on a body, and when a vector diagram is drawn to represent them, a closed polygon is obtained, then those forces or velocities will be in complete equilibrium and their resultant equals zero.

7. Resolution of a Vector :

We have seen that two velocities v_1 and v_2 acting on a body are equivalent in their effect to a single velocity v obtained by the vector addition of v_1 and v_2. This addition can be made by the parallelogram of velocities or by the triangle method. In the (Fig 3.8) v_1 and v_2 are the sides of the parallelogram and v their resultant is its diagonal. Conversely if a velocity v is acting on a body, it is possible to replace it by the velocities v_1 and v_2 without producing any difference in the motion of the body. In fact the effect of single velocity v is exactly similar to the combined effect of the two velocities v_1 and v_2. The velocities v_1 and v_2 are called the components of the single velocity v. In the (Fig. 3.8) v_1 and v_2 are inclined at an acute angle to each other. The process of finding the components of a velocity along any particular direction is called the resolu-

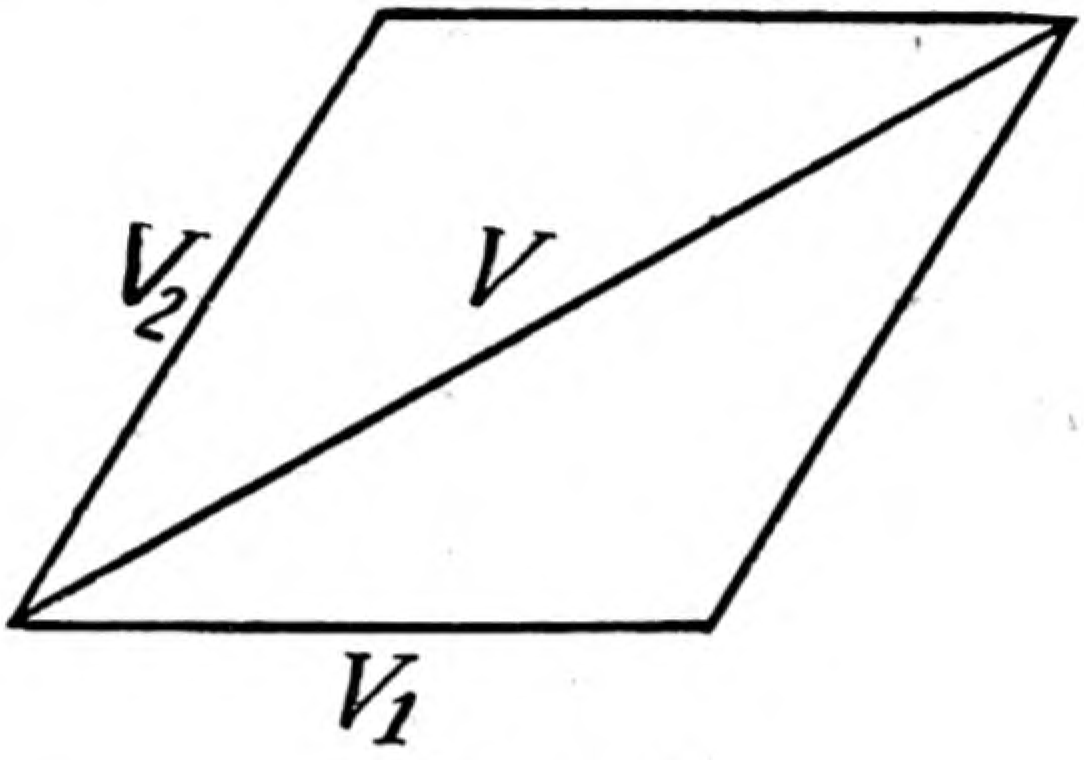


Fig. 3.8

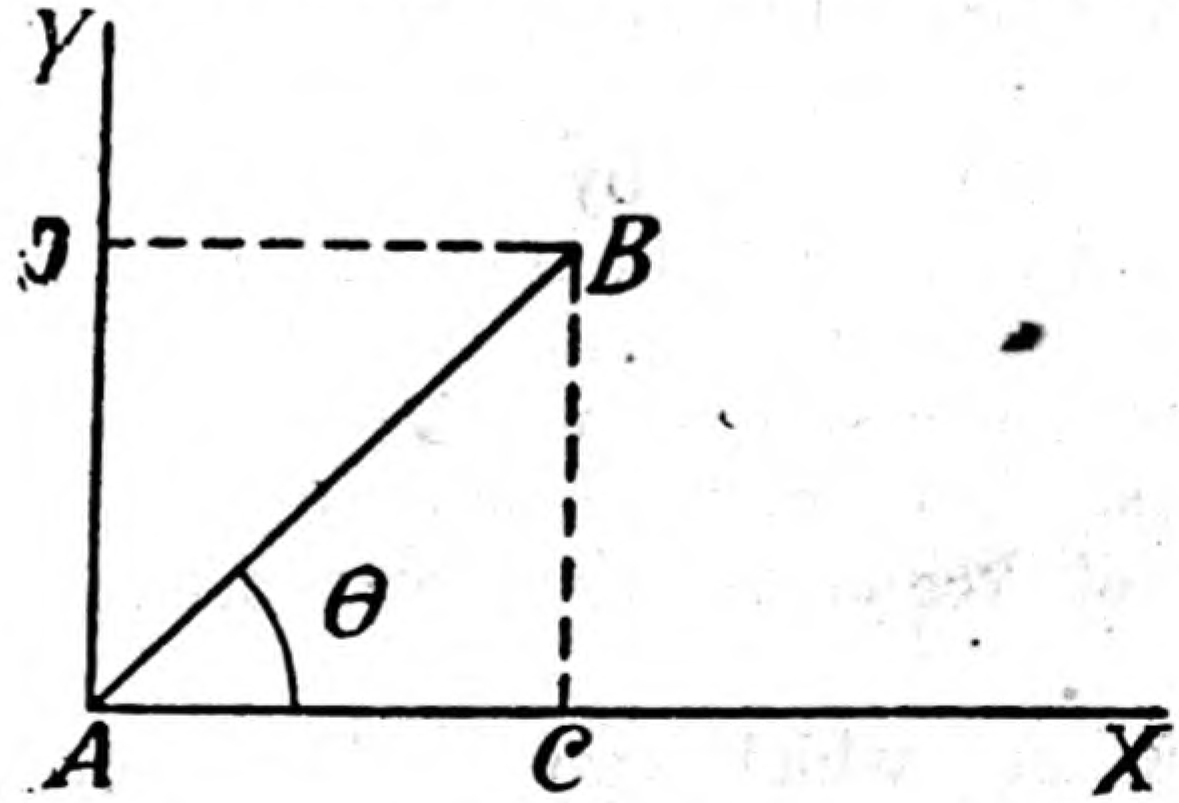


Fig. 3.9

tion of the velocity into its components. A case of special interest is when the two components are at right angles to one another. Such perpendicular components are called rectangular components. Suppose the line AB, (Fig. 3.9) represents a velocity v and we want to find out its rectangular components, one of them along the line AX which makes an angle θ with AB, and the other in the perpendicular direction AY. From the point B drop BC perpendicular to AX, and BD perpendicular to AY. Then it is obvious that the velocities represented by AC and AD have a resultant equal to AB or v. AC and AD are called the rectangular components of AB.

Since $$\frac{AC}{AB}=\cos\theta,$$

$$AC=AB\cos\theta$$

or $$v_x=v\cos\theta\ ;\text{ and since}$$

$$\frac{AD}{AB}=\frac{BC}{AB}=\sin\theta,$$

Therefore, $$AD=AB\sin\theta$$

or $$v_y=v\sin\theta$$

Thus the rectangular components of a velocity (or a vector v) in the directions making an angle θ with it, and perpendicular to that, are $v\cos\theta$ and $v\sin\theta$.

8. Projectile Motion.

When a stone is thrown in horizontal direction, it is acted upon by the downward acceleration as of a falling body in addition to its forward motion. The two velocities, consisting of the uniform forward velocity, and the accelerated downward velocity, act independently of each other; and the path taken by the stone is obtained by the composition of these two velocities. To consider a particular case, suppose that the stone is thrown forward horizontally from a point o, with a velocity of 50 feet per second. The downward acceleration of gravity acting on it is 32 feet per second per second.

In the beginning, the stone has a forward velocity of 50 feet per second, and downward velocity is zero. After 1 sec., it still has the same forward velocity of 50 feet per second, but its downward velocity has increased to 32 feet per second. After 2 seconds the forward velocity is still 50 feet per second, but the downward velocity becomes 64 feet per second. Neglecting the resistance due to air friction, the forward velocity remains constant and the downward one increases by 32 feet per second every second. We can plot the position of the stone after each second and joining the various points obtained we obtain a curve which is the path taken by the stone. Calculating the distance fallen by the formula

$h=\frac{1}{2}\,gt^2$, we see that the distance fallen in 1 second

$$=\frac{1}{2}\times 32\times 1^2=16 \text{ feet.}$$

Therefore, at the end of 1 second, the stone has moved forward by 50 feet fallen downward by 16 feet. In the Fig 3.10

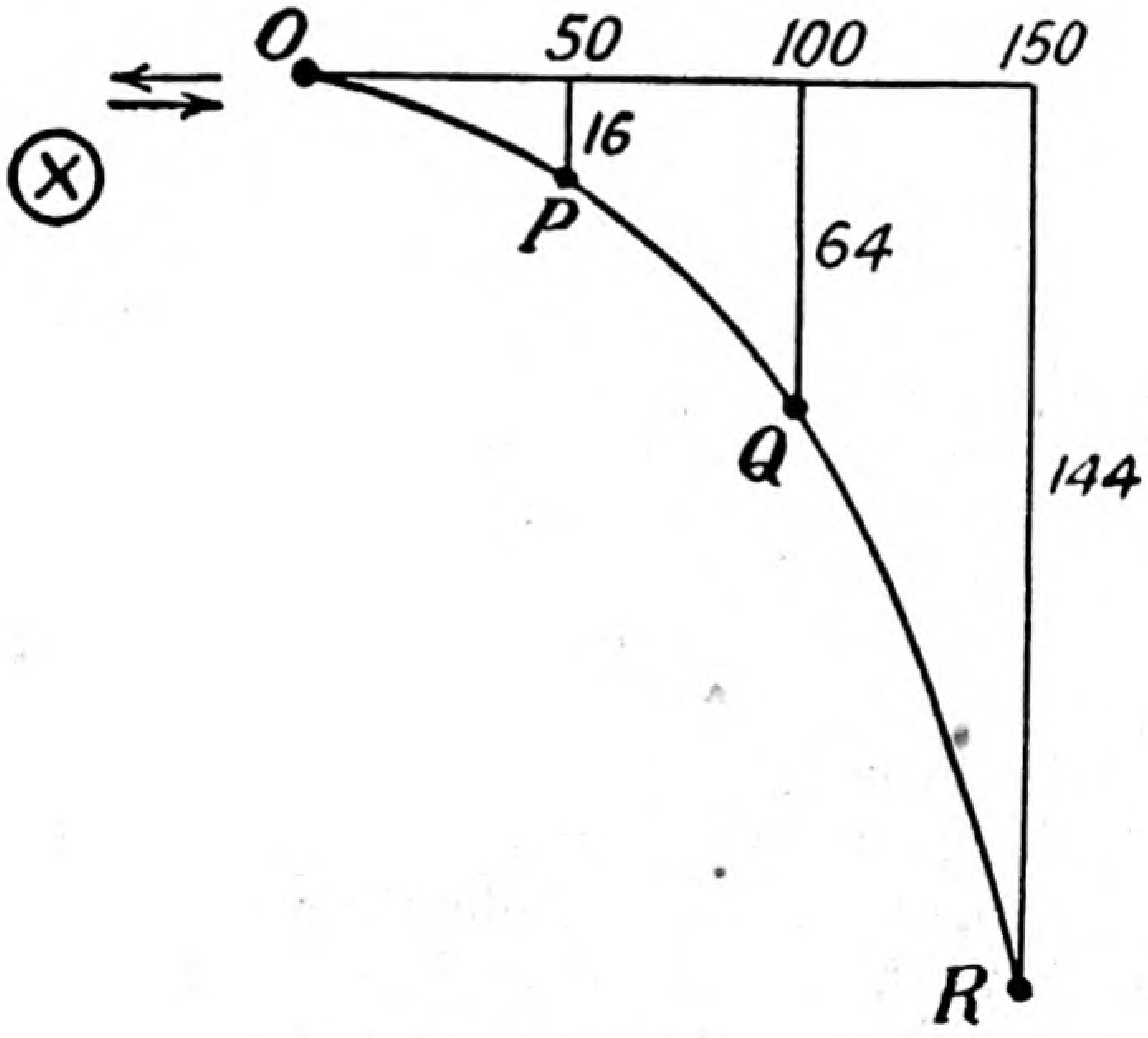


Fig 3.10

its position is shown by the point P. After 2 seconds it has moved forward by 100 feet and fallen downward by $\frac{1}{2}\times 32\times 2^2=64$ feet, and its position is represented by the point Q. Similarly after 3 seconds it has moved forward by 150 feet and fallen downward by $\frac{1}{2} \times 32 \times 3^2 = 144$ feet and is represented by the point R. The line OPQR is curved and a curve of this kind is called a parabola.

Due to the resistance of the air, however, the actual path, especially in the case of cannon firing, is not a parabola, nor is the maximum range obtained as would be the case if air friction is neglected. In modern army gunning all these factors have to be taken into account. The scientific subject of motion of projectiles is of great importance in gunning and bombing operations of the army, and is called 'Ballistics'.

A similar case is that of a bomb dropped from an aeroplane and aiming at a target. The moment the bomb is released it possesses a horizontal velocity equal to the volocity of the aeroplane. As it falls it acquires an increasing downward velocity, but maintains its forward velocity. The bomb gunner has to determine the proper time when the bomb should be released, so that taking all velocities into consideration, the bomb may hit the target. If, as is often done in firing a cannon, or a rifle gun, or hammer throwing, the ball or the bullet is shot in a direction, which is inclined upwards, the shot rises up, as it also moves forward, attains a maximum height, and then begins to fall, but all the time moving with a forward velocity.

The velocity, with which the shot is fired, can be resolved into a horizontal component, and a verticle component. If θ is the angle which the direction of initial velocity v makes with the horizontal, then the horizontal component is $v.\cos\theta$ and the vertical component $v.\sin\theta$. To determine the path of the projectile we may consider these two velocities to act independently on the body. The horizontal velocity will move it forward, and the vertical velocity will carry it up. Neglecting air friction the horizontal velocity remains constant, but the vertical one decreases till it becomes zero, at the highest point reached, when the body begins to fall down with an increasing downward velocity, finally striking the earth. The path taken is shown in

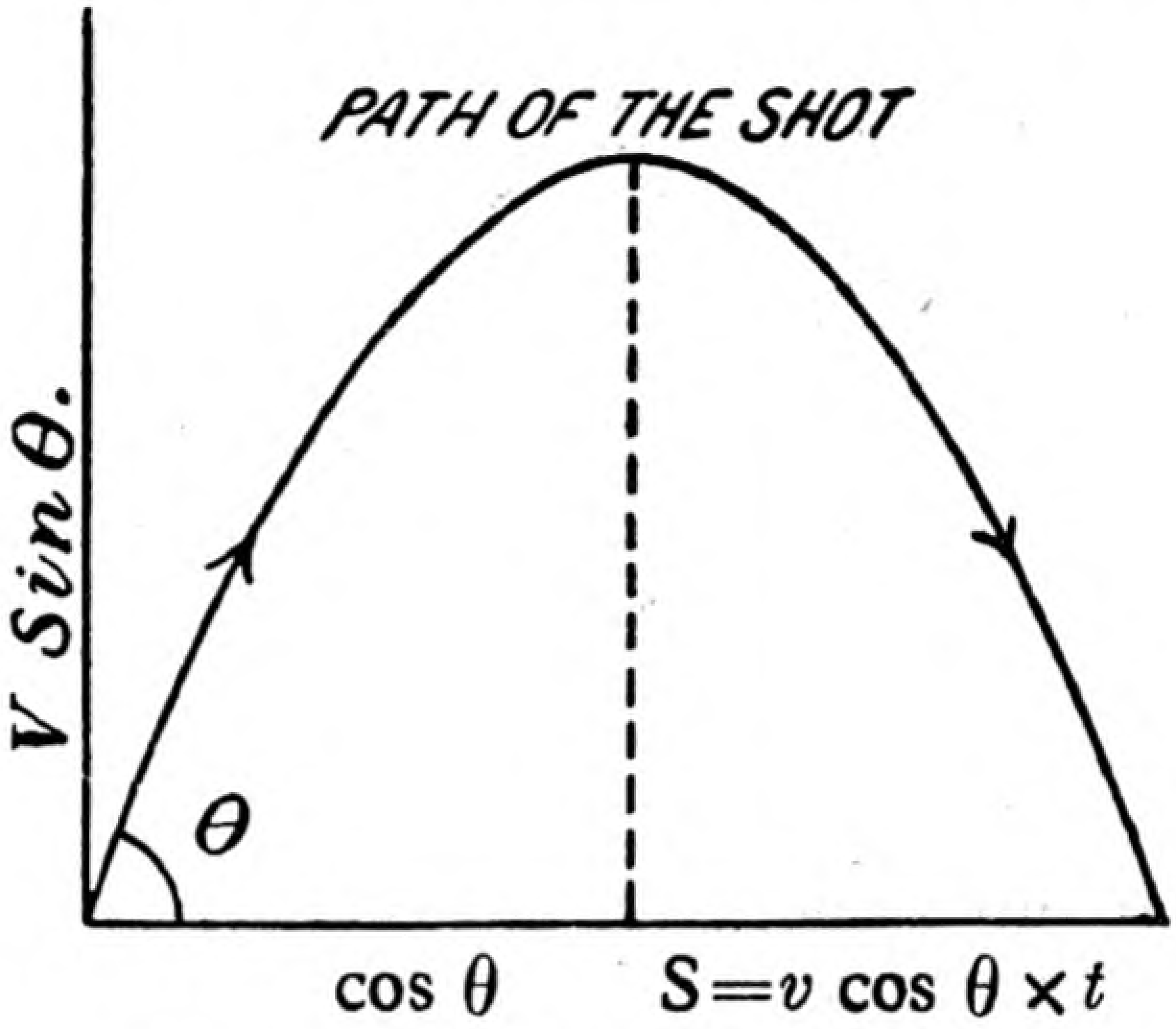


Fig. 3.11 Path of a projectile

Fig. 3.11, which is a parabola. The range or the maximum distance moved forward depends both on v and on θ. For a particular value of v the range is maximum when θ is 45°. This is why when throwing the shot or hammer, the athlete should throw it up at an angle of 45° to the horizontal.

9. Relative Velocity :

Ordinarily, when we speak of the velocity of a body, we understand its velocity with respect to the surface of the earth, the latter being supposed to be at rest. However we know that the earth is not at rest because it is rotating on its axis and also revolving in its orbit around the sun. A point on the equator of the earth rotates through a circle of 25000 miles circumference in 24 hours. The earth is moving in its orbit at about 18 miles per second. Therefore, if we were to consider the motion of a body such as a railway train, not with respect to earth but with respect to space, its motion will be very complicated because the motion of the earth will have to be compounded with the motion of the train. Motion and velocity are therefore relative quantities, that is to say, we only express the motion of a body with respect to or relative to another body which we suppose to be at rest ; although in reality there is no body at rest in the universe. Similarly, suppose two persons are sitting in a train on opposite seats. We would say that they are at rest with respect to one another or relative to one another although we know that both of them are being carried by the train. They are therefore moving with respect to the earth, but at rest relative to each other. Therefore a body may be at rest relative to some object and moving relative to other objects. This means that our ideas of rest and motion are only relative and there is no absolute rest or absolute motion in the universe. The velocity of a body depends not only on its motion of the observer, or what we say in scientific language the velocity of a moving body depends on the frame of reference of the observer.

Let us consider another example of a man sitting near the window of a train and dropping a stone from his hand. If he looks at the stone to him it will appear that it has fallen to the ground in a vertically downward direction. If a person was standing on the earth near the railway track when the stone was dropped, he will see that the stone moves, along a parabolic path as it falls to the earth, because the dropping stone has a forward velocity, and also a downward velocity as of a projectile. Thus the velocity of a body is a relative quantity depending on the velocity of the observer, or the velocity of a body A relative to a body B depends on the velocity of B as well as A.

As another example consider the case of two ships S_1 and S_2 leaving a port P at the same time. Suppose the ship S_1 sails towards the east with velocity of 30 miles per hour, and the ship S_2 sails from the port towards north at a speed of 40 miles per hour. Their relative positions after one hour are shown in figure 3.12.

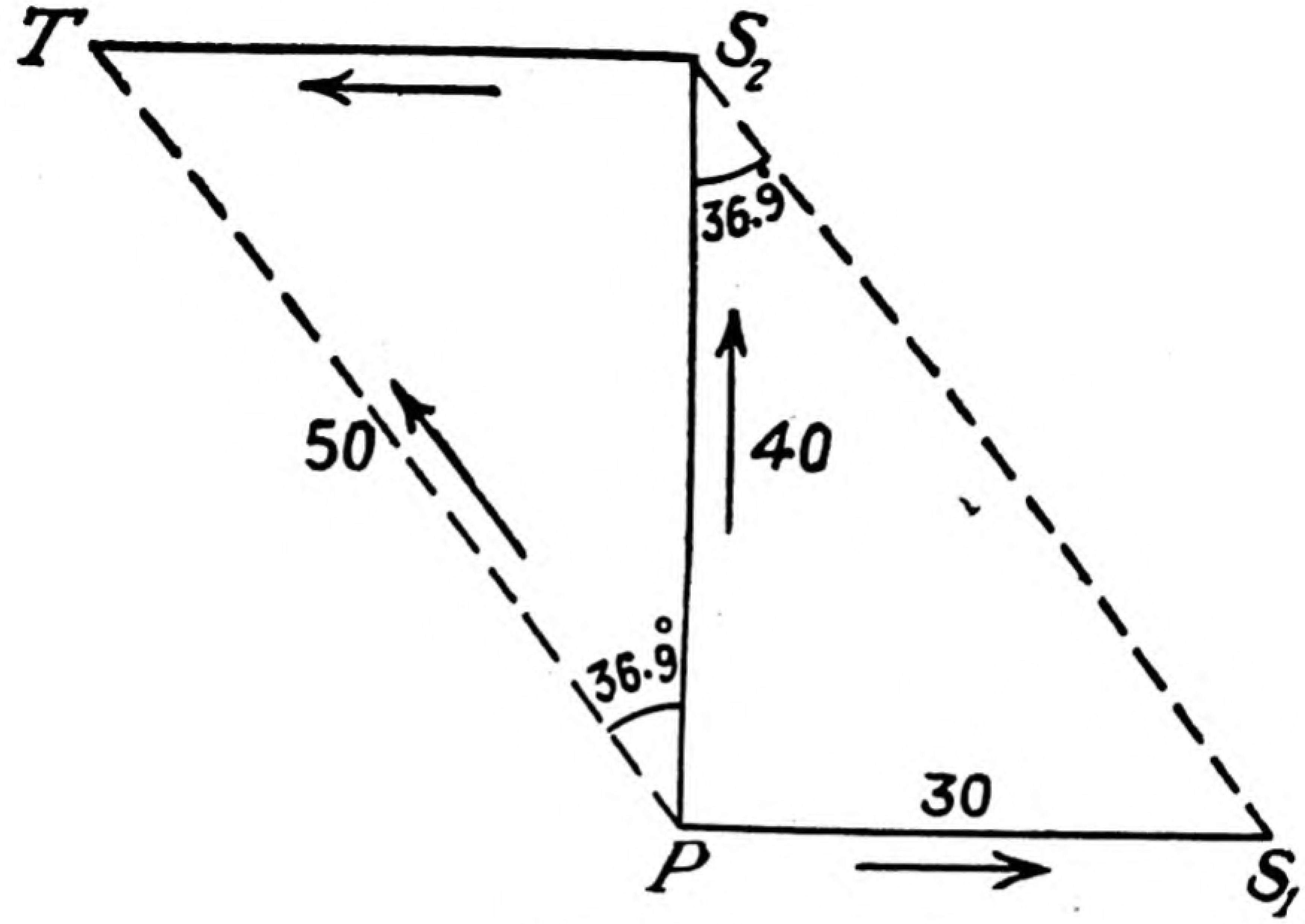


Fig. 3.12

To an observer in S_1 the ship S_2 will appear to be travelling away from S_1 with a speed of 50 miles per hour and in a direction 36.9° west of north. For an observer in S_2 the velocity of S_1 relative to him will be 50 miles per hour in a direction 36.9° east of south. We may find the relative velocity by supposing that on each of the ships a velocity of 30 miles/hr towards west is imposed. This velocity is equal and opposite to that of S_1 and its effect will be to bring S_1 to rest. The resultant of the imposed velocity and the true velocity of S_2 is shown by the line P T in the figure, which represents to scale the relative velocity of S_2 with respect to S_1. This is a general rule for finding the relative velocity of a body B with respect to another body A. Impose on both of the bodies a velocity equal and opposite to that of A. As a result A will come to rest, and the resultant of the imposed velocity and actual velocity of B will be its relative velocity with regard to A.

An interesting example of relative velocity is the case of a man walking in the rain. Although the rain drops may be falling vertically, but they appear to the walking man as if striking on him from an inclined direction. To protect himself, therefore, he has to hold his umbrella some what inclined in the forward direction. The angle at which the umbrella is to be held depends on the velocity of the man and the rain.

Suppose the man is moving with a velocity V_m represented by the line M A, and the rain is falling vertically with a velocity V_r represented by the line M B (fig. 3.13).

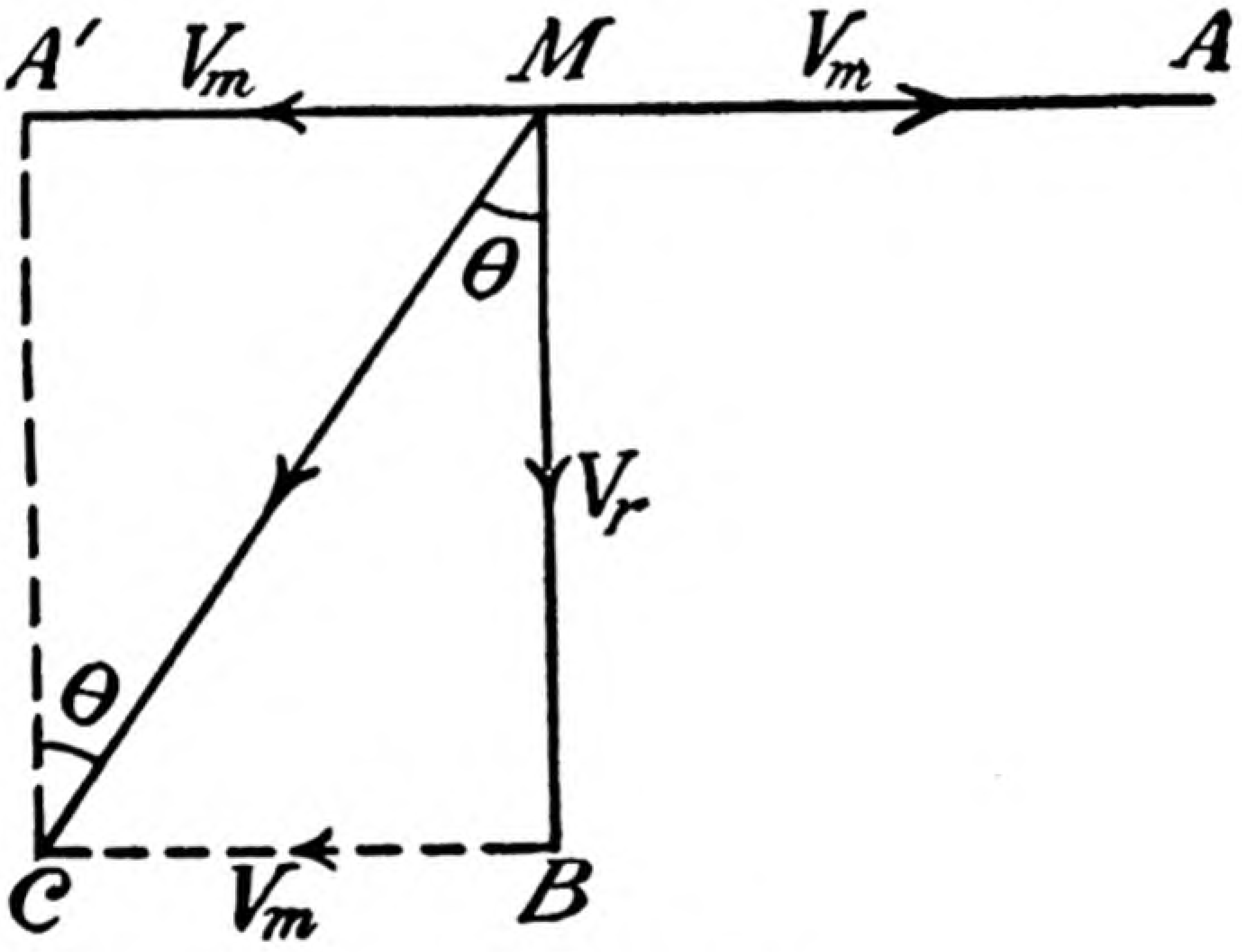


Fig. 3.13

To find relative velocity of rain with respect to man, impose a velocity of V_m along M A′ on the man as well as rain. Due to this imposed velocity man comes to rest. The resultant of MB and MA′ is MC which is the relative velocity of rain with respect to the man. Therefore the rain appears to strike the man along the line MC making an angle θ with the vertical.

$$\tan\,\theta = \frac{BC}{BM} = \frac{V_m}{V_r}.$$

Next let us consider the case of motion of a body with respect to the motion of wind which strikes it. Figure 3.14 illustrates the case of a car moving through air. In fig. 3.14 A the car is shown having a velocity of 10 m. p. h. due east in still air, and the air appears to strike the car

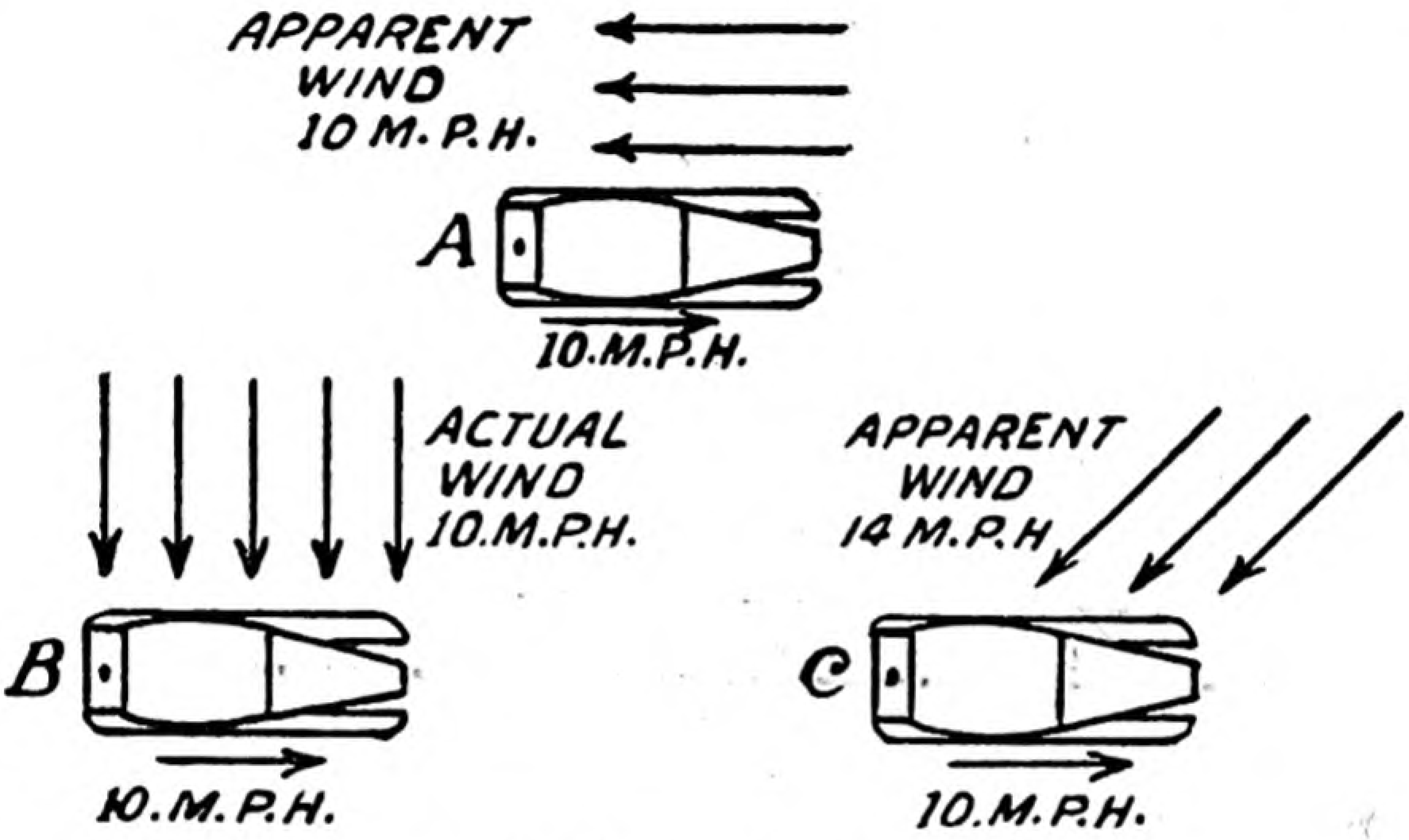


Fig. 3.14 Motion of a car in wind

from east with a velocity of 10 m. p. h. or the relative velocity of wind with respect to car is 10 m. p. h. from east. Figure B shows the actual velocity (10 m. p. h.) of the wind coming from north as the car goes east at 10 m. p. h. In fig C the relative velocity between the car and wind is shown. The wind appears to come from the north-east direction with a velocity of about 14 m. p. h.

According to the theory of relativity developed by Albert Einstein, one of the greatest mathematical physicists, all physical quantities of length, mass and time are relative depending on the velocities of the object and the observer. In the case of bodies moving with very large velocities of many thousands of miles per second, appreciable changes in mass and length take place, although for small velocities the changes are too small to be observed. The only absolute physical quantity on this theory is the velocity of light in vacuum. For his remarkable theory, which forms the foundation of modern physics, he was awarded the Nobel prize in 1921.

Summary

A vector quantity can be represented by straight line. The length of the line on some scale denotes the magnitude while an arrowhead on the line gives the direction of the vector.

Vectors are added by the law of parallelogram of vectors.

Law of parallelogram of velocities. If two velocities acting on a point are represented in magnitude and direction by two straight lines drawn from that point and a parallelogram is drawn with these lines as adjacent sides, then the diagonal of the parallelogram through that point gives the resultant velocity in magnitude and direction.

If v_1 and v_2 be the two simultaneous velocities of a point and θ be the angle between them, the resultant velocity R of the point is given by the formula.

$$R=\sqrt{v_1^2+v_2^2+2v_1v_2\cos\theta}.$$

If the resultant R makes an angle α with the direction of v_1 then

$$\tan\alpha=\frac{v_2\sin\theta}{v_1+v_2\cos\theta}.$$

when $\theta=0°$

$R=v_1+v_2$ and $\alpha=0$.

when $\theta=90°$

$R=\sqrt{v_1^2+v_2^2}$ and $\tan\alpha=\frac{v_2}{v_1}$

and for $\theta=180°$

$R=v_1-v_2$ and $\alpha=0$

Triangle of velocities. If the velocities of a point are represented in magnitude and direction by two sides of a triangle taken in order then their resultant is represented in magnitude and direction by the third side of the triangle taken in the opposite order.

The triangle of velocities can be expressed in the following form also. If three velocities, acting simultaneously on a body, can be represented in magnitude and direction by the three sides of a triangle taken in order, then the three velocities are in equilibrium, and the body remains at rest.

Polygon of velocities. If a number of velocities acting on a body are represented in magnitude and direction by the sides AB, BC, CD, DE of a polygon taken in order then their resultant is completely represented by the closing side AE of the polygon taken in the opposite order. Or we can say that if a number of velocities are acting together on a body, and when a vector diagram is drawn to represent them, a closed polygon is obtained and then those velocities will be in equilibrium and their resultant equals zero.

A velocity can be resolved into two rectangular components along any two directions perpendicular to each other. Component of velocity v along the direction OX is $v \cos \theta$ and $v \sin \theta$. Component along OY is $v \sin \theta$, where θ is the angle between direction of v and the line OX. (figure 3.15).

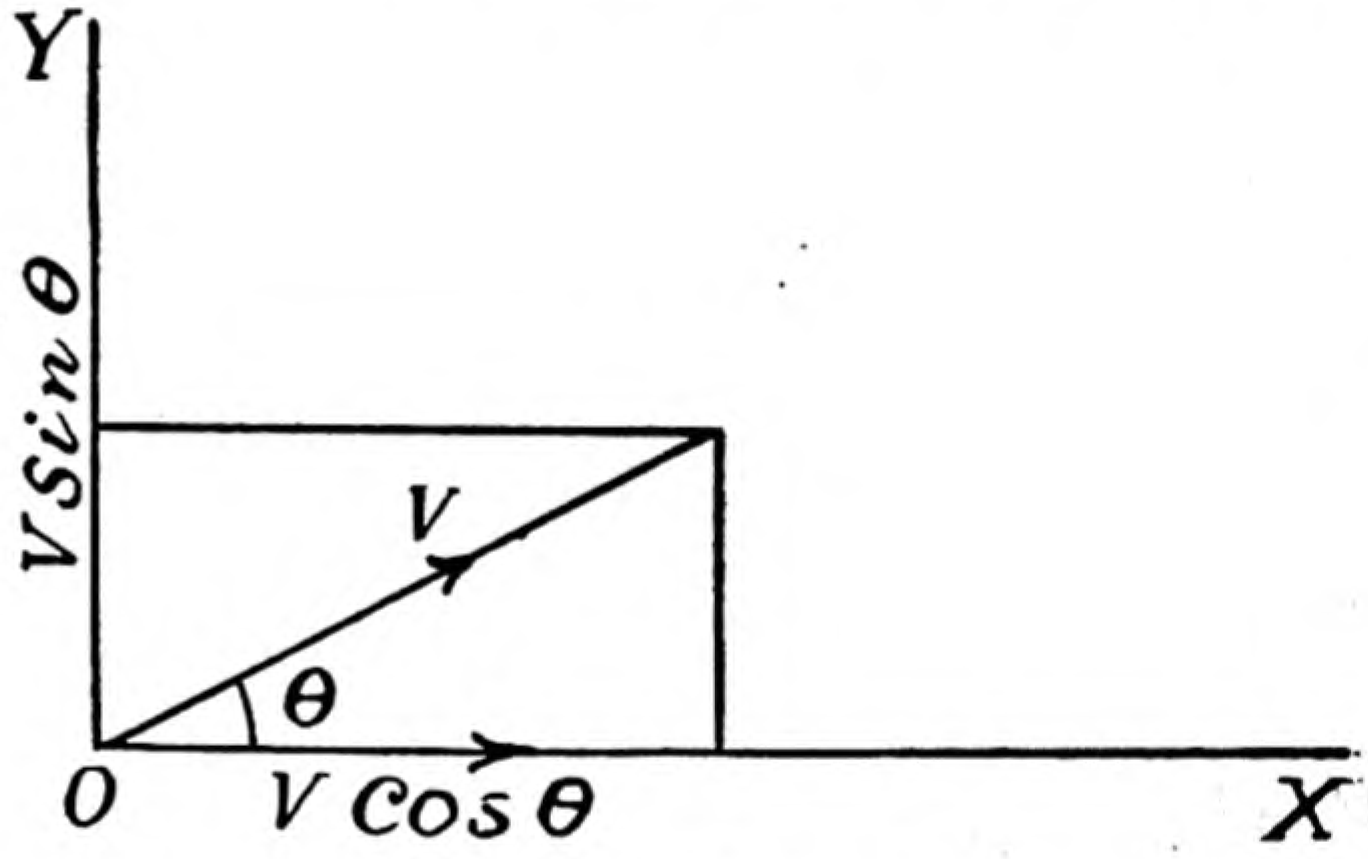


Fig 3.15

The path of a body, having a uniform velocity in the horizontal direction and falling under the action of gravity, is a parabola.

The relative velocity of a body B with respect to another body A is determined by imposing on A as well as on B a velocity equal and opposite to that of A. Thus A is brought to rest and the resultant of the imposed velocity and the actual velocity of B gives the relative velocity of B with respect to A.

Exercises

1. Calculate the magnitude and direction of the resultant of two velocities u and v inclined at an angle θ. What is the resultant, if $\theta=0$?

Two equal velocities have a resultant equal to either. At what angle are they inclined to each other ?

[*P. U. 1952*].

In this case $v_1=v_2=R=\sqrt{v_1^2+v_2^2+2\,v_1\,v_2\cos\theta}$

$$v_1=\sqrt{v_1^2+v_2^2+2\,v_1^2\cos\theta}=v_1\sqrt{2+2\cos\theta}$$

or $$1=\sqrt{2+2\cos\theta}$$

Squaring $$1=2+2\cos\theta$$

or $$\cos\theta=-\tfrac{1}{2}$$

∴ $$\theta=120^\circ$$

2. One of the resolutes of a velocity of 50 miles/hr is a velocity of 44 ft/sec. Find the other component.

$$50 \text{ miles/hr}=\frac{22}{15}\times 50=\frac{220}{3}\text{ ft/sec}$$

One component $=44$ ft/sec.

$$\therefore \text{ other component}=\sqrt{\left(\frac{220}{3}\right)^2-(44)^2}=58.67 \text{ ft/sec}$$

3. A river is flowing at the rate of 5 miles/hr. A swimmer swims across the river with a velocity of 4 miles/hr. Find the resultant velocity of the man in the river. If the river is half a mile wide, how long will he take to cross it ? How far below the starting point will he reach the opposite bank.

The velocities of the man and the river are at right angles to each other. Therefore $\theta=90^\circ$ and

Resultant velocity $\sqrt{4^2+5^2}=6.4$ miles/hr.

The velocity of 4 miles/hr takes the man across the river while the velocity of water of 5 miles/hr takes him down the stream.

∴ Time to cross the river $=\frac{1}{2}\times\frac{1}{4}=\frac{1}{8}$ hr $=7\frac{1}{2}$ minutes.

Distance travelled down the stream in $7\frac{1}{2}$ minutes $=5\times\frac{1}{8}$ $=\frac{5}{8}$ miles $=1100$ yards.

4. A man can row a boat at the speed of 6 m. p. h. in still water. If a stream is flowing at 3 m. p. h., in what direction should he row so that the boat may go right across the stream. Also calculate the resultant velocity of the boat.

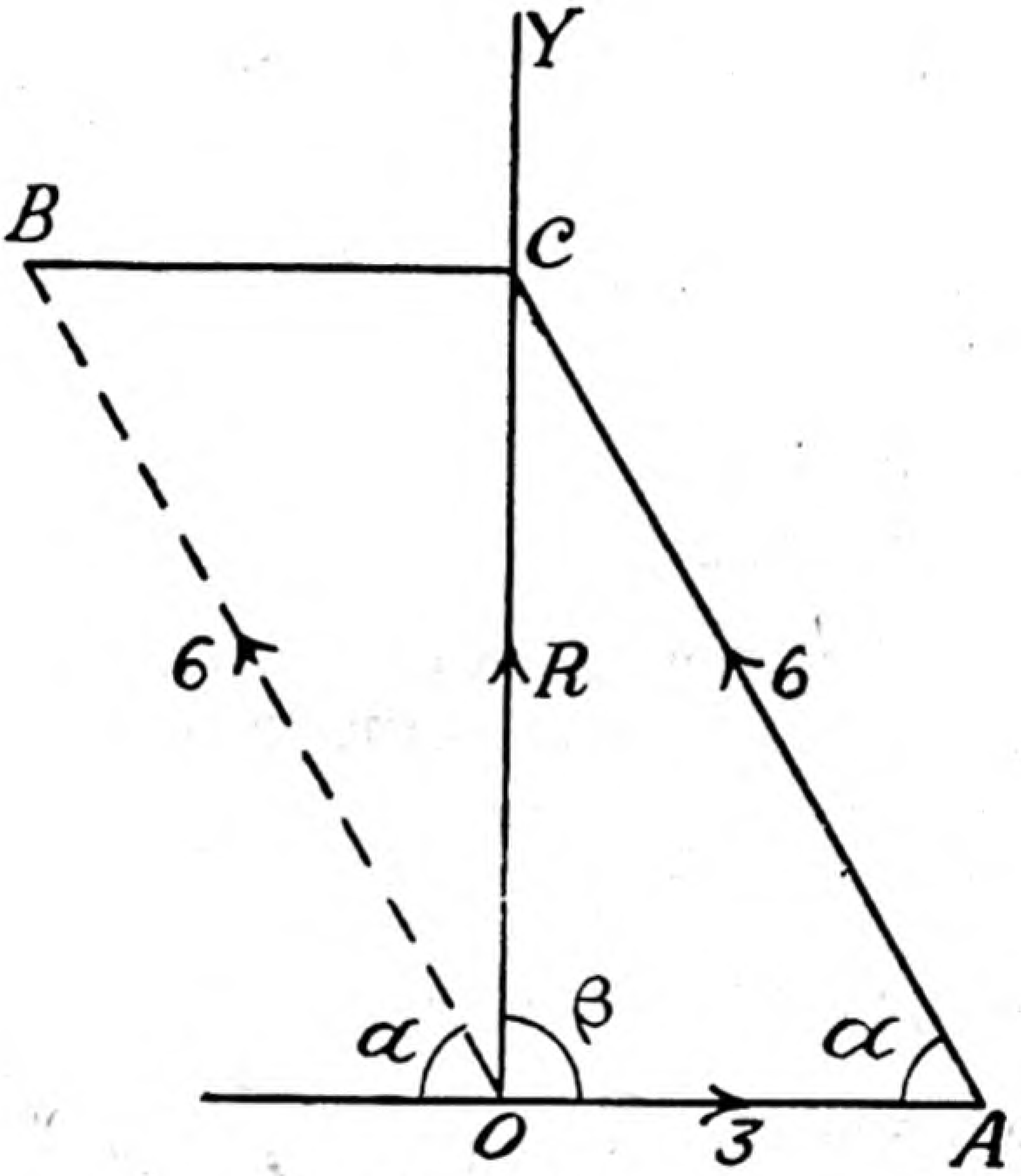


Fig 3.16

Represent the velocity of 3 m. p. h. by the line OA on some scale. Draw OY perpendicular to OA. Then the resultant velocity of the boat lies along OY. With A as centre and radius equal to AC representing velocity of 6 m. p. h. on the same scale, draw an arc; cutting OY in C. Join AC and complete the parallelogram of velocities OACB. In this parallelogram OA is v_1, OB is v_2 and OC is the resultant R in magnitude and direction.

From the figure $\frac{\text{AO}}{\text{AC}}=\cos\alpha=\frac{3}{6}\ \therefore\alpha=60°$

Therefore the man should row upstream making an angle of 60° with the bank.

Again $\frac{OC}{AC}=\sin 60=\frac{\sqrt{3}}{2}$

$\therefore$ Resultant velocity $OC=AC.\frac{\sqrt{3}}{2}=\frac{6\times\sqrt{3}}{2}=5.2$ miles/hr

Alternative solution :

Suppose the velocity v_2 of the boat makes an angle of θ with the direction of the velocity v_1 of the stream. Then the direction of the resultant is given by the relation

$$\tan\beta=\frac{v_2\ \sin\ \theta}{v_1+v_2\cos\theta}$$

B is given to be 90°

$$\therefore \tan\beta=\tan\ 90^\circ=\frac{v_2\ \sin\ \theta}{v_1+v_2\cos\theta}=\text{as. }\infty$$

Or $v_1+v_2\cos\theta=0,$

Or $\cos\theta=-\frac{v_1}{v_2}=\ -\frac{3}{6}=-\frac{1}{2}$

$\therefore\ \theta=120.$

The boat should be rowed in a direction making an angle of 120° with the direction of the stream.

$$\text{Resultant}=\sqrt{v_1^2+v_2^2+2\ v_1\ v_2\cos\theta}$$
$$=\sqrt{3^2+6^2+2\times3\times6\times\cos 120}$$
$$=\sqrt{9+36-18}=\sqrt{27}=5.2 \text{ miles/hr.}$$

5. A car is running at 30 miles per hour in the direction 40° east of north. Find the components of its velocity in the north and east directions.

30 cos 40° = 22.98 miles per hour towards north.
30 sin 40° = 19.28 miles per hour towards east.

6. A man can swim 5 miles an hour in still water. If a river be flowing with a speed of 3 m. p. h, find the direction in which he must strike in order that his resultant motion be perpendicular to the flow of water. What will be his resultant velocity in this direction.

(i) upstream making an angle of 53.1° with the bank.
(ii) 4 m. p. h.

7. A man is running in rain with a velocity of 8 miles/hr. If the rain be falling vertically with a velocity of 10 miles/hr, find the relative velocity of the rain with respect to the man.

12.8 miles/hr. Rain strikes him from front making an angle $\tan^{-1}\frac{4}{5}$ (38.7°) with the vertical.

8. Two ships A and B start together from the same port. A sails 25 m. p. h. towards east. B sails 30 miles per hour due north east. Find the distance between them after two hours. 43.1 miles.

9. A man is driving his car due east with a velocity of 30 miles per hour. The wind appears to strike him from north with a velocity of 20 miles per hour. Determine the true velocity of the wind with respect to the earth.

Relative velocity of wind=20 miles/hr.=OB in figure.

If OA in fig 3.17 represents a velocity of 30 miles due

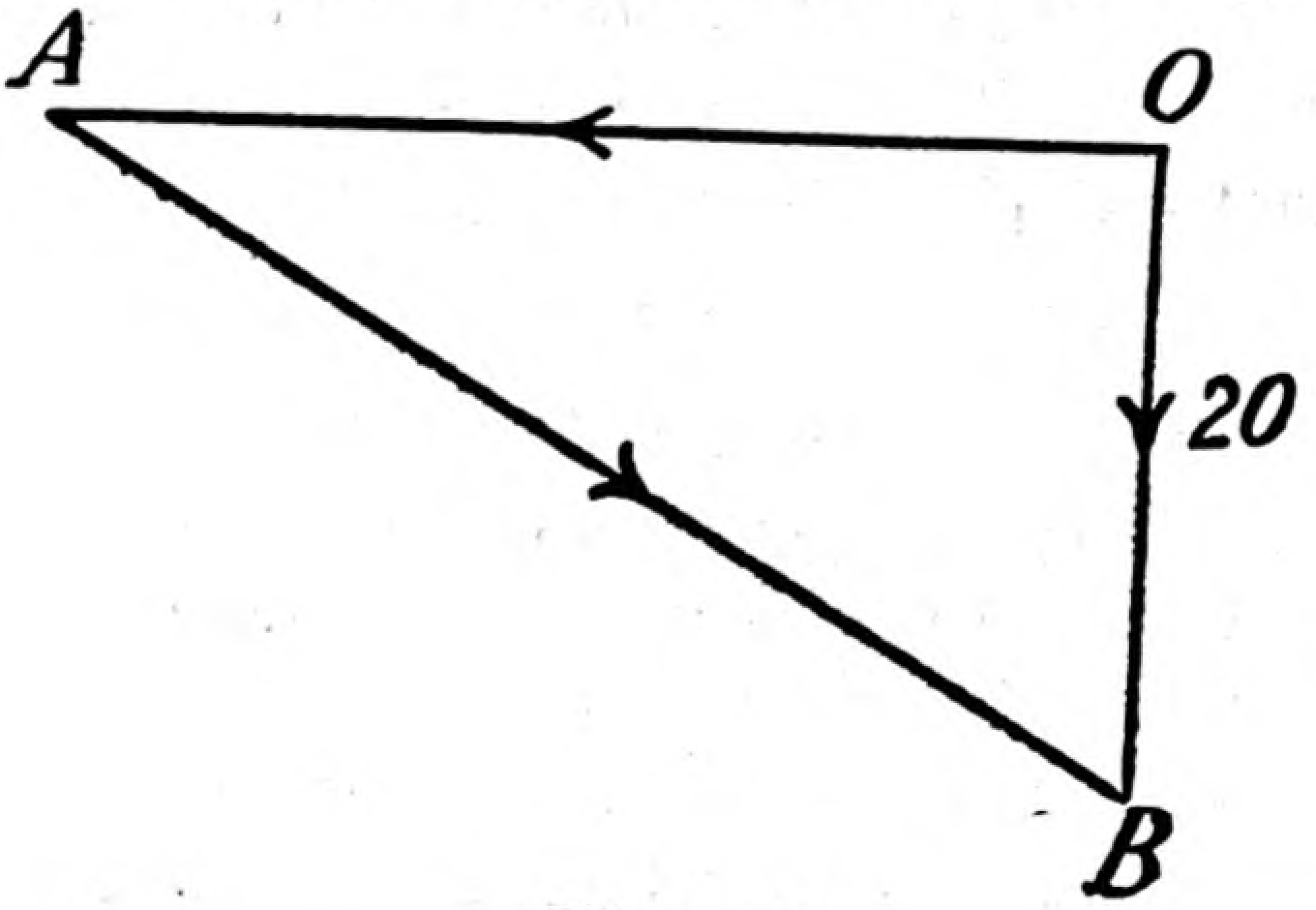


Fig 3.17

west, that is opposite to the velocity of the car, then OB is the resultant of OA and the true velocity of the wind. Join AB. From the triangle of velocities OA and AB are the two components of the resultant OB. Therefore AB is the true velocity of the wind.

$$AB=\sqrt{OA^2+OB^2}=\sqrt{30^2+20^2}=36 \text{ m. p. h.}$$

$$\tan B=\frac{OA}{OB}=\frac{30}{20}=1.5 \qquad B=56.3^\circ$$

So the wind is blowing from a direction 56.3° west of north.

10. An aeroplane is flying due north with a velocity of 80 miles per hour. If the wind be blowing from east at 20 miles an hour, find the resultant velocity of the aeroplane.

82.5 m. p. h. in the direction 14° west of north.

11. If the three sides of a triangle taken in order represent in magnitude and direction the simultaneous velocities of a particle, prove that the particle will remain at rest.

12. An aeroplane, which is flying in a horizontal direction at 120 m. p. h. drops a bomb from a height of 400 feet. If the resistance of air be neglected, what path will the bomb follow? What time will it take to reach the ground? What horizontal distance will it travel before striking the ground? With what velocity will it strike the ground? (*a*) parabola. (*b*) 5 seconds (*c*) 880 ft. (*d*) 237.8 ft/sec.

13. A bullet is fired in a horizontal direction from the top of a tower 100 feet high, with a velocity of 700 ft/sec. Calculate the distance of the point from the base of the tower, where the bullet will strike the ground. 1750 feet.

CHAPTER IV

MOMENTUM AND FORCE.

1. Force.

We are all familiar with the common ideas of a force. When we lift a weight, or push a cart, or pull a rope we are exerting a force. If a force is applied to a body, which is free to move, it produces motion in that body, as also a force is needed when a moving body is to be stopped. Quite often one or more forces may act on a body without producing any motion in it. In such cases, there are present other forces, which tend to cancel the effect of the applied force, and the body remains in equilibrium.

Force is defined scientifically as a cause, which when acting on a body, changes or tends to change its state of rest or uniform motion in a straight line. This implies that if no force were acting on a moving body it will continue to remain at rest or move with a uniform and unchanging velocity in a straight line.

2. Momentum.

This is a property possessed by a material body on account of its mass and its velocity. To drive a nail we strike at it with a hammer, and to make the blow effective, the hammer is given a good speed as it strikes. In this case the mass of the hammer as well as its velocity are needed. These quantities determine the momentum.

The momentum of a body is defined as the product of its mass and its velocity.

$$M = m.v$$

If the velocity of a body changes, its momentum also undergoes a corresponding change. The physical principles governing the changes in velocity of bodies, and the forces acting on them have been the subject of investigation for many centuries, but the results of these investigations have been summed up by Newton (1642-1727) in his famous laws of motion. These laws are the foundation of Dynamics, which deals with the motion of bodies and the forces producing that motion.

3. Newton's Laws of motion :

1st Law :—Every body continues in its state of rest, or uniform motion in a straight line, unless it is acted upon by some impressed force.

2nd Law :—Change of momentum per unit time, that is, the rate of change of momentum of the moving body is proportional to the impressed force, and takes place in the direction along which the force acts.

3rd Law :—Action and reaction are always equal and opposite.

The discovery of the first two laws was made by Galileo towards the end of the sixteenth century, and the underlying idea of the third law, was also known before the time of Newton, but he was the first to state the three laws in the above form in his important book 'PRINCIPIA', or Principles of Natural Philosophy, in 1686.

According to the first law if no external force or forces act on a body, then, it remains at rest or if it is moving, it continues to move with a uniform velocity in a straight line. Whereas, it will be obvious to the student that a body which is at rest continues to remain in this state, and to make it move, some force must act upon it, but it is not obvious, in the first instance, how a body moving with a uniform velocity will always continue to move with the same velocity, if no forces are acting upon it. Our ordinary experience is, that if a moving body is left to itself it slows down in motion till it comes to rest. We have to remember that an ordinary moving body is acted upon by opposing forces, such as the force of friction of the surface on which it moves, and the resistance of the air which has to be displaced out of its path during motion. If the force of friction is reduced, such as on a very smooth surface, we find that a body moves much farther than on a rough surface, and we conclude that if all friction and air resistance were done away with, a moving body will continue to move without changing its velocity for an indefinite time.

The property of the inability of a material body to change its state of rest or uniform motion is called INERTIA.

We easily notice that, heavier is the body, the more difficult it is to change its state of rest or uniform motion and we, therefore, find that inertia is proportional to the mass of a body.

According to the second law of motion, the impressed force produces a change in the momentum of a body, the rate of change of momentum being proportional to the force. If a force F acts on a body of mass m for time t and as a result changes its velocity from v_1 to v_2, then the change in momentum is (mv_2-mv_1). The mass of the body is supposed not to change with its velocity. Recent views, however, are that at extremely high velocities of say a hundred thousand miles per second or more, the mass of a body increases with increase of velocity. This view is put forth by the great modern theoretical physicist, Einstein, in his Theory of Relativity. For common velocities, as we meet with every day experience, however, there is no change in the mass depending on velocity. We, therefore, express the second law as

$$\mathrm{F} \propto \frac{mv_2-mv_1}{t}$$

$$\propto m\frac{(v_2-v_1)}{t}$$

$$\propto m.\,a.$$

or $$\mathrm{F}=k.m.a. \qquad \ldots \quad (1)$$

The proportionality, $\mathrm{F} \propto m.a$, is changed into an equation by multiplying with a constant, k, called the constant of proportionality.

4. Units of Force.

Equation (1) is the basis of the scientific measurement of a force. We have already seen, that the unit for mass of a body is a gram in the C. G. S. system, and a pound in the F. P. S. system; and the unit of acceleration in these systems is 1 centimeter per second per second, and one foot per second per second.

If now we choose a unit of force, as that force which when applied to a mass of 1 gram, produces in it an acceleration of 1 centimeter per second per second, then equation (1) will be written as

$1 = k.1.1$, and it is obvious that the value of k must equal 1 for the equation to be satisfied. This unit of force is called a DYNE

The unit of force on F.P.S. system is one, which when acting on a mass of one pound, produces in it an acceleration of 1 foot per second per second. We see that the constant of proportionality k is again equal to unity. This unit of force on the F.P.S. system is called a POUNDAL.

If, therefore, we use either a dyne or a poundal as the unit of force, equation (I) can be written as $F = m.\ a$....(II)

This important equation is the mathematical expression of the second law of motion.

It has been referred in the first chapter that in addition to the C. G. S. and F. P. S. systems of units another system called m. k. s. system having the meter, the kilogram, and the second as the units of length, mass, and time is also coming into use, on the m. k. s. system, the unit of force is that force which when acting on a mass of 1 kilogram produces in it an acceleration of 1 meter per second per second. This unit is called a Newton, in honour of the greatest scientific genius.

Since 1 kilogram equals 1000 grams, and 1 meter equals 100 centimeters, it follows that 1 Newton $= 10^5$ dynes.

The dyne and the poundal are called absolute units of force.

5. Gravitational units of Force :

The force, which the earth exerts on a mass of 1 gram, equals g dynes, because if the mass is allowed to fall, it possesses an acceleration of g cms. per second per second. This force is called 1 gram weight. Taking the average value of g as 980 centimeters per second per second, it is seen that 1 gram's weight equals 980 dynes.

Similarly the force, with which a mass of 1 pound is pulled by the earth, is called 1 pound weight.

A force of 1 pound's weight produces an acceleration of 32.2 feet per second per second in a mass of 1 pound and, therefore, 1 pound's weight=32.2 poundals. The gram weight and the pound weight are also units of force, and their multiples, as a kilogram weight or a ton's weight, are used by engineers, and are called gravitational units of force.

The gram weight and the pound weight vary slightly at different places, because the value of g changes. The acceleration of gravity has a value of 978 centimeters per second per second at the equator and 983 centimeters per second per second at the poles. But for purposes of calculation the average values above given are used.

6. Impulse :

If a force F acts for a short time t, the change in the momentum produced is given by the second law. Since $\mathrm{F}=\dfrac{mv_2-mv_1}{t}$, we have $\mathrm{F}.\,t=mv_2-mv_1$. The product F. t is called the impulse of the force F. As an example, when a hammer strikes on the head of a nail, its impulse is used in driving the nail into the wooden block. Similarly when a cricketer catches a fast cricket ball, the momentum of ball is stopped in a short time, and the ball exerts an impulse on the hands of the cricketer.

Example : A hammer weighing 2 lbs and moving 16 feet per second strikes against a nail. Find the impulse given ?

The change of momentum is $2 \times 16 = 32$ units. Since this equals F. t, the units for impulse are poundals seconds. Dividing by 32.2 we get pounds seconds.

$$\text{Therefore, impulse is} = \frac{32}{32.2} = 0.99 \text{ pounds seconds.}$$

7. Conservation of Momentum :

This principle also follows from the laws of motion. In fact it is inherent in these laws. According to this principle, the total momentum of a system of bodies remains unchanged as a result of impact or interaction between the bodies in the system, if no other external force is acting on them. A special and important case is that of the firing of a gun or rifle. Before the explosion of the gun powder, the gun and the bullet are at rest. When the powder explodes, a very strong pressure is exerted on the bullet, which escapes from the barrel in the forward direction with a high velocity and possessing a momentum m_1v_1, where m_1 is the mass of the bullet and v_1 its velocity of escape. The gun recoils in the backward direction with such a velocity v_2 that the backward momentum m_2v_2, where m_2 is the mass of the gun equals the forward momentum of the bullet and, therefore, the total momentum of the system, rifle plus bullet, remains unchanged after the firing.

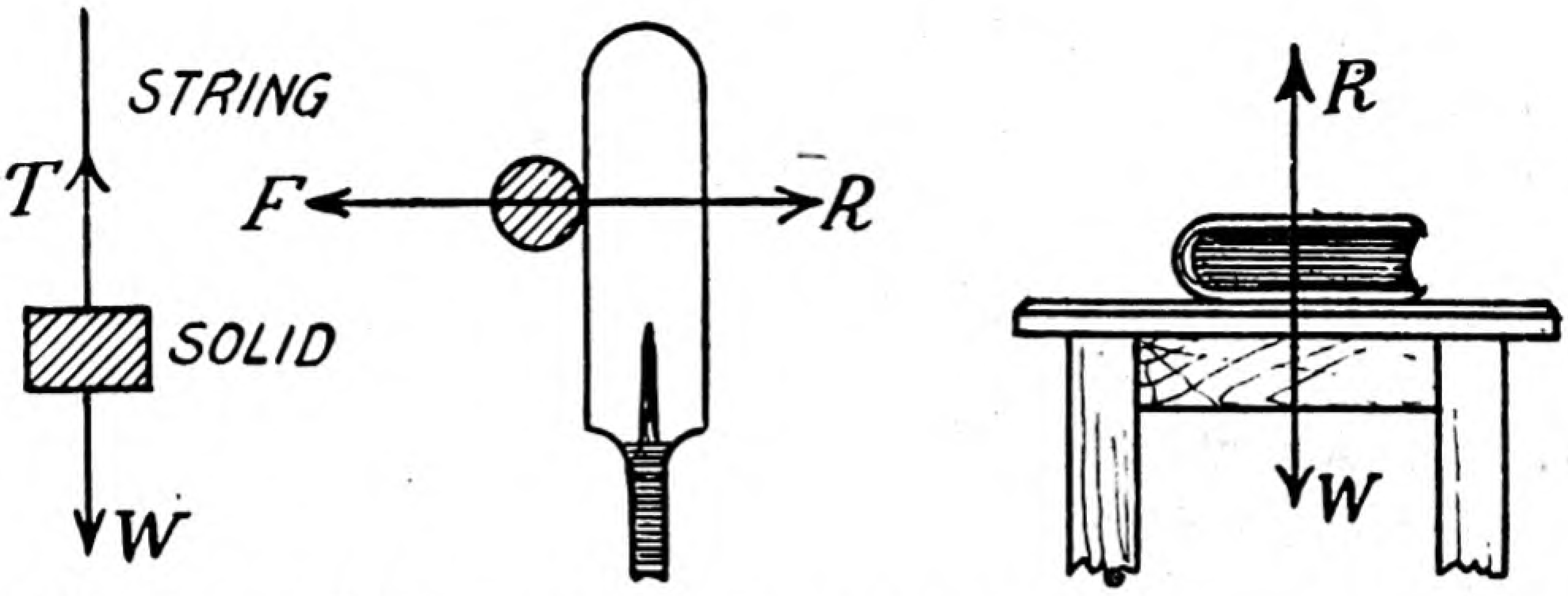
Example : A rifle which weighs 2 kilograms fires a bullet of mass 20 grams with a forward speed of 1 kilometer per second. What is the speed of recoil of the rifle ?

Suppose v is the speed of recoil. Then $2000 \times v$ is the backward momentum of the rifle, and this must be $= 20 \times 1000$, which is the forward momentum of the bullet. Equating these we have $v = 10$ centimeters per second.

Care should be taken that all quantities are expressed in similar units. In the above case we have expressed mass grams and velocities in centimeter per second.

8. Third Law of motion :

The forces called action and reaction in this law, refer to forces acting between two different bodies at their point



W, the weight of the solid is the force of action of the solid on the string. T, the tension is the force of reaction of the string on the solid. W=T

F is the force of action of bat on the ball. R is the force of reaction of ball on the bat. F=R.

W, weight of the book is the force of action of the book on the table. R is the force of reaction of the table on the book. W=R.

Fig 4.1

of contact figure 4.1. If a solid body is suspended from a string which is tied at the upper end to a support, the body is pulled downward by the earth, and it exerts a downward force equal to its weight on the string. A tension is produced in the string which pulls the solid body upward with an equal force to that, with which the string is pulled downward. These two forces we can call as action and reaction, which act in a pair between two bodies. Similarly a book resting on a table presses on the table with its weight and the table exerts an equal upward thrust upon it. When a cricketer strikes the ball with his bat, the bat exerts a force on the ball, and the ball exerts an equal force on the bat. The ball is driven forward with a large velocity, while the bat is slowed down as a result of the force acting upon it. When a man jumps from the ground, the action and reaction between the man and the ground are equal, but while man springs forward by a sufficient distance, the earth is displaced by a negligible amount, because of the great mass it possesses compared to that of the man.

9. First and Third Laws are only special cases of the second Law of Motion.

It obviously follows from the equation, $F=m.a.$, that if no impressed force is acting on the body, i. e., if $F=0$, then a or acceleration must be $=0$. This means that in the absence of an applied force a body continues in its state of rest or uniform motion, which is the first law of motion. The first law, therefore, is contained in the second law and is only a special case of the second law.

The third law can also be considered as a special case of the second law if we look upon the two bodies, between which action and reaction take place, as a single system. If no external force acts on this system then the momentum of the system should remain constant. It, therefore, follows that if one body of the system presses the other body with a force F which we call action, the other body would press the first one with an equal and opposite force F, called the reaction. If the reaction were not equal to action, then the internal force F of the system will alter the momentum of the whole system without any external force, which is against the second law.

Let us consider the gun and the bullet as one system. The momentum of the system is zero when the gun is at rest before firing the bullet. The momentum of the system should remain zero as long as no external force acts on it. Due to the internal force (explosion) of the system, the ignition of gunpowder pushes the bullet forward. In order that algebraic sum of the momenta of the system (gun and bullet) should remain zero, the gun must be pushed back with an equal force making action and reaction in the system equal and opposite. Similarly if the bat and the ball are considered as one system, the momentum of the system before and after hitting the ball should remain constant, since no external force acts on the system. Therefore the action of the bat imparts momentum to the ball, and an equal and opposite reaction of the ball diminishes the momentum of the bat by an equal amount so that there is no change in the momentum of the system as a whole.

Summary

Momentum of a body is equal to the product of its mass and velocity.

Momentum, $M = mv$.

Laws of motion.

First Law. Every body continues in its state of rest or of uniform motion in straight line, unless it is acted upon by some impressed force. This law is also called the Law of Inertia.

Second Law. The rate of change of momentum of a body is proportional to the force impressed upon it, and takes place in the direction along which the force acts.

Third Law. Action and reaction are always equal and opposite.

Dyne is the unit of force in the metric system. It is the force which, when it acts on a mass of one gram, produces an acceleration of one centimeter per second per second.

Force of one gram wt $= g$ dynes $= 980$ dynes.

Newton is the force that produces an acceleration of one meter per second per second in a mass of one kilogram

Newton $= 10^5$ dynes

Unit of force in the F.P.S. system is a poundal. It is the force which, acting on a mass of one pound, produces in it an acceleration of one foot per second per second.

Force of one pound wt. $= g$ poundal $= 32$ poundals.

Impulse of a force $=$ Force $\times$ time $= F \times t$

$=$ change of momentum due to the force.

$= m(v_2 - v_1)$.

Principle of Conservation of Momentum. The total amount of momentum of a system of bodies remains unchanged by impact or interaction between the bodies if no external force acts on the body.

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Exercises

1. What is the momentum of a 100 lbs shell, if its speed is 2000 ft. per second?

2×10^5 lbs ft./sec.

2. A body of mass 30 lbs is acted upon by a force of 8 pounds weight for 6 seconds. How much distance has the body moved from its initial position of rest? What is the final momentum?

$$\text{Force} = 8 \times 32 = 256 \text{ lbs} = ma = 30a.$$

$$\therefore \quad a = \tfrac{256}{30} = \tfrac{128}{15} = 8{\cdot}53 \text{ ft/sec}^2$$

$$\text{Distance} \quad S = v_1 t + \tfrac{1}{2} a t^2$$

$$v_1 = 0 \text{ and } t = 6 \text{ sec.}$$

$$\therefore \quad S = \frac{128 \times 36}{2 \times 15} = \mathbf{153{\cdot}6 \text{ feet.}}$$

$$\text{Final velocity after 6 seconds} = a\,t = \frac{128 \times 6}{15} = v_2$$

$$\therefore \text{ Final momentum} \quad = m.v_2 = \frac{128 \times 6 \times 30}{15}$$

$$= \mathbf{1536 \text{ lbs ft./sec.}}$$

3. Determine the force which must be applied to an automobile weighing one ton, and travelling at the rate of 30 *m.p.h.*, in order to bring it to rest in a distance of 20 yards?

$$v_2^2 - v_1^2 = 2\,a\,s$$

$$v_2 = 0,\ v_1 = 44 \text{ ft./sec. and } s = 60 \text{ ft.}$$

$$\therefore \ -44 \times 44 = 2 \times 60 \times a$$

$$\text{or} \qquad a = -\frac{44 \times 44}{120}$$

Negative sign only shows that acceleration is opposed to the direction of motion.

$$\text{Force} = ma = \frac{2240 \times 44 \times 44}{120} \text{ lbs}$$

$$=\frac{2240\times44\times44}{120\times32}=1129{\cdot}3 \text{ lbs wt.}$$

4. A motor car weighing 400 lbs is moving with a velocity of 30 miles per hour. By the application of brakes it is brought to rest in a distance of 40 feet. Find the average force resisting the motion. (*Del. U.*) 302·5 lbs wt.

5. A constant force acts for 3 seconds on a mass of 16 lbs., and then ceases to act. During the next three seconds the body describes 81 feet. Find the magnitude of the force in poundals. (*P. U.*)

The body moves with a uniform speed after the force has ceased to act.

$\therefore$ Speed generated by the force in 3 seconds $=\frac{81}{3}$

$=27$ ft/sec.

$\therefore$ Momentum generated in three seconds $=16\times27$.

Hence Force = Rate of change of momentum $=\frac{16\times27}{3}$

$=144$ lbs. $=4.5$ lbs wt.

6. Travelling with a speed of 500 meters per second, a bullet, the mass of which is 25 gms., hits the side of a mound and penetrates to a depth of 5 meters. Determine (*i*) the time taken by the bullet in stopping, (*ii*) the average force exerted against it.

$$\text{Average speed}=\frac{v_1+v_2}{2}=\frac{500+0}{2}=250 \text{ m./sec.}$$

$$\therefore \quad \text{Time}=\tfrac{5}{250}=\tfrac{1}{50} \text{ sec.}$$

change of mementum = Impulse = Force × time.

$$\therefore \quad m\,(v_1-0)=F\times t.$$

$$25\times50000=F\times\tfrac{1}{50}.$$

$$F=25\times50000\times50=625\times10^5 \text{ dynes.}$$

7. If a force of 200 pounds weight acts on a car for a period of 5 seconds and starting from rest produces in it a velocity of 10 miles per hour. Determine the mass of the car. 2181·8 lbs.

8. A bullet is fired from a gun with a speed of 400 meters per second. If the mass of the bullet and the gun are respectively 10 grams and 5 Kilograms, determine the speed of recoil of the gun. 80 cm./sec.

9. State Newton's laws of motion.

بندوق کی گولی۔

A projectile weighing 560 lbs., is fired from a gun weighing 40 tons, with a velocity of 1600 ft. per sec. Find the velocity of recoil. 10 ft./sec.

10. A body of mass 300 grams is at rest. What force will you apply to the body to move it through 200 cm. in 10 seconds? 1200 dynes.

11. State the principle of conservation of momentum. How is it deduced from Newton's third law of motion. State the relation for the recoil of gun.

A force acts for 8 seconds on a body of mass 100 lbs. after which the force ceases and the body describes 80 feet in the next 5 seconds. Find the magnitude of the force.

(*P. U. 1950.*) 200 lbs.

12. Two men jump from a roof. One of them lands in sand while the other falls on a concrete floor. Which of them is likely to be injured more?

13. A man, weighing 160 lbs., is sitting in a lift. What will be his apparent weight (*a*) when the lift is going up with an acceleration of 8 ft./sec.^{2}, (*b*) when the lift is going down with an acceleration of 8 ft./sec.^{2}, (*c*) when the lift is going down with an acceleration of 32 ft./sec.^{2} and (*d*) when the lift is going up or down with a uniform velocity?

The man will press the lift with a force equal to his apparent weight and by Newton's third law, the reaction R of the floor of the lift will be equal to the apparent weight of the man.

mass of the man $= m = 160$ lbs.

(*a*) Two forces act on the man, the reaction R poundals upward and pull of the gravity mg poundals downwards fig. 4·2. The resultant of these two forces produces an acceleration of 8 ft./sec.^{2} in the upward direction.

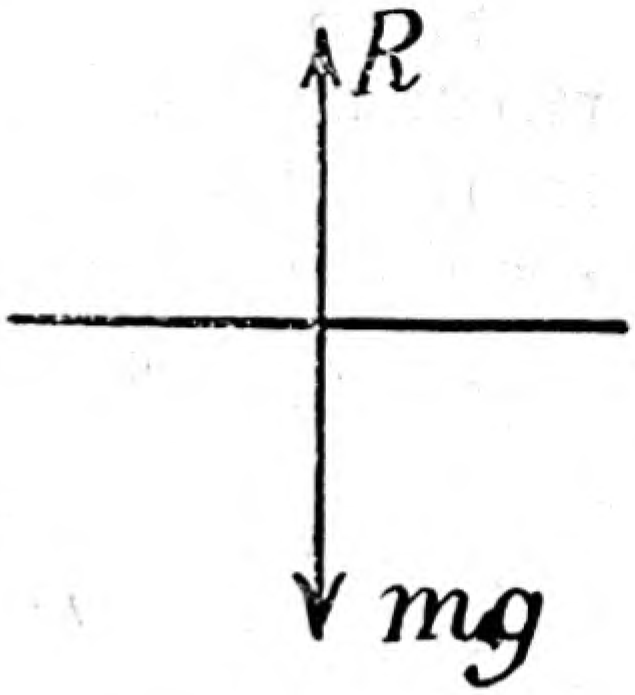


Fig. 4.2

$\therefore \quad R - mg = ma = 8m$

or $\quad R = m(g+a) = 160(32+8)$

$= 160 \times 40$ lbs. $\quad \overset{5}{\cancel{16}}$

$\therefore$ apparent weight$= \dfrac{16\cancel{0} \times 40}{\underset{2}{3\cancel{2}}} = 200$ lbs. wt.

(*b*) In this case the resultant of R and mg produces an acceleration of 8 ft./sec.^{2} in the downward direction.

$\therefore \quad mg - R = ma = 8m.$

or $\quad R = m(g-a) = 160(32-8)$

$= 160 \times 24$ lbs.

$\therefore$ apparent weight$= \dfrac{160 \times 24}{32} = 120$ lbs.

(*c*) The resultant of R and mg in this case gives an acceleration of 32 ft./sec.^{2} in the downward direction.

$\therefore \quad mg - R = 32\,m = mg$

$\therefore \quad R = 0.$

$\therefore$ Apparent weight$=0.$

(*d*) When the lift moves with a uniform velocity, there is no resultant.

$\therefore \quad R - mg = 0$

$R = mg.$

$\therefore$ apparent weight$=160$ lbs.

14. A lift is going up with an acceleration of 10 ft./sec^2. What will be the apparent weight of a 20 lb. body, suspended from a spring balance in the lift?

26.25 lbs. wt.

15. Find the apparent weight of a man, weighing 140 lbs. on earth, when he is sitting in a lift that is

(*a*) going up with an acceleration of 6 ft./sec.^{2}

(*b*) going down with an acceleration of 6 ft./sec.^{2}

(*c*) falling freely under the action of gravity.

(*d*) going up or down with a uniform velocity.

(*a*) 166·25 lbs. (*b*) 166.75 lbs. (*c*) 0, (*d*) 140 lbs.

CHAPTER V

STATICS AND EQUILIBRIUM OF BODIES

1. Definition.

Statics is that branch of mechanics in which we deal with forces in a state of equilibrium with each other. Since the forces are in equilibrium, their resultant is zero and the body acted upon by the forces remains at rest. It is a much older subject than dynamics, because the builders of ancient buildings, bridges, and structures must have had some knowledge of Statics as the basis of durable structure work.

Force being a vector quantity, the combined effect of two or more forces is found by the laws of addition of vectors. The laws of the parallelogram, the triangle, and the polygon of vectors have already been given and all apply to the case of forces.

2. Experimental verification of the laws of parallelogram and triangle of forces :—

A drawing board is set up in a vertical position by suitable clamps. Two pulleys are fixed near the two upper corners of the board. A strong string is taken and passed over the groove in the pulleys and small pans attached to the ends of the string. Another piece of string is tied to the middle point of the strings which pass over the pulleys, and a similar pan tied to its end. By placing suitable weights in the three pans the knotted point is brought somewhere as shown in the fig. (5.1), so that the three forces are acting along each arm of the string and these forces meet at the point o. Care should be taken, that the pans are hanging freely without touching the board, and the pulleys have as little friction as possible. A sheet of paper of sufficient size is now fixed by drawing pins to the board.

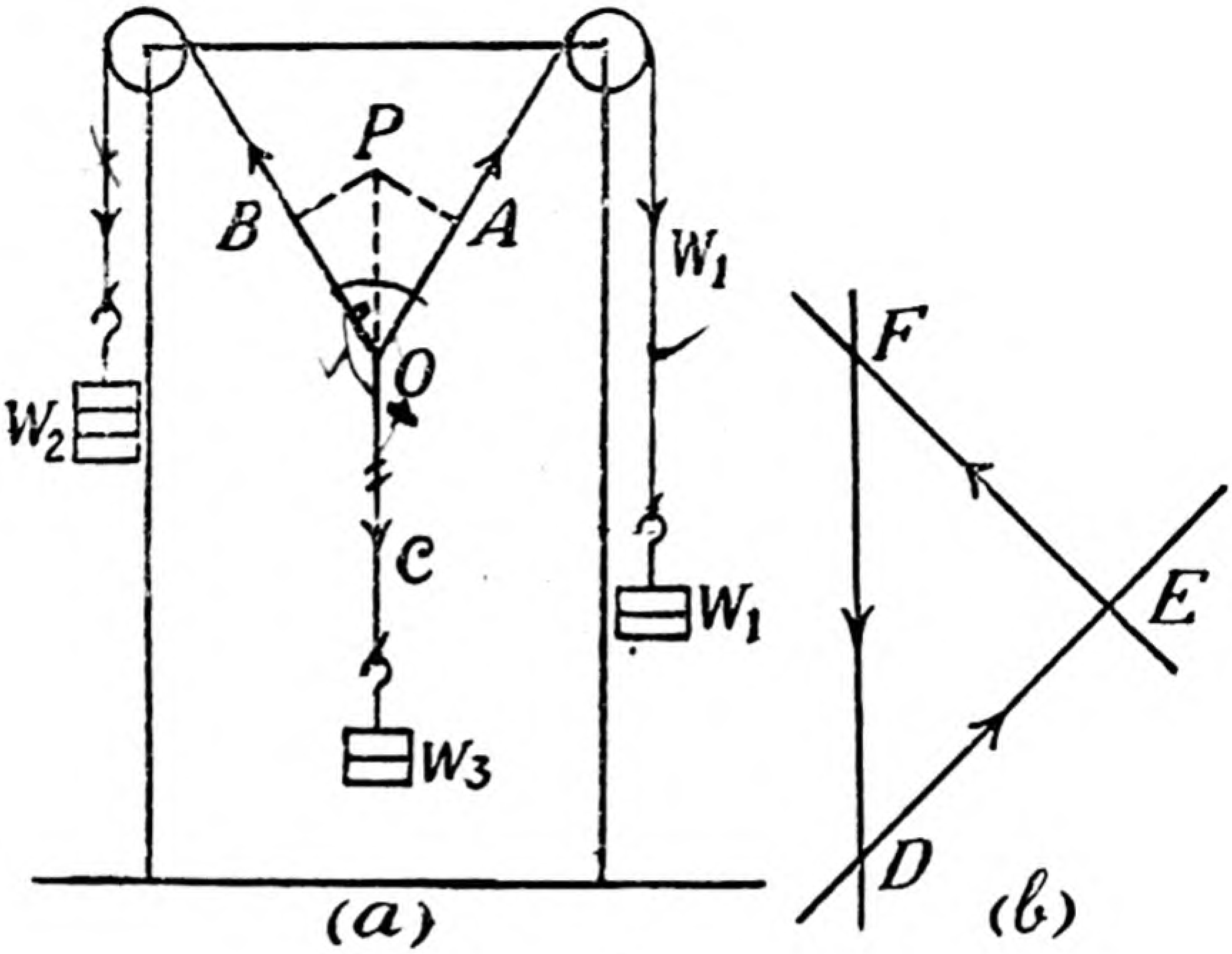


Fig. 5.1

The three forces meeting at the knotted point O possess magnitudes equal to the weights including that of the pan, suspended from the ends of the three strings. The direction of the strings is marked on the paper by suitable means. A glass plate will be found useful when marking the direction. Having marked the direction of the strings the three weights W_1, W_2 and W_3 suspended from their ends, are measured with a reliable spring balance. The apparatus is now dismantled, and a vector diagram for the three forces in equilibrium is drawn on the paper.

The three lines which have been marked are produced to meet at the common point O. A suitable scale is chosen for representing the three forces, by appropriate lengths of the three straight lines, OA representing W_1, OB representing W_2, and OC representing W_3. From the point A, a straight line is drawn parallel to OB, and from the point B another straight line parallel to OA meeting in the point P to form a parallelogram OAPB. The scale chosen for representing the forces should be such, as to give a large parallelogram permissible by the size of the paper. Join

OP and measure its length. OP being the diagonal of the parallelogram is the resultant of the forces W_1, and W_2 represented by OA and OB. Convert the length OP into the corresponding force and the value will be found to be very nearly equal to the force W_3, and opposite in direction to OC. However, due to errors of experiment OC and OP will not be exactly opposite, but CO produced will be found to make a small angle with OP. The force W_3 is equal and opposite to the resultant of W_1 and W_2. We can call W_3 the EQUILIBRANT of the forces W_1 and W_2, because it is equal and opposite to their resultant.

It can also be verified that the force W_3 equals $\sqrt{W_1^2+W_2^2+2\ W_1\ W_2 \cos\angle AOB}$, but it is obviously opposite in direction to the resultant OP.

The resultant of two inclined forces acts on the piece of stone or pebble projected from a boy's sling. When the rubber bends are pulled, there is a force acting along the length of each band, and when the sling is released the resultant of the two forces throws the pebble forward with a high velocity, (fig. 5.2).

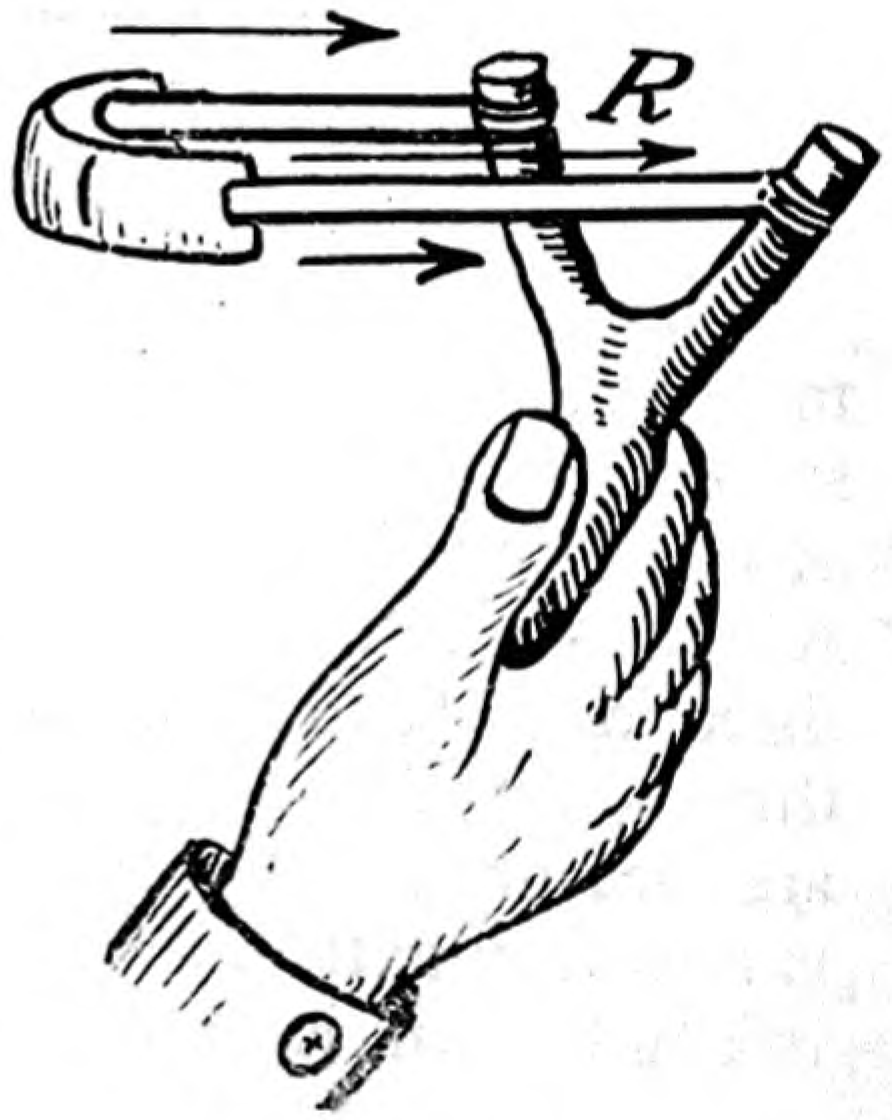


Fig. 5.2

3. Triangle of Forces.

On the same paper draw three straight lines DE, EF and FD respectively parallel to OA, OB and OC, enclosing between them a triangle DEF of a *fair size*. Measure each side of this triangle and show that the *ratio of each force to the length of the corresponding side of the triangle* is equal in all the three cases. That is

$$\frac{W_1}{DE}=\frac{W_2}{EF}=\frac{W_3}{FD}.$$

If arrows are drawn on the sides of the triangle in the direction of the forces, (as shown in fig 5.1b), the three arrows are all in the same order. This proves that whenever three forces acting on a particle are in equilibrium, they can be represented by the three sides of a triangle taken in order. This statement is called the law of triangle of forces.

4. Lami's Theorem.

If we measure angles AOB, BOC and COA in fig 5 (*a*) we can also show that

$$\frac{W_1}{\sin\angle BOC}=\frac{W_2}{\sin\angle COA}=\frac{W_3}{\sin\ \angle AOB}.$$

This is taken as a proof of the Lami's theorem which states that if three forces acting on a body keep it in equilibrium then each force is proportional to the sine of the angle between the other two forces.

5. Polygon of Forces.

The law of polygon of forces can also be verified by an experimental arrangement similar to that discussed in the last article. In this case instead of having three segments of the string from which weights are suspended, we can have four or five segments tied to the same point, and instead of two, more pulleys can be attached to the drawing board to enable the strings to pass over them for suspension of weights. This arrangement is illustrated in Fig. (5.3). Five forces f_1, f_2, f_3, f_4, f_5 are all acting on the common point O, and are in equilibrium. Draw straight lines AB, BC, CD, DE to represent on a suitable scale the magnitude and the direction of the forces f_1, f_2, f_3, and f_4. It will be found that the closing side EA of the polygon ABCDE is parallel to the force f_5, and its length is such as to represent this force f_5 on the same scale as chosen for other forces.

Thus the experiment proves the law of polygon of forces which may be stated that if a number of forces

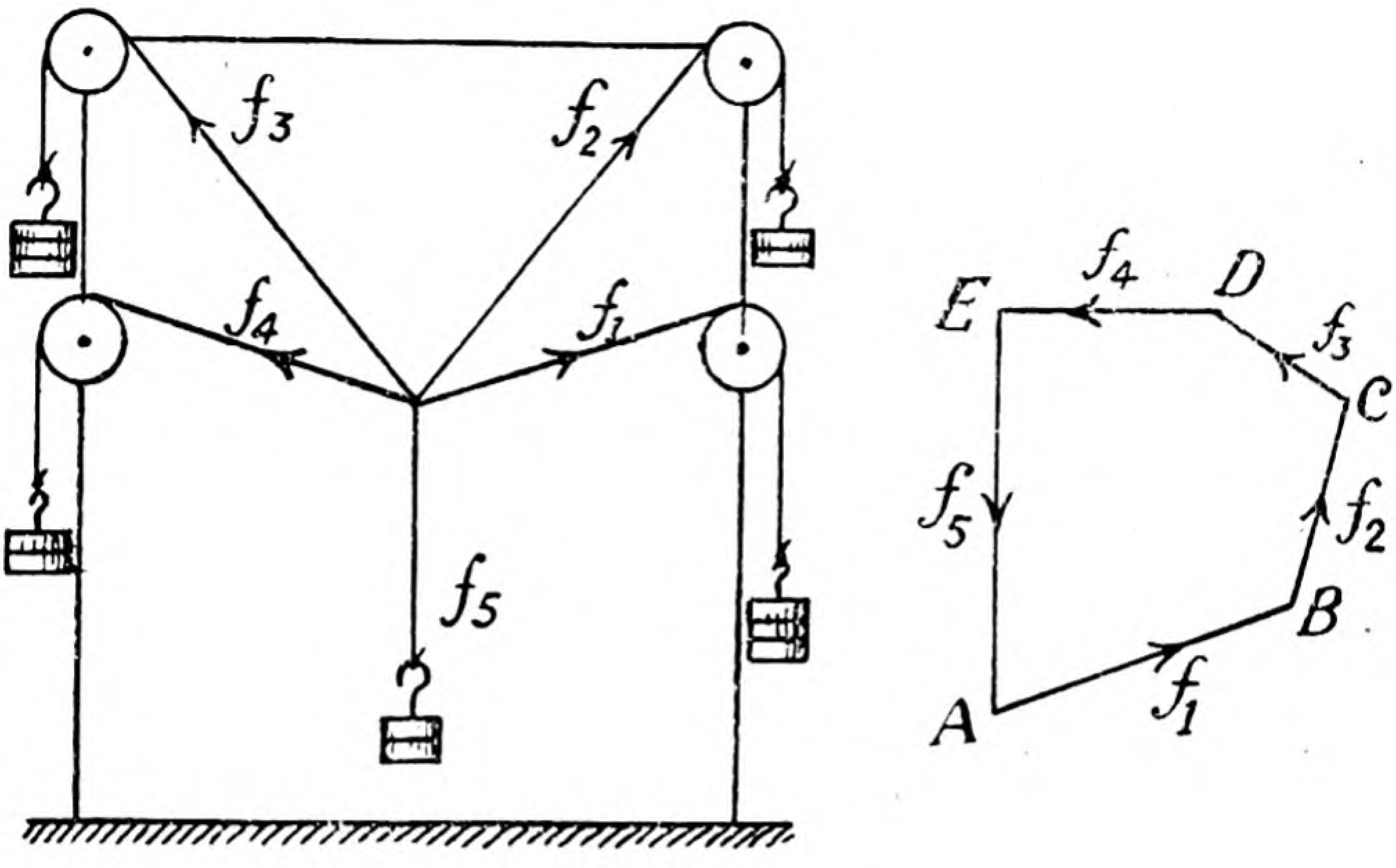


Fig. 5.3

acting on a particle, be in equilibrium, then they can be represented in magnitude and direction by the sides of a closed polygon taken in order. ABCDE is the polygon of forces, for the five forces f_1, f_2, f_3, f_4 and f_5 acting on the particle at O.

Since in this experiment the force f_5 is balancing the combined effects of the forces f_1, f_2, f_3 and f_4 acting at O, it is therefore equal and opposite to their resultant. This means that if forces f_1, f_2, f_3, and f_4 only were acting on the particle at O, their resultant would be given by the side AE (not EA) of the polygon ABCDE. Hence the law of the polygon of forces can also be stated in the following way. If a number of forces, acting simultaneously on a point O, are represented in magnitude and direction by the sides AB, BC, CD and DE of a polygon ABCDE taken in order, then the closing side AE taken in the opposite order, represents the resultant in magnitude and direction.

6. Resolution of Forces.

As explained in the last chapter, any vector quantity can be replaced, so far as its effect is concerned, by a pair of vectors, which will together produce the same effect and which are called the components of the vector. A force f acting on a

Ghulam Mohammad Chela

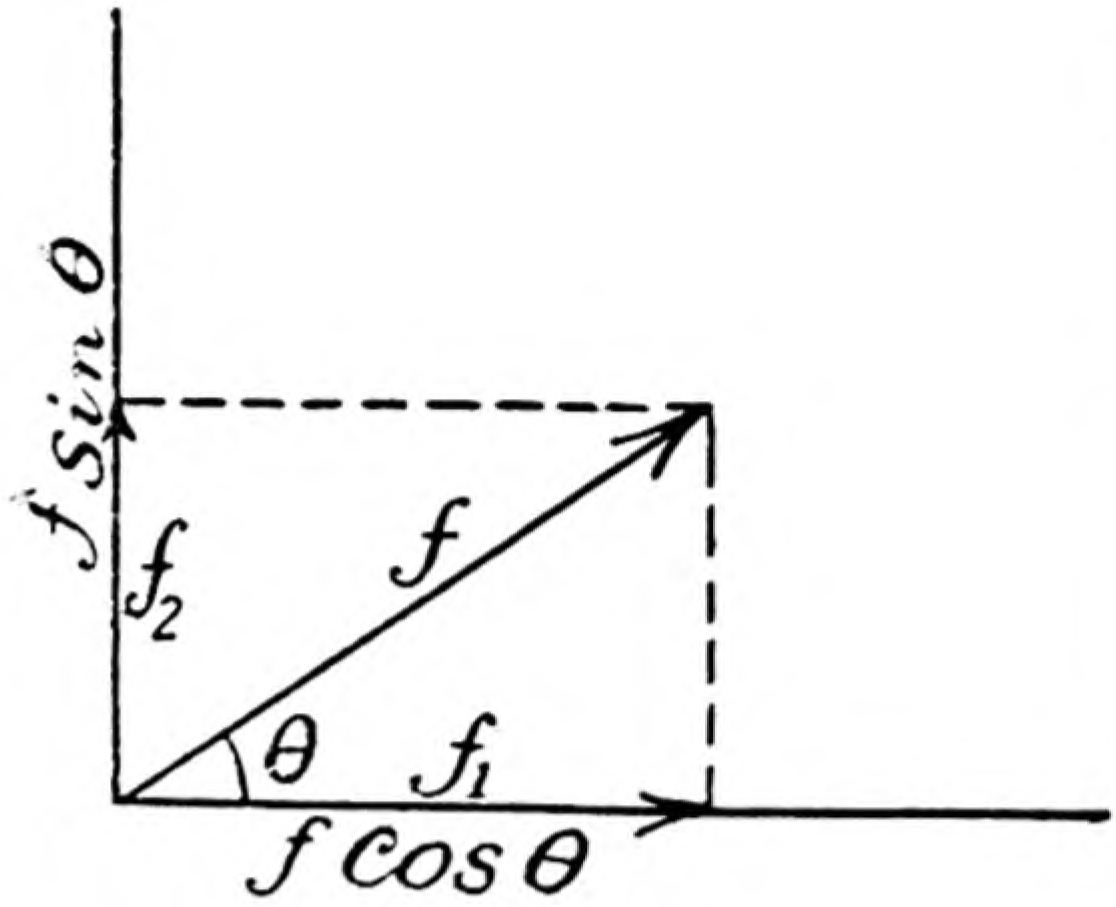


Fig. 5.4

body can be resolved into two mutually perpendicular directions, one of them making an angle θ with the direction of the force. The resolved component in this direction is $f_1 = f \cos \theta$, and perpendicular to this direction is $f_2 = f \sin \theta$ (fig 5.4). The resolution of a force into the components is very useful in analysing and understanding the effect of one or more forces upon a body. The following cases will help in understanding the application of this principle.

1. A boat is being towed through a canal or a river. Often when a large cargo is to be conveyed by a boat, instead of rowing the boat which requires much effort, the boat is towed by a horse or men moving on the bank and pulling on a rope fixed to the boat. In the fig. 5.5 OH is

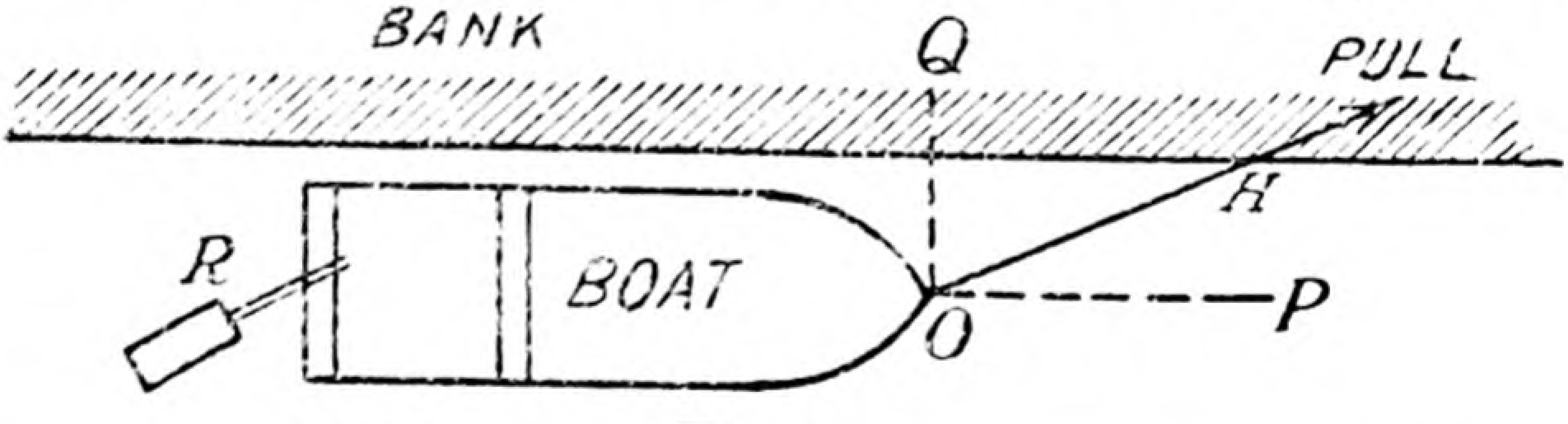


Fig. 5.5

the direction of the rope which is the direction in which pull is applied by the horse or the men. This pull may be resolved into two forces OP and OQ, the former parallel to the direction of the stream, and the latter at right angles to that. The effect of the force OQ, in moving the boat is small since it acts over the large sides of the boat, which presents considerable resistance in moving through water. This force is also counteracted by adjusting the steer R at a suitable angle. The forward force OP propells the laden boat through the water. This method is quite common in inland waterways.

2. *Action of a Lawn Mower* :—

Lawn mowers are used for clipping or mowing the grass so as to make it short and even. A man presses on the mower by means of a handle OM. The force is applied in the direction of the handle and can be resolved, (Fig. 5·6) into two components OP & OQ of which OP acts in the forward direction and moves the lawn mower, while OQ is acting in the downward direction and serves to increase the pressure exerted on the ground. If, however, the lawn mower is pulled by the handle, the force applied is now acting in the direction OM′ and resolved into OP′ and OQ′. Of these OP′ moves the lawn mower forward, while OQ′ tends to lessen the weight with which the mower is pressing on the ground. It will be noticed that it should be easier to work the lawn mower by pulling at it, than by pushing it with the handle. The same reasoning is applied in using a lawn roller. For a similar reason, when a prambulator is to be moved through a sandy surface, it is easier to pull it than to push it.

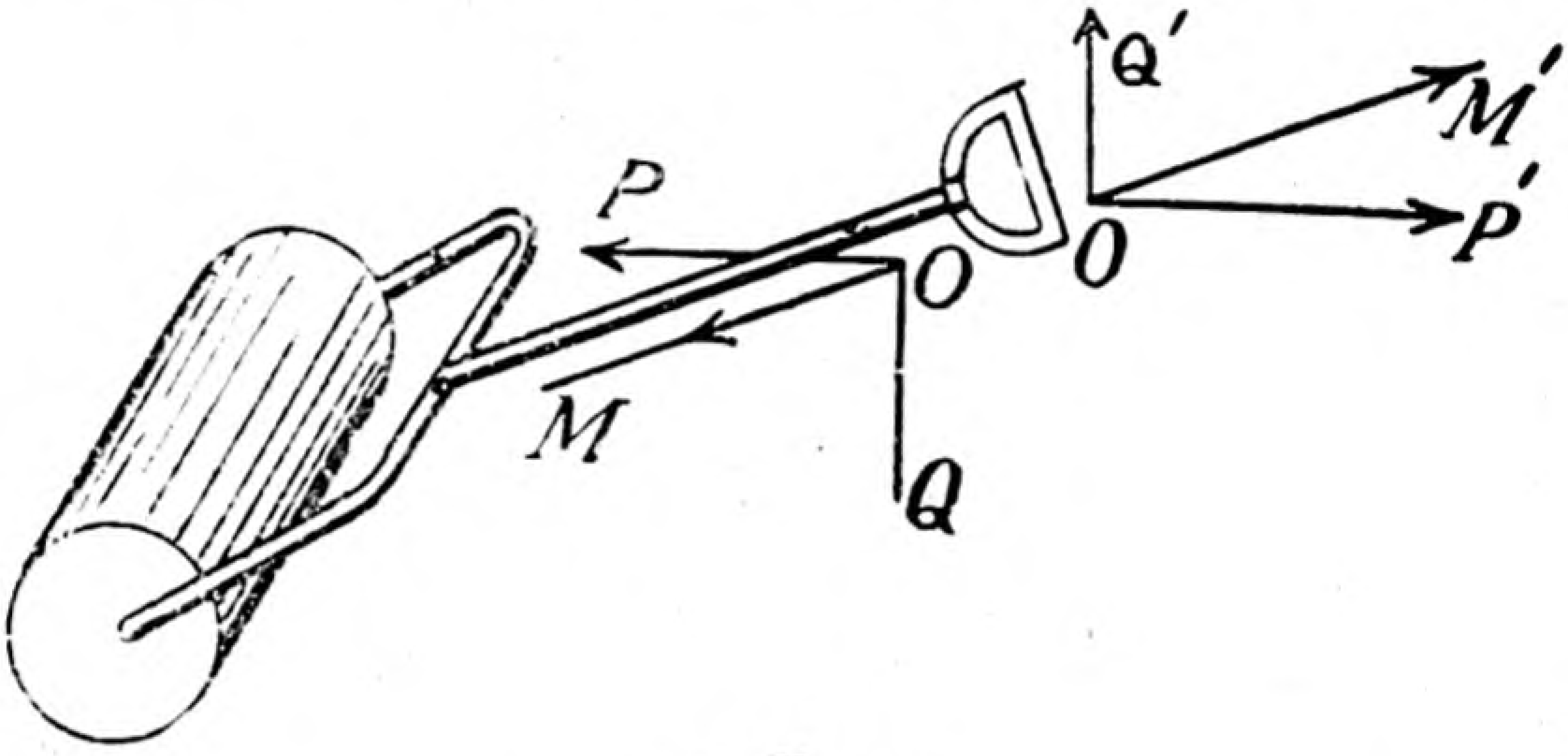


Fig. 5.6

7. Action of sails in Propelling Boats.

The force which wind exerts on the sails of a boat has been used for ages for driving boats on water. The wind strikes against the surface of the sails and produces a strong

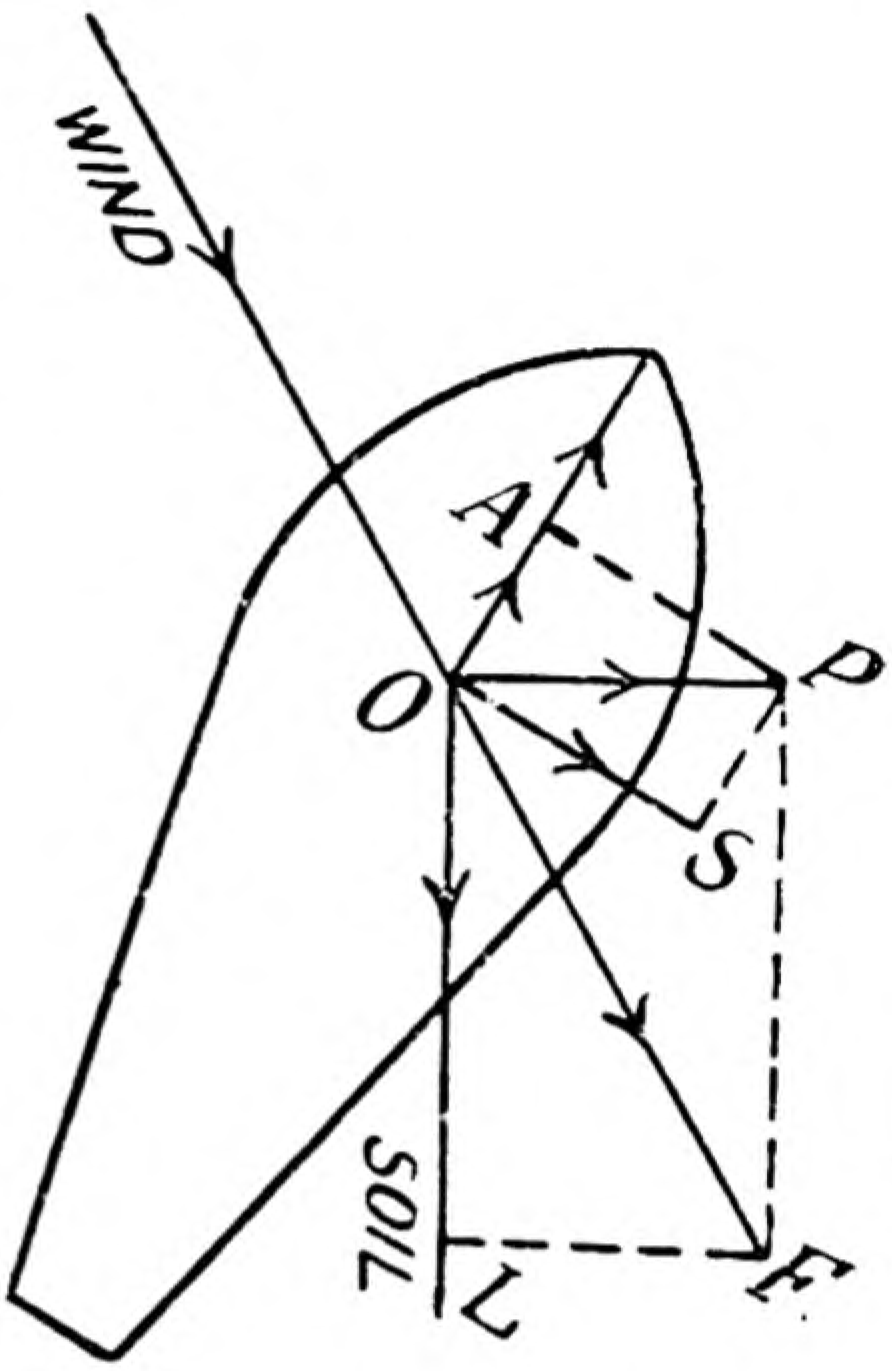
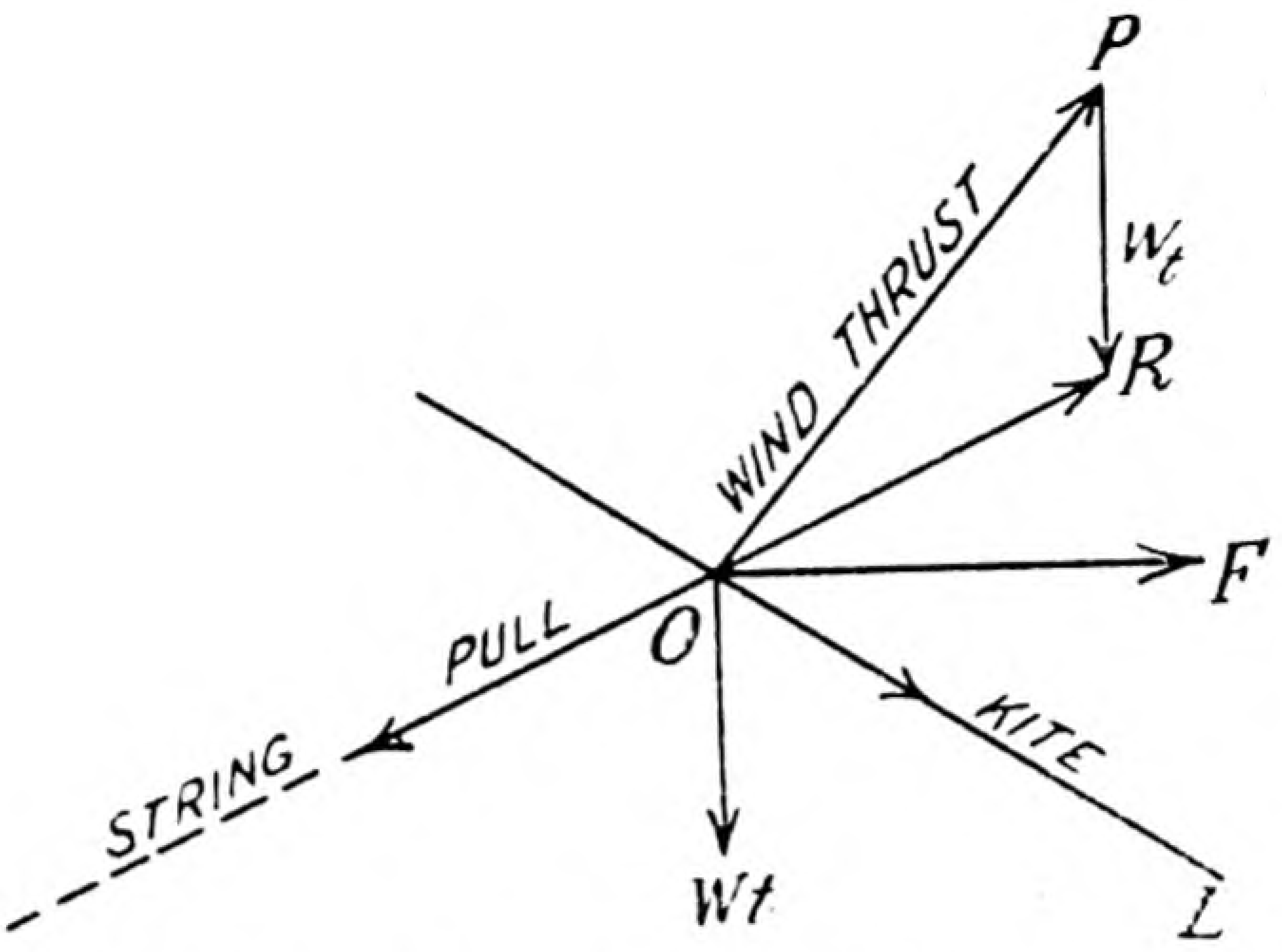


Fig. 5·7

force in its own direction represented by OF in the Fig. 5·7. This can be resolved into the components OP perpendicular to the surface of the sail and OL parallel to it. The latter causes no force action on the boat, while the force OP tends to propel the boat. The force OP may be further considered resolved into OS and OA. The component OS tends to move the boat sideways through the water, but as the resistance to this motion is very great, the effect of the force OS is actually small. The component OA is acting along the length of the boat in the forward direction. The shape of the boat is such, that the water resistance in this direction is much less, with the result that the boat moves in the direction OA, which is inclined to the direction of the wind. By adjusting the rudder and the sails, the boat can be moved in any direction. By following a zig-zag path it is possible to make the boat move even against the wind.

8. Flying a Kite.

Three forces are involved in the equilibrium of a kite, the weight of the kite, the pull or the tension in the string, and the wind pressure or wind thrust on its surface. The wind strikes on the surface of the kite, and the force OF which it exerts may be resolved into OL and OP (Fig 5·8). The former acts along the edge of the kite, that is parallel to its surface and produces no effect, white OP exerts a perpendicular force (wind thrust) to the surface. The weight *Wt* is acting vertically downward and the tension in the string acts along its length. The wind thrust may be supposed to act at a single point O, called the centre of pressure. For equilibrium the resultant of any tow



Pull is anti resultant of Wind and weight Forces.

Fig. 5·8

forces is equal and opposite to the third. Thus the resultant of OP and *Wt* i.e., OR must be equal and opppsite to the tension in the string.

9. Equilibrium of forces supporting a picture.

Pictures are often suspended by a string attached to two points on the upper rod of the frame. This string is then suspended from its middle point from a peg as in (Fig. 5·9). There are three forces acting in this case, *i.e.*, the tension T in each half of the string, and the weight W of the picture acting at its centre of gravity in the vertically downward direction. The three forces pass through the same point O and the condition of equilibrium can be easily found by resolving the tension T in the vertical and the horizontal directions. If the angle between the two segments of the string is called 2 θ, then the vertical component of each tension T, is

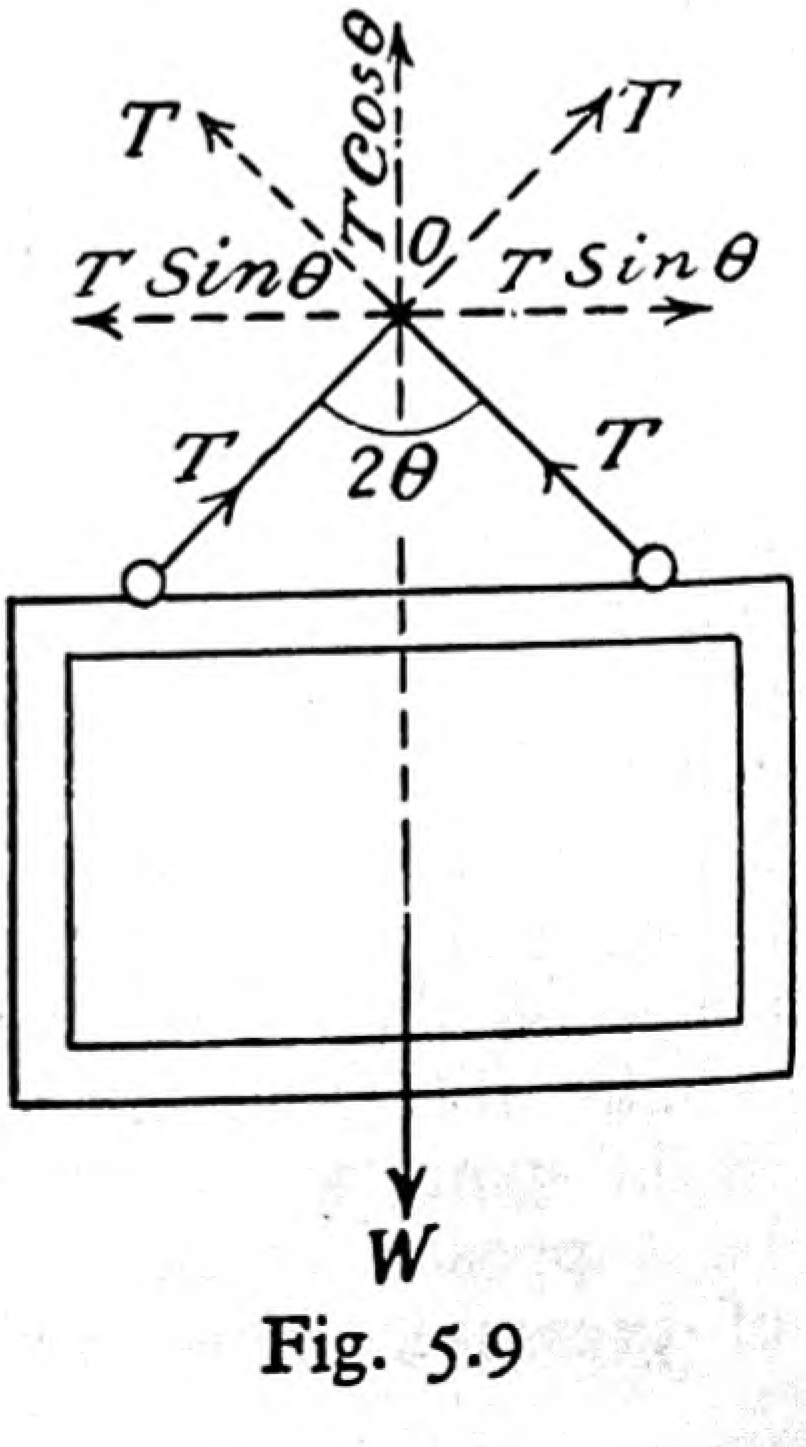


Fig. 5·9

T cos θ and a horizontal component T sin θ. Thus the total upward force on the picture is 2 T cos θ and this should be equal and opposite to W, whereas the two forces T sin θ are equal and opposite to each other and are neutralized.

10. The Crane.

Crane is used for hauling or lifting heavy weights, and transporting them, for loading and unloading of cargo ships and wagons etc. The weight to be raised is attached to a strong steel rope, called tie rope, which is suspended from the end of a stout iron pole called the jib. The jib is hinged at its lower end against a heavy and massive frame. The tie rope is wound or unwound when the weight is to be raised or lowered. In a small crane the rope may be wound on the windlass (wheel and axle) by manual work, whereas in a large crane an engine is used for the purpose. The forces acting are the weight W, the tension in the tie rope, (T) and the force along the jib (R). These three forces pass through the same point and being in equilibrium can be represented by the sides of a triangle taken in order. This triangle is drawn in the (Fig 5.10.) as A, B, C. Line AB represents the weight, AC the tension in tie rope, and CB the reaction along the jib. It is seen that the force along the rope is greater than the actual weight.

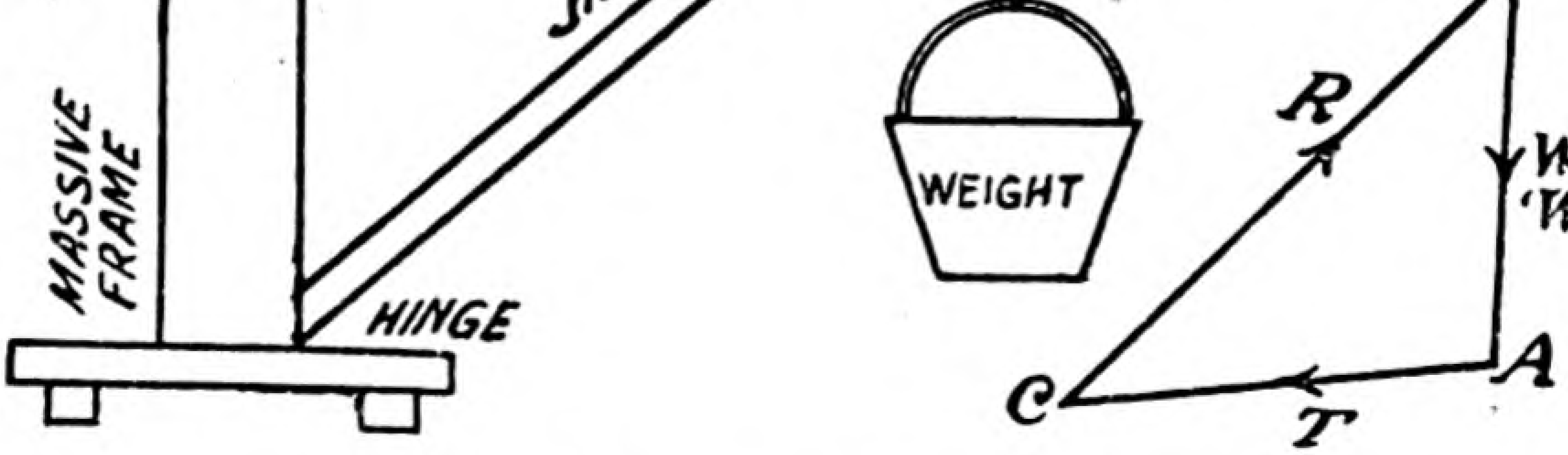
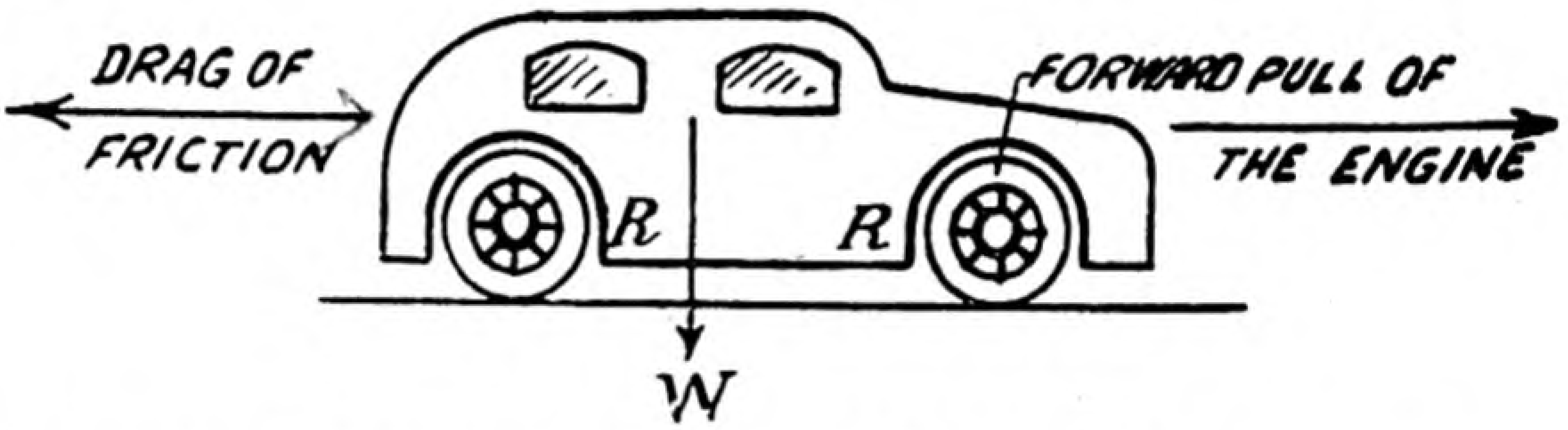


Fig. 5.10 A crane

11. Dynamic Equilibrium.

Forces acting on a body are in Equilibrium not only when the body remains at rest, a condition called static equilibrium ; but if it moves with a uniform velocity, then also the different forces acting on it are in equilibrium.

In the case of an automobile moving with a uniform velocity, the different forces are (i) Its weight W, (ii) The forward thrust of the engine on the back wheel, (iii) The support (R) of the ground acting on all the wheels in the upward direction and (iv) the opposing force due to wind resistance and friction between the ground and the wheels (fig. 5.11.)



Balance of forces on an automobile in uniform motion.

Fig. 5.11

The engine has to apply a force just to overcome the wind resistance and the ground friction when the car is moving at a uniform speed. If, however, the speed is to be increased, then the engine has to apply a force both to produce acceleration, and to overcome wind resistance and friction. The same considerations apply to a train travelling at a uniform speed. The engine has to overcome the friction between the wheels and the rails, and the wind resistance. When the train is accelerating greater pull or force is needed to produce the acceleration.

In the case of an aeroplane also, when cruising at a constant speed, the pull of the propeller just counterbalances the drag due to the flow of air past the wings and the body of the plane.

Summary.

Law of Parallelogram of Forces. If two forces acting on a point be represented in magnitude and direction by two lines drawn from the point and a parallelogram be constructed with these lines as adjacent sides, then the resultant is represented in magnitude and direction by the diagonal of the parallelogram through that point.

Triangle of forces. If two forces acting on a point are represented in magnitude and direction by two sides of a triangle taken in order, then the resultant is represented by the third side of the triangle taken in the opposite order.

Or,

If three forces acting on a particle are in equilibrium, they can be represented by the three sides of a triangle taken in order.

Lami's theorem. If three forces acting on a body keep it in equilibrium then each force is proportional to the sine of the angle between the other two forces.

Polygon of Forces. If a number of forces acting on a point are represented by the sides of a polygon, taken in order, then their resultant is represented by the closing side of the polygon taken in the opposite order. *Or,*

If a number of forces acting on a point are represented in magnitude and direction by the sides of a closed polygon taken in order, the forces will be in equilibrium.

A force can be resolved along two mutually perpendicular directions. The component in a direction making an angle θ with the direction of the force F, is F cos θ and the component perpendicular to this direction is F sin θ.

Exercises.

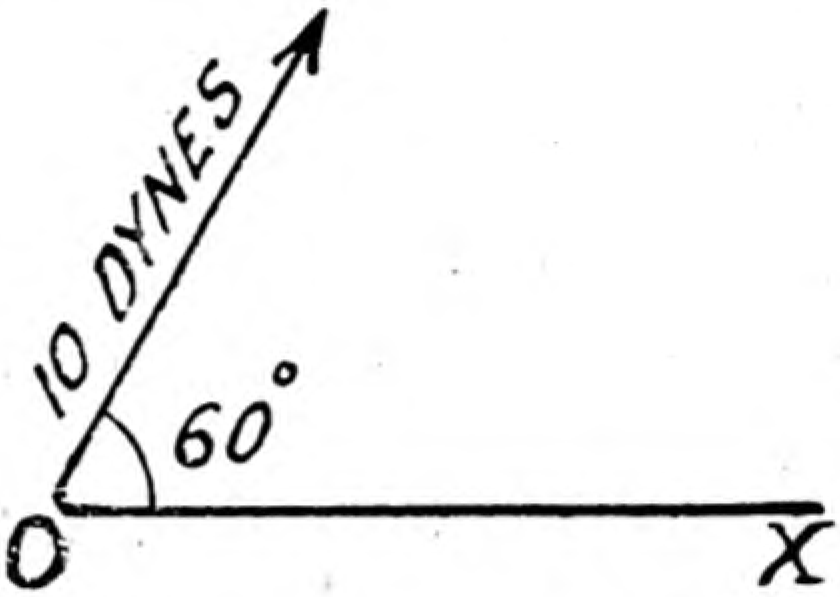
1. State the law of parallelogram of forces.

When three forces acting at a point are in equilibrium, show that each is proportional to the sine of the angle between the other two. *(P. U. 1940.)*

2. State and show how you will verify experimentally, the principle known as "the triangle of forces."

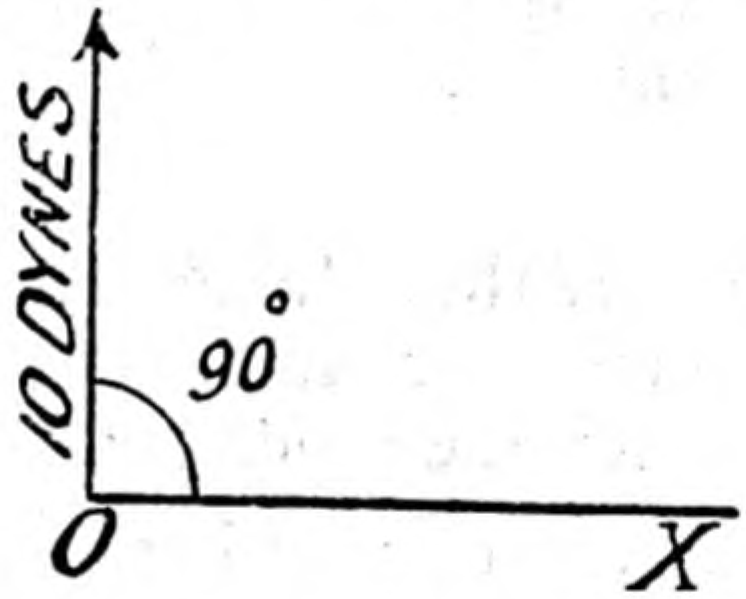
What will be the components of a force of 10 dynes along directions making angles of 60 degrees and 90 degrees with the direction of the force ?

(*i*)



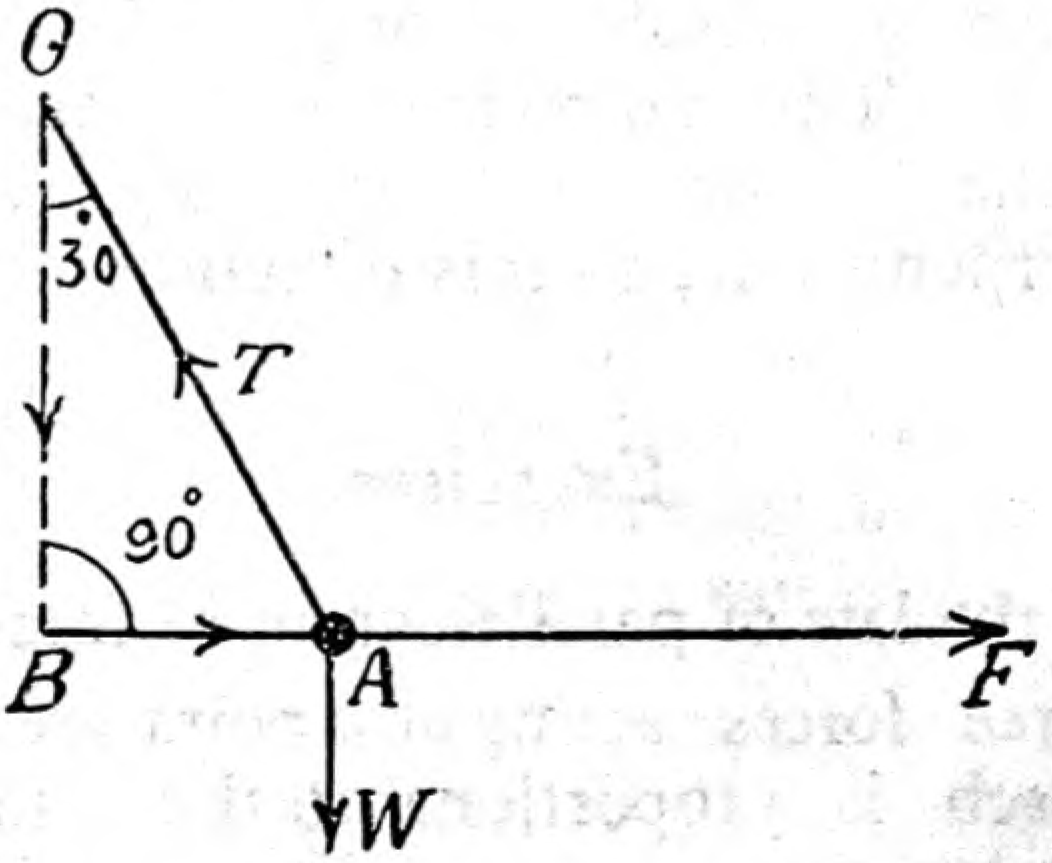
Component of 10 dynes along OX=10 cos 60=5 dynes.

(*ii*)



Component of 10 dynes along OX=10 cos 90=0 dyne.

3. Determine the horizontal force that is required to displace a mass of 30 grams suspended by a string until the string makes an angle of 30° with the vertical. Also calculate the tension in the string in this position.



Three forces act on the body in the position A.

Horizontal force F acting along BAF.

W=30 gram *wt* acting vertically downward due to the pull of the earth.

Tension T acting along the string AO.

The three forces are in equilibrium and are parallel to the sides of the △OBA taken in order.

Therefore OBA is the triangle of forces.

$$\therefore \quad \frac{F}{BA}=\frac{W}{OB}=\frac{T}{AO}$$

$$F=\frac{W\times BA}{OB}=30 \tan\ 30°=\frac{30}{\surd 3}$$

$$=17{\cdot}32\ gm.\ wt.$$

$$T=\frac{W.AO}{OB}=\frac{W}{OB/AO}=\frac{W}{\cos 30}$$

$$=\frac{2\times 30}{\surd 3}=34.64\ gm.\ wt.$$

Alternative solution

Resolve T along the horizontal and verticle directions.

The vertical component should balance W.

$$\therefore \quad T \sin 60=W=30\ gm.$$

$$\therefore \quad T=\frac{30}{\sin 60}=\frac{30\times 2}{\surd 3}=34{\cdot}64\ gm.\ wt.$$

The horizontal component should balance F

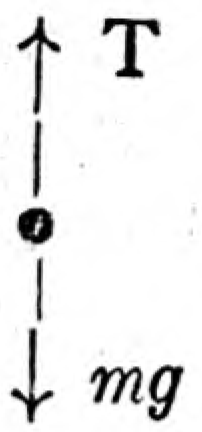
$$\therefore \quad T \cos 60=F \text{ or } F=\frac{T}{2}=\frac{34{\cdot}64}{2}=17.32\ gm\ wt.$$

4. Find the angle between two equal forces, when their resultant is equal to either of them. 120°.

When are two forces equal (*a*) statically, and (*b*) Dynamically ?

5. A mass of 10 grams is suspended by a string and is falling with a uniform acceleration of 400 cm./sec.^{2} Find the tension of the string. [g=980 cm./sec.^{2}]. (P.U.)

There are two forces acting on the body; tension T acting upward and weight mg acting downward. The resultant of these two forces produces an acceleration of 400 cm/sec^2 in the downward direction. Mass m is 10 grams.



$$\therefore mg - \text{T} = m \times 400$$

$$\therefore \qquad \text{T} = m(g - 400) = 10 \times 580 \text{ dynes}$$

$$[g = 980 \text{ cm./sec.}^2]$$

$$= \frac{10 \times 580}{980} = 5{\cdot}92 \text{ gram } wt.$$

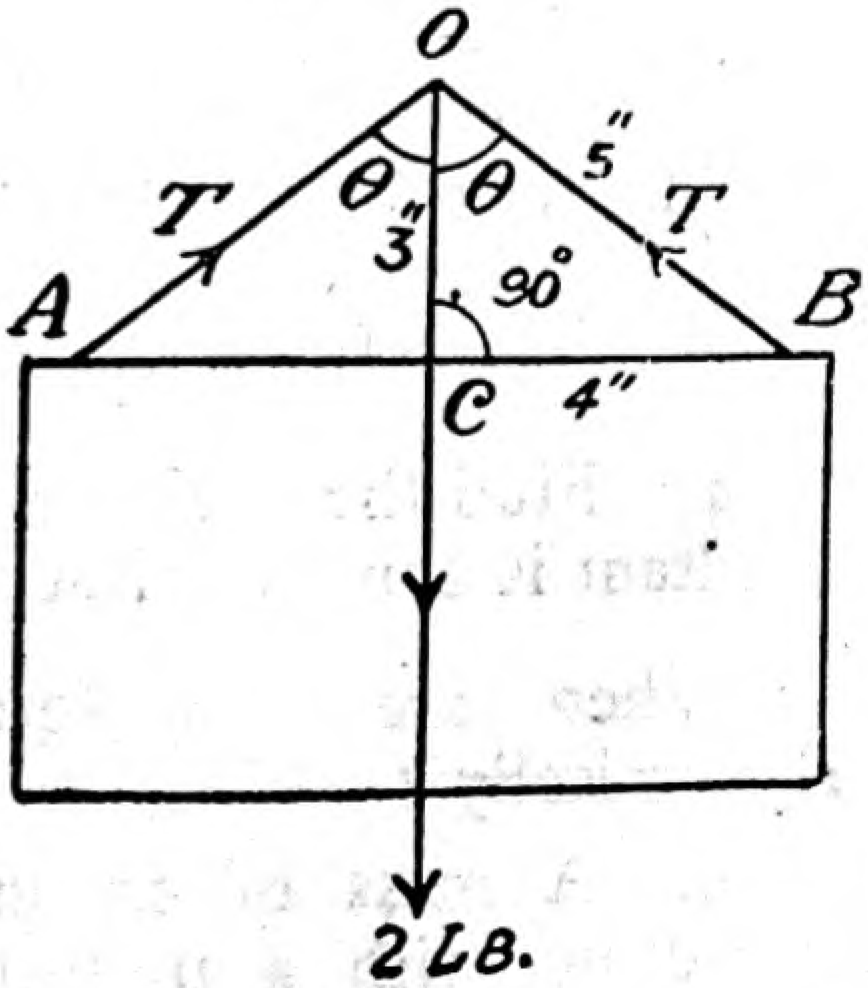
6. A man pulls a weight along the ground exerting a force of 40 lbs. Find the effective horizontal component, if the rope makes an angle of 45° with the ground.

28·3 lbs.

7. A force of 15 ℔ wt. is to be resolved into two components of 10 ℔ wt. and 12℔ wt. Draw by geometrical construction the lines of action of these components.

8. A string 10 inches long is tied to the frame of a picture at two points A and B 8 inches apart. The string is hung from a smooth peg so that the line AB is horizontal. If the weight of the string be 2 ℔. find the tension in the string.

String AOB passes over the peg O. Since AB is horizontal the vertical line OC is perpendicular to AB and bisects it.



Three forces are acting on the picture, 2 lb. wt. acting vertically downward tension T along BO and tension, T along AO.

Vertical components of T should balance the weight 2 lb.

∴ $2T \cos \theta = 2$ or $2T \times \dfrac{OC}{AO} = 2$

or $2T \times \frac{3}{5} = 2$ (since BC $=4''$, OC$=3''$)

or $T = \frac{5}{3} = 1\frac{2}{3}$ lb. wt.

CHAPTER VI

CIRCULAR MOTION OR ROTATION.

1. Uniform Circular Motion.

So far we have discussed the motion of a body along a straight path, but in general a moving body may traverse a curved path, and a simple case of such motion is when the path is circular. If the moving body moves over a circular path covering equal arcs in equal intervals of time, however small, the body is said to perform uniform circular motion. Although the magnitude of velocity of such a body remains constant, its direction is continually changing. The change of the direction of velocity causes an acceleration in the body, just as truly as, when the magnitude of the velocity changes without changing the direction; and a force must act on the body in uniform circular motion to produce this acceleration.

The velocity of a body at any instant is obtained by drawing a tangent to the circle at the point where the body is situated at that instant, and making the length of the tangent to represent on scale the magnitude of the velocity.

A simple case of circular motion is that of a stone tied to the end of a string, and being whirled by holding the other end of the string in the hand. Suppose the length of the string is R centimeters, and the revolving stone makes one revolution in T seconds, then assuming the speed to be uniform, the speed of the stone in its circular path is $\frac{2\pi R}{T}$ centimeters per second. We call this the speed of the revolving body, because, the direction of motion is constantly changing; as a result of which the velocity of the body is variable, although the magnitude of velocity (speed) is constant.

2. Angular Velocity.

In the case of revolving bodies the rate of revolution of the body about the axis can also be denoted by describing the rate at which any radius is rotating round the axis. In one complete revolution the angle turned through is 2π radians or 360°. Let the time taken for a revolution be T seconds. Therefore, the rate at which the body is rotating is $\frac{2\pi}{T}$ radians per second. This is called the angular velocity, ω (Omega), and defined as the angle turned through by any radius in the body per unit time.

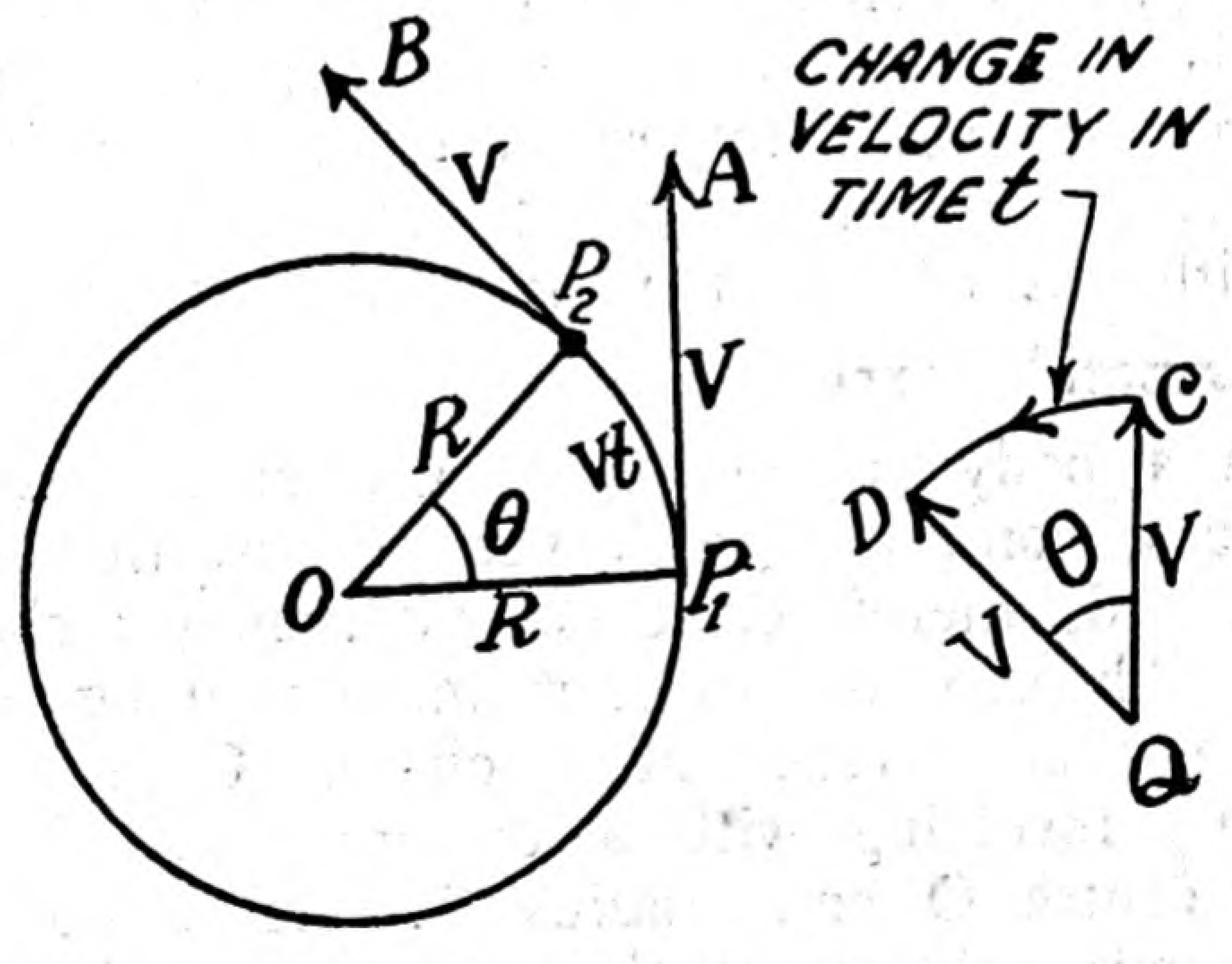
Since the linear speed of a point at a distance R from the axis of rotation is $\frac{2\pi R}{T}=v$ and $\frac{2}{T}=\omega$. We see that $v=R.\omega$.

The angular speed is, often, also given as revolutions per second (r. p. s) and since in one revolution the angle turned is 2π radians, we can convert revolutions per second into radians per second. Suppose a revolving body makes n revolutions per second, then its angular velocity can be written either as n (r. p. s) or $2\pi n$ radians per second.

3. Centripetal Force.

When a body is moving along a curve, even though it may have a constant speed, yet its direction keeps on changing and, therefore, it is undergoing a change in its velocity. The simplest case of motion along a curve is along the circumference of a circle. Consider a body, (Fig. 6.1.) revolving with a constant velocity V in a circle having a centre O and radius R. At a point P_1 the instantaneous velocity of the body is given by the tangent P_1A. After a small interval of time t, the body has revolved through an angle $\theta=\omega.t$ and reached the position P_2. The arc $P_1 P_2=v\times t$. Its velocity now is given by the tangent P_2 B. We notice that the lengths of the tangents P_1A and P_2B are equal, but their directions are different being inclined at the angle θ.

To determine the change in the velocity, we construct the triangle of velocities, QCD. A line QC is drawn parallel to P_1A, to represent the magnitude of velocity V of the body on an appropriate scale. This is the velocity possessed by the revolving body at the point P_1. A second line QD is drawn to represent the velocity of the body at the point P_2 after a time t. If we join CD then this represents the change in the velocity, as the body moves from P_1 to P_2 through the angle $CQD=\theta$. For a small value of θ the triangle QCD is seen to be similar to OP_1P_2. It is obvious that since the sides QC and QD are respectively perpendicular to OP_1 and OP_2, the third side is perpendicular to P_1 P_2 and, therefore, the direction of change of velocity is such that it is directed towards the centre of the circle. We thus see, that a body revolving with a constant speed along a circular path is acted upon by an acceleration directed towards the centre of the circle. We can calculate the magnitude of this acceleration by the following simple treatment.



Uniform motion in a circle

Fig 6.1

Since the triangles QCD and OP_1 P_2 are similar, their corresponding sides are proportional. Therefore,

$$\frac{CD}{QC}=\frac{P_1\ P_2}{OP_1}\ \text{Or}\ \frac{CD}{V}=\frac{Vt}{R}\ \text{or}\ \frac{\text{Change of velocity}}{t}=\frac{V^2}{R}$$

i.e. Acceleration$=\frac{V^2}{R}$. Further since a body possessing an acceleration a must be acted upon by a force $F=.\ m.a$, the above acceleration $\frac{V^2}{R}$ can only be produced by a force $m\frac{V^2}{R}$ where m is the mass of the body revolving in a circle of radius R with a constant speed V. This force must be acting in the same direction as the acceleration, that is to say, towards the centre of the circle. Therefore, the revolving body is constantly being acted upon by the force $\frac{mv^2}{R}$ directed towards the centre of the circle, and called the CENTRIPETAL FORCE, which means a force towards the centre.

We have seen that, according to the Newton's law of motion, a moving body if left to itself moves along a straight line, and in order to move the body along a curved path a force has to act upon it. In the case of circular motion this force is given by $\frac{MV^2}{R}$. We have learnt that the velocity V equals R ω, therefore, we can write Centripetal force F as

$$F=\frac{MV^2}{R}=\frac{M(R\omega)^2}{R}=MR\omega^2,$$

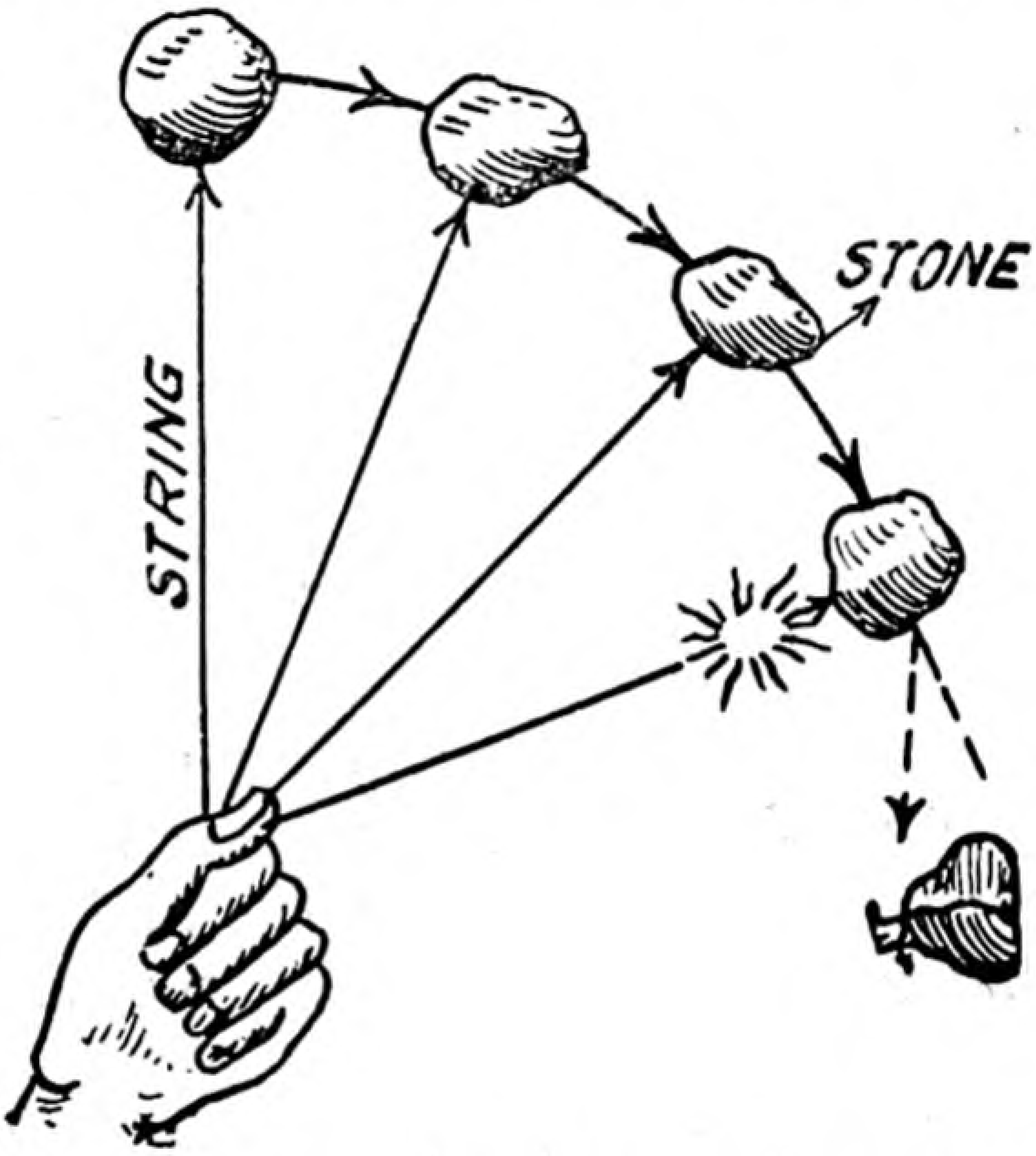
where ω is the angular velocity, and V the linear speed of the revolving body.

If the Centripetal force ceases to act the revolving body no longer moves in the curve, but shoots away along a straight line tangent to the curve.

The earth and other planets revolve around the sun, and the centripetal force in this case is the gravitational pull which the sun exerts upon the planets.

4. Centrifugal Force.

Centripetal force and the centrifugal force are in the nature of action and reaction. In the case of a stone tied to the end of a string, and revolving in a circle, the hand has to exert a force on the string pulling it inwards, which is the centripetal force; but the hand is acted upon by an outward force which is a reaction upon it, and is called the centrifugal force. In the same way the two forces are acting in the string, and on the stone. If the string breaks, the stone will fly away from the centre along a tangent to the circle (fig. 6.2.) If a revolving body is free the centrifugal force tends to take it away from the centre.



If the string breaks the stone flies off.

Fig. 6.2

If we are travelling in a motor car which goes round a curve, we experience a force acting in a direction away from the centre of the curved road and the magnitude of this centrifugal force is quite considerable, as the following case will show :—

Example :—Determine the centrifugal force acting upon a man sitting in a car, weighing 160 lbs., and which goes round a curve of radius 60 yards at a velocity of 45 miles per hour.

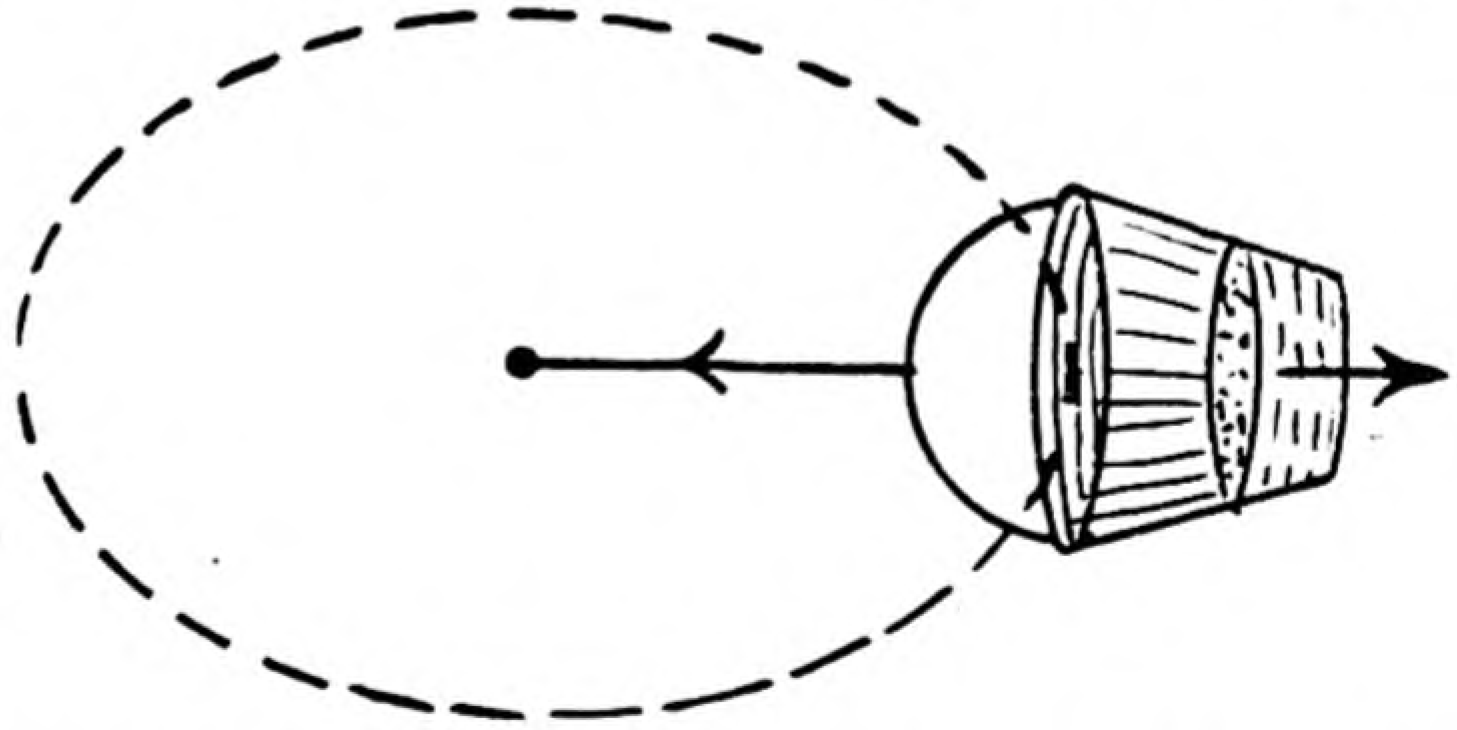
45 miles per hour = 66 feet per second.

$$F = \frac{MV^2}{R}$$

$$= \frac{160 \times (66)^2}{180} \text{ Poundals.}$$

$$= \frac{160 \times (66)^2}{180} \times \frac{1}{32} = 121 \text{ Pounds weight.}$$

An interesting display of strong centrifugal forces is that of a motor cycle rider, who can ride on the cycle at a high speed on the vertical walls of a strong circular enclosure, like a well. The trick rider first of all rides on an inclined surface, and then increasing his speed ultimately whirls round the vertical walls of the enclosure. The cycle and the rider are horizontal but do not fall down. In this case the centrifugal force, with which the cycle and the rider are pressed against the walls of the enclosure is very much greater than the force of gravity acting on them, and therefore, holds them perpendicular to the walls of the enclosure i.e., in a horizontal position. Similarly water placed in a bucket which is being revolved in a horizontal circle does not fall down, as the centrifugal



Centrifugal force acts on a liquid revolved in a vessel and prevents it from spilling.

Fig. 6.3

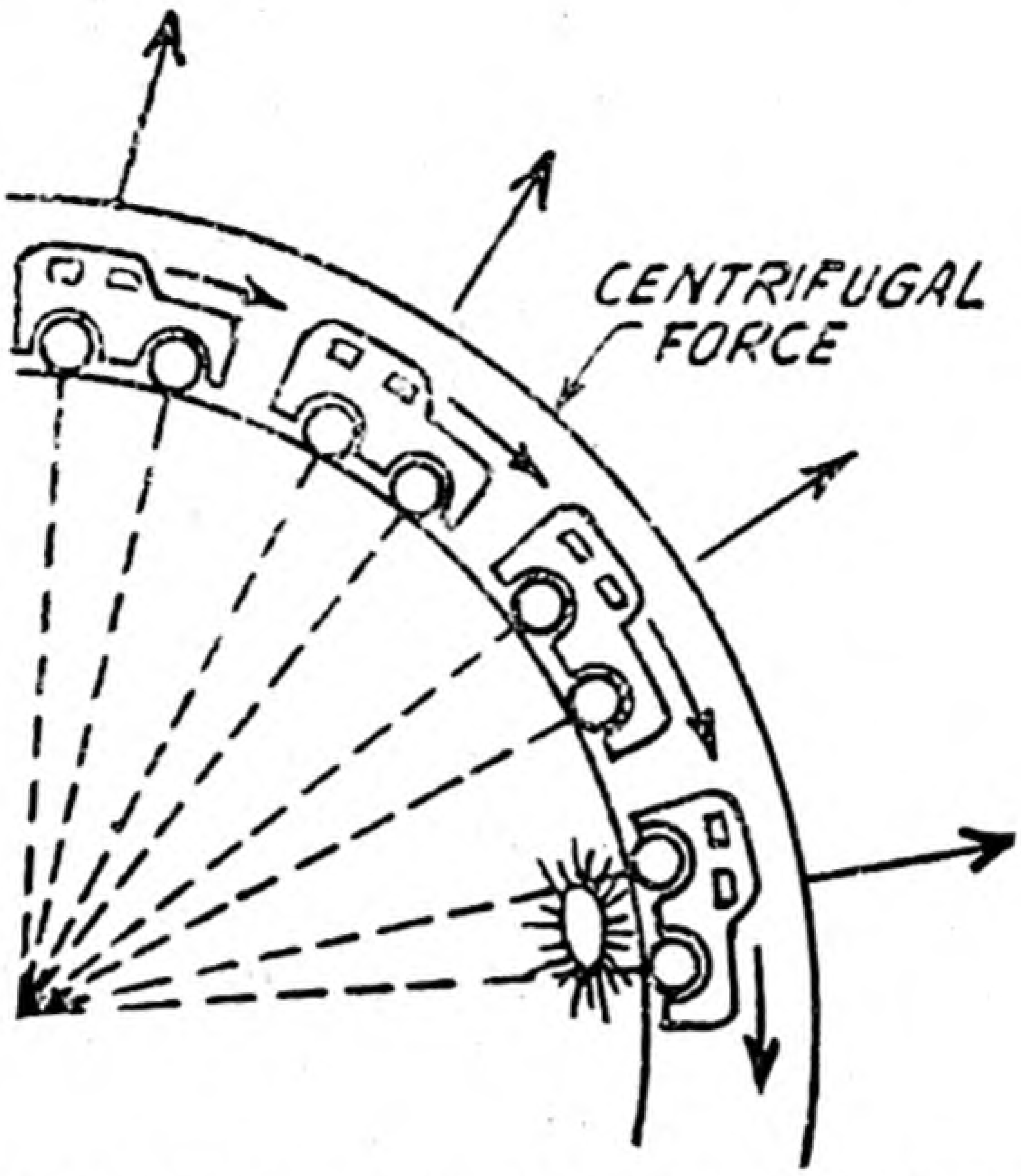
force which presses it against the end of the bucket becomes stronger than the force of gravity (fig. 6.3.)

There are many industrial applications making use of the strong centrifugal forces acting on revolving bodies. In the cream separator, milk is revolved and the heavier liquid component is acted upon by a stronger centrifugal force, than the lighter cream component, which rises to the surface. Separation of crystals from mother liquers is done by centrifuging, when the crystals being heavier than the liquid, settle at the bottom. The drying of clothes is done by revolving the drum having holes at the bottom in which the clothes are placed. When revolved at high speed, the clothes and the moisture in them are acted upon by strong centrifugal forces, but moisture being liquid escapes through the capillaries in the clothes, and the holes in the drum, leaving the clothes dried.

An apparatus called centrifuge is used for separating fine suspensions in liquids such as colloidal particles and bacteria. Here the tube containing the suspension is revolved at a speed of many thousand revolutions per second. In the Ultra-centifuge still higher speed revolutions are used and even gases of different density can be separated.

5. Banking of Roads.

We have seen that when a motor car or a bicycle is going round a curved road at a considerable speed, a centrifugal force acts upon them and this force is apt to cause side slipping or skidding of the vehicle. (Fig. 6.4) On a level road the weight of the car or the bicycle acts vertically downward, and the thrust or the reaction of the road is acting vertically upwards, whereas the centrifugal force acts in the outward direction to the curvature of the road. These forces ordinarily do not produce a balance, and that



When friction "Breaks" the Car slips off.
Fig. 6.4

is why the bicycle rider generally leans inward to the curve, when going around it.

When the cyclist leans inwards, he is inclined to the vertical at some angle θ, and the reaction of the road is

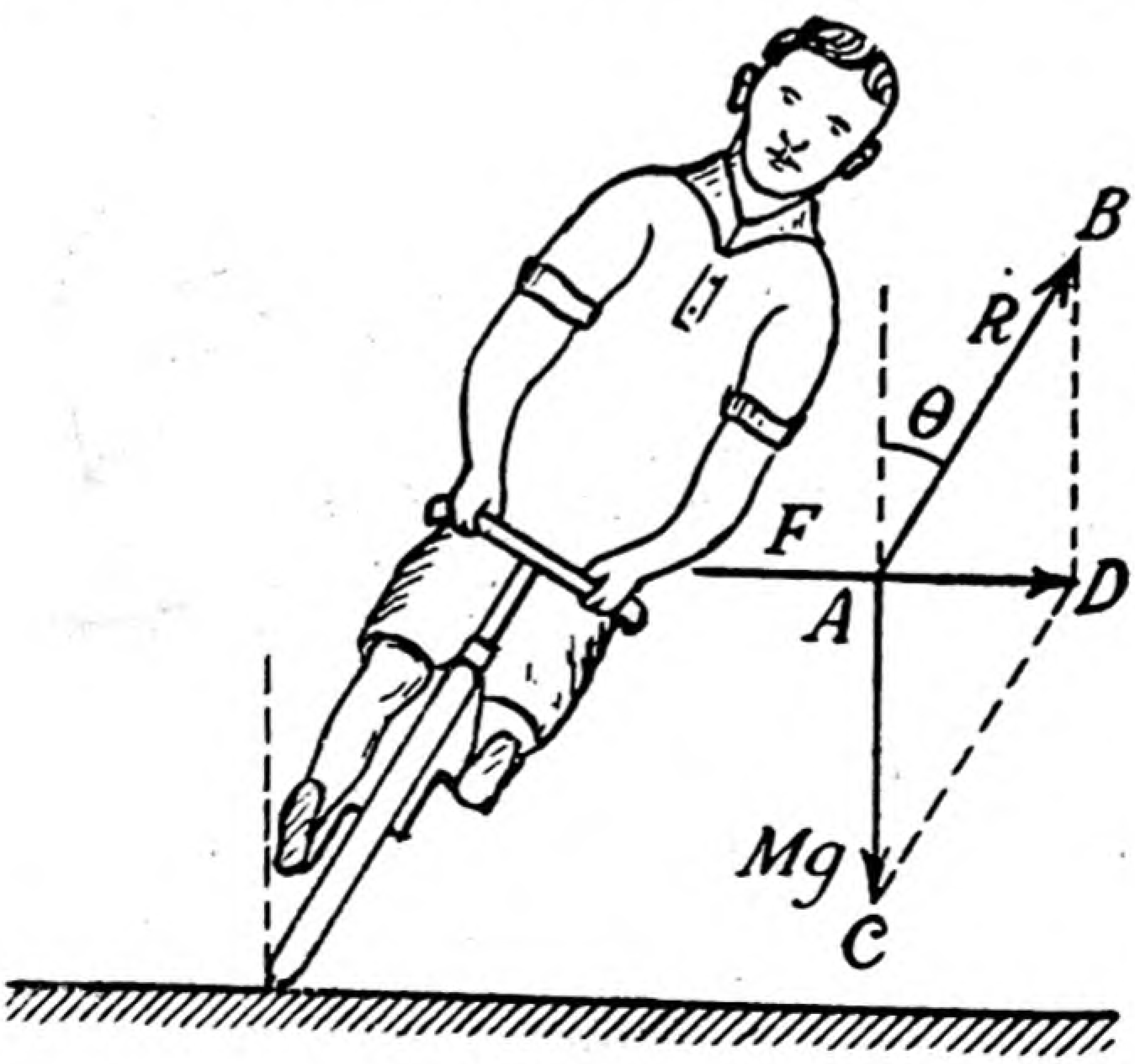
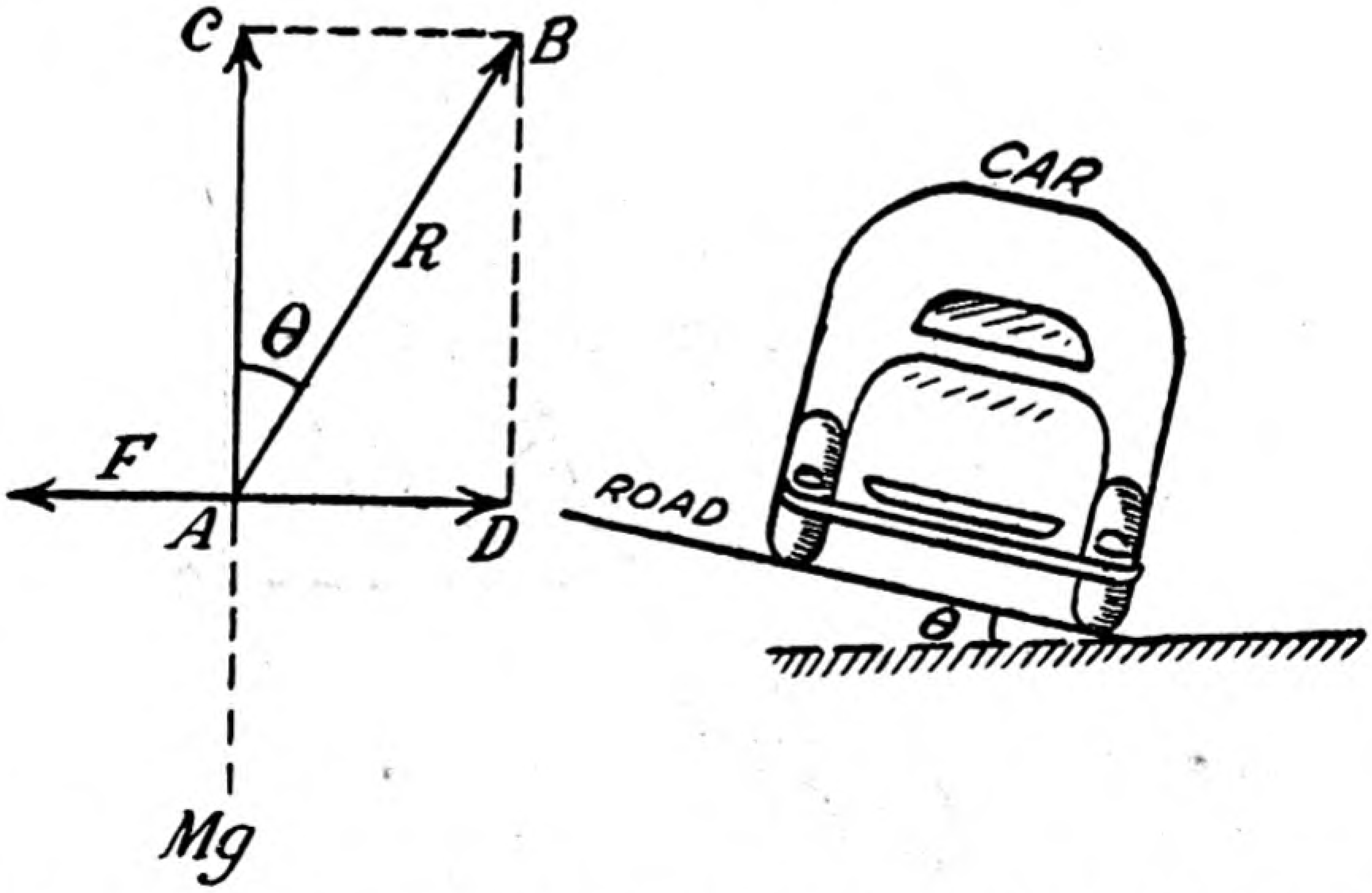


Fig. 6.5

therefore also inclined at θ to the vertical. In the fig. 6.5, AB represents the reaction of the road and AC the weight of the bicycle and the rider acting at the centre of mass. The resultant of these two forces becomes AD, which opposes the centrifugal force F and therefore prevents the slipping. In order to avoid the side slipping, the roads, where they possess a curve are made inclined instead of being flat on level. The outer side of the curved road is raised higher than the inner side and this is called the banking of the roads. The angle of inclination of the road θ, (Fig. 6.6) is such that for a particular velocity the three forces i. e., the weight of vehicle, (Mg.), the thrust of the road perpendicular to its surface (R) and the centrifugal force (F) tend to balance each other. The thrust of the road being inclined to the vertical can be resolved into two components, i. e., a vertical one (AC) which acts opposite to the weight of vehicle, and a horizontal one (AD) acting inwards and tending to oppose and neutralize the effect of the outward centrifugal force (F) and thereby producing a state of equilibrium between the forces, and preventing the possible skidding of the car on the curve track. These forces are illustrated in the Fig. 6.6.



A car rounding a curve. The road is banked or inclined,

Fig. 6.6

It is seen that

$$\tan\theta=\frac{CB}{AC}=\frac{AD}{AC}=\frac{F}{Mg}=\frac{\frac{Mv^2}{r}}{Mg}=\frac{v^2}{rg}.$$

Thus θ angle of banking is calculated for a particular speed and it depends on the radius of curvature of the curved path.

Similarly railway trains, where they have to travel along a curve, produce a strong sideward force on the rails, and the track is banked with the outer rails raised higher. This tends to counterbalance the strong outward force on the track.

6. Change in Weight of a body at Different Places of the Earth.

A body weighs slightly more at the poles than at the equator by about 1 part in 300. As it is taken from the equator to the poles its weight appears to increase. The decrease of weight at the equator is partly due to the greater equatorial radius of the earth, and partly to the greater centrifugal force which acts on the body at the equator than at other places. This centrifugal force is acting in an opposite direction to the inward pull of the earth on the body, and tends to reduce its weight. A body on the equator describes nearly twenty five thousand miles in twenty four hours, in the course of one rotation of the earth. On the other hand, at the poles there is no centrifugal force. The apparent weight of a body may be written as $\left(mg-\frac{mv^2}{r}\right)$. Calculated from this equation a weight of one ton (2240 lbs) has a centrifugal force of about 8 lbs. at 7hrs. equator and its weight becomes reduced by this amount.

As is well known the earth is an oblate spheroid in shape instead of being spherical. The equatorial diameter of the earth is greater than the polar diameter by nearly 13½ miles. This bulging of the earth at the equator is also due to the greater centrifugal force acting on its equatorial parts. This force has been acting through ages and ages and when the earth was semisolid it produced the bulging effect.

Summary.

Angular velocity, ω, of a body rotating about an axis is the number of radians turned by it in one second.

$$\omega = \frac{\text{Angle in radians}}{\text{time}} = \frac{\theta}{t}.$$

$= 2\pi n$ where n is the number of revolutions made by the body in one second.

$= \frac{v}{r}$ where v is the linear velocity and r is the radius of the circle in which the body is rotating.

Centripetal force is the force, that is directed towards the centre, and acts on a body so that it may move with a constant velocity in a circle. For a body of mass m, moving with a constant velocity v in a circle of radius r, the

centripetal force $= \frac{mv^2}{r}$.

Centrifugal force is the reaction to the centripetal force and is therefore equal and opposite to it. It is the force with which a body moving in a circle, tends to fly away from the centre. Centrifugal force is also equal to $\frac{mv^2}{r}$.

Railway tracks and roads are banked at all curves in order to counteract centrifugal force.

Exercises.

1. A shaft 8 inches in diameter is rotated in a lathe, so that the surface linear speed is 200 feet per minute. What is its angular speed? 10 radians/sec.

2. A car travelling at a speed of 30 m. p. h., goes round a curve, whose radius is 200 feet. Calculate the

centrifugal force acting on a man weighing 140 lbs. riding in the car.

42.35 lbs. wt.

3. A stone of mass 2 pounds and attached to a string of length 4 feet is revolved in a circle. What is the maximum velocity of revolution, which can be given to the stone, if the maximum tension the string can stand is 4 lbs. weight.

16 ft/sec.

4. Find the value of centripetal force when a particle moves in a circle with a uniform speed.

How will the centripetal force change when

(*a*) the speed of the particle is doubled?

(*b*) the radius of the circle is doubled?

5. A car weighing 2400 lbs. is going at uniform speed of 30 m.p.h. on a circular road of radius equal to 400 ft. Find

(*a*) the angular velocity of the car in radians per second.

(*b*) the acceleration.

(*c*) the centripetal force.

(*d*) the centrifugal force.

(*a*) .11 radian/sec. (*b*) 4.84 ft./sec.^{2}

(*c*) 363 lbs. wt. (*d*) 363 lbs. wt.

6. A mass of 100 grams is tied at the end of a string one meter long and whirled in a vertical circle at 30 revolutions per minute. Calculate the centripetal force and the tension in the string when the mass is at the highest and the lowest positions.

(*a*) 100.6 gm. wt.; (*b*) .6 gm. wt., (*c*) 200.6 gm. wt.

CHAPTER VII

WORK, ENERGY AND POWER

1. Work.

The word, work, as used in everyday life, means the doing of some job. Thus when a boy solves his arithmetic problems, he is said to be working. A watchman standing at his post is considered to be working, and is paid for this work. But the scientific definition of work is entirely different from the everyday sense. If a force acts upon a body and as a result motion of the body takes place, then the force is said to perform work on the body. If, however, the force just acts and produces no motion of the body, then it does not do any work.

Suppose a man lifts a weight. He is said to have done work against the force of gravity. The greater the height to which he raises the weight, the more is the work he has to do. But if the man only supports a weight without raising it he is doing no work. A man, standing with a suit case in his hand, is doing no work, but when he lifts the suit case he does work. Similarly when a body is pushed along the ground, work is done against friction.

2. Measurement of work.

The work performed depends on two factors, that is, on the force acting and on the distance moved by the body in the direction of the force. In Physics, work is defined as the product of the force, F, acting and the displacement S taking place in the direction of the force.

$$\text{Work} = W = F \times S.$$

A force F may act on a body which is not free to move in the direction of the force, and its motion may take place along a line which makes an angle θ with the direction of the force. As an example we have the case of a man pushing a lawn mower. The motion takes place parallel to the ground while the force is applied in the direction of the handle, (figure 7.1). Similarly when a sledge is pulled with a rope, the direction, in which the force is applied, is inclined to the surface of the ground along which motion takes place, (figure 7.2). In such cases the work done is given by multiplying the component of the force (F cos θ), resolved parallel to the direction of motion and the distance, S moved by the body.

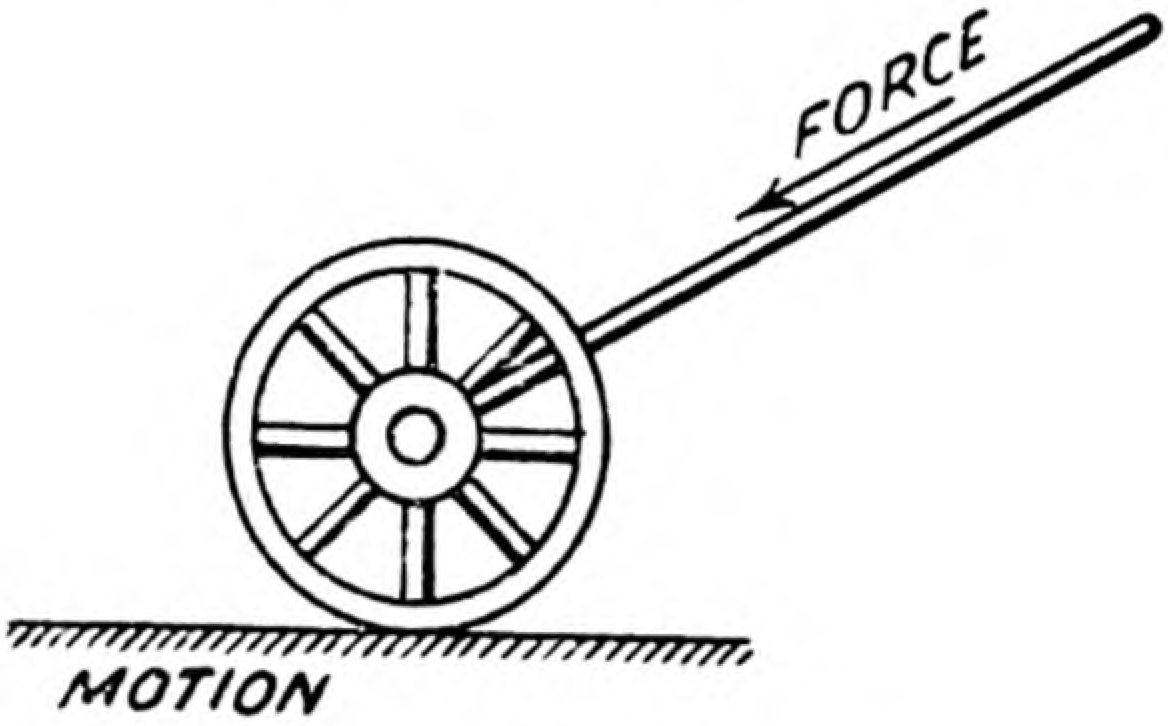
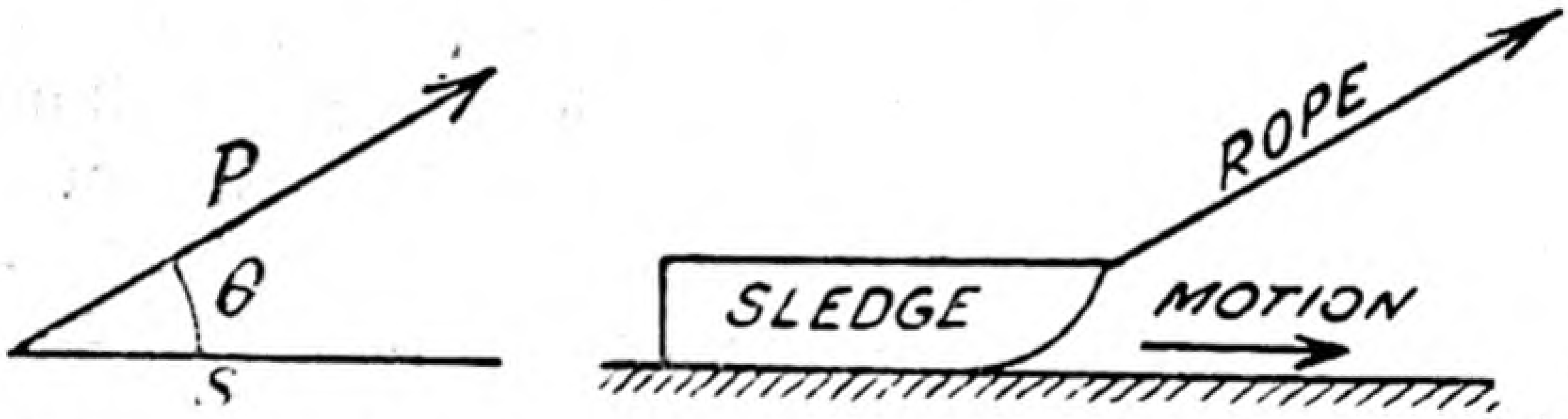


Fig. 7.1



$W = F \cos \theta \times S$

Fig. 7.2

$$W = F. \cos \theta \times S.$$

We can look upon this equation as

$$W = F \times S \cos \theta,$$

that is the work done equals the product of the force applied and the resolved part of the displacement, S cos θ, in the direction of the force, fig. 7.3.

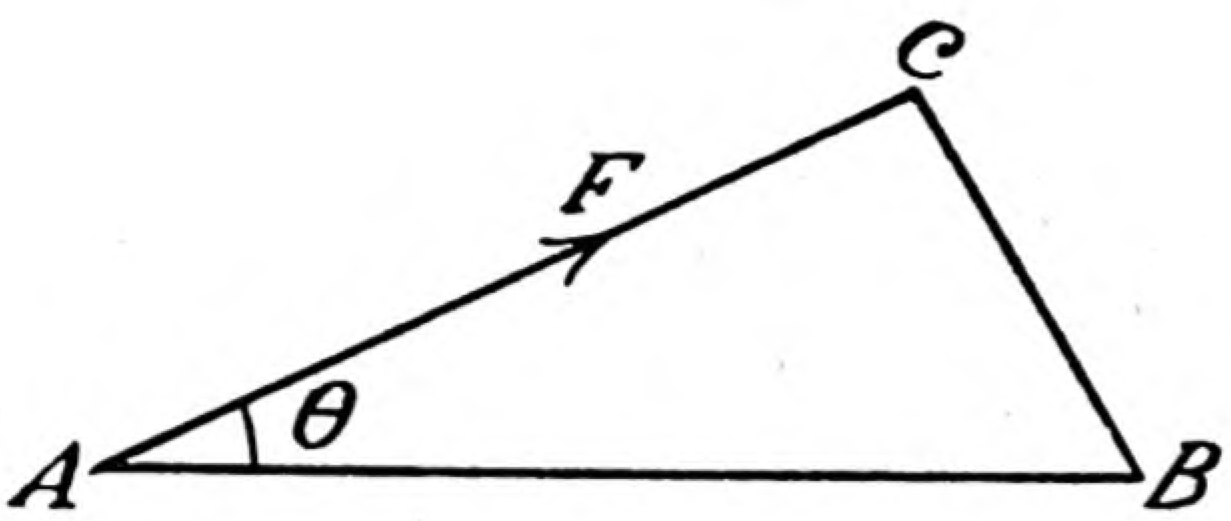


Fig. 7·3
Actual distance AB = S

Resolved part of distance AB along the direction of F = AC = S cos θ.

Therefore work done when the body moves from A to B under the action of force F acting along AC = F × AC = F. S cos θ.

3. Units of Work.

In scientific studies, the metric or the C.G.S. unit of work, most commonly used, is called the erg. It is the amount of work done when a force of one dyne produces a displacement of one centimeter in a body, along the direction of the force. The erg is a very small unit of work. The work done in raising the eyelids is about one erg. A bigger unit which is commonly used for practical purposes is ten million (10^7) times bigger than the erg and is called a Joule.

$$\text{or } 1 \text{ joule} = 10{,}000{,}000 \text{ ergs.}$$
$$= 10^7 \text{ ergs.}$$

Another unit of work used in the metric system is a gram-centimeter, which is the amount of work done when a mass of one gram is raised vertically through a distance of one centimeter.

Since one gram wt. = 981 dynes.

$$\text{Work of one gram-centimeter} = 1 \text{ gram wt.} \times 1 \text{ cm.}$$
$$= 981 \text{ dynes} \times 1 \text{ cm.}$$
$$= 981 \text{ ergs.}$$

The unit of work which the engineers use, is the work done when a mass of one kilogram is raised vertically through a vertical distance of one meter and is called a kilogram-meter.

One kilogram-meter = 100,000 gram-centimeters.

$$= 98{,}100{,}000 \text{ ergs.}$$

$$= 98{\cdot}1 \times 10^6 \text{ ergs.}$$

Absolute unit of work on the F.P.S. system is the work done by a force of one poundal, when it causes a displacement of one foot along the direction of its application. This unit is called a foot-poundal. A bigger unit of work than the foot-poundal (ft. lbl.) is a foot-pound (ft. lb.). An amount of work equal to one foot-pound is done when a body weighing one pound is lifted vertically through a distance of one foot. Since a force of one pound or one pound's weight equals 32·2 poundals (approximately 32 poundals), a work of one foot-pound is equivalent to 32.2 (approximately 32) foot-poundals.

$$1 \text{ ft. lb.} = 1.355 \text{ joules} = 1.355 \times 10^7 \text{ ergs.}$$

$$1 \text{ ft. lbl.} = 42{\cdot}1 \times 10^4 \text{ ergs}$$

$$1 \text{ joule} = .738 \text{ ft. lb.}$$

Example. 1. A box is pushed through a distance of 20 feet along a horizontal floor. If the frictional force between the surface of the box and the ground is 30 pounds weight, how much work is done?

$$\text{work done} = \text{Force} \times \text{distance}$$

$$= 30 \times 20 = 600 \text{ foot-pounds.}$$

Example. 2. A bicycle weighs 20 kilograms. What is the work done if it is carried to the top of a roof 4 meters high?

$$\text{work done} = \text{Force} \times \text{distance}$$

$$= 20 \times 4 = 80 \text{ kilogram—meters}$$

$$= 80{,}000 \times 100 \text{ gram centimeters}$$

$$= 981 \times 8 \times 10^6 \text{ ergs}$$

$$= 784.8 \times 10^7 \text{ ergs} = 784.8 \text{ joules.}$$

Example. 3. A car weighing 2000 lbs. is going at 30 m. p. h. when brakes are applied. If the car comes to rest in 5 seconds, calculate the work done in stopping the car.

Applying the formula

$$v_2 = v_1 + a\,t,$$

$v_2 = 0$, $v_1 = 44$ ft/sec. and $t = 5$ sec.

$$\therefore \quad a = \frac{v_2 - v_1}{t} = -\frac{44}{5} \text{ ft/sec.}^2$$

Force acting on the car $= ma = 2000 \times \frac{44}{5} = 17600$ lbls.

Distance covered in 5 seconds $= 44 \times 5 - \frac{1}{2} \times \frac{44}{5} \times 5 \times 5$

$= 110$ ft.

work done $=$ force × distance.

$= 17600 \times 110$ ft. lbls.

$$= \frac{17600 \times 110}{32} = 60{,}500 \text{ ft-lbs.}$$

4. Power. We have seen that to lift a certain weight, work has to be done against the force of gravity. A labourer who raises a load of bricks to the top of a building, does a certain amount of work, but his employer would require that he should carry the load up in a reasonable time so that his rate of work is not too slow. In order to increase the rate of work more labourers may be employed or some suitable machinery may be used. Similarly if the speed of a vehicle or a train is to be increased, the engine has to work at a greater rate to do the work in shorter time. Thus the rate of work is as important as the amount of work done by an agent.

The rate of doing work is called power.

$$\text{Power} = \frac{\text{work}}{\text{time}} = \frac{F \times S}{t} = Fv = \text{Force} \times \text{velocity}.$$

Thus the shorter the time in which a given work is done, the greater is the power.

5. **Units of power.** The common unit of power is one foot-pound per second and a bigger unit, called the horse power (H. P.) is 550 foot-pounds per second or 550 × 60 *i.e.* 33,000 foot-pounds per minute. This unit of power was introduced by the great British engineer, James Watt (1736-1819), the inventor of the steam engine. Horses were used to haul up coal, water etc., from the coal mines in Britain. When Watt invented his steam engine, he took it to the owners of coal mines and proposed that the engine should be used instead of horses. He was asked how many horses would be displaced by his engine. Watt, therefore, started experiments to find the rate of work done by a well-bred horse. He found that an average horse raised 150 lbs of coal through a height of 220 feet in one minute, and could keep on working at this rate. Horse's rate of work or power is therefore $150 \times 220 = 33{,}000$ ft. lbs per minute and this amount came to be known as horse power.

The H. P. of a motor cycle is about 2 to 5, that of a car about 10 to 30 and of a truck or bus from about 50 to 100. The H. P. of a sea ship turbine may be many thousands and of an aeroplane about a thousand. A man on the average can work at about $\frac{1}{10}$ H. P., while over short periods an athlete can develop as much as a horse power.

The unit of power on the metric system is unit work per second *i.e.*, one erg per second. This however is too small a unit, and the one commonly used is 10^7 ergs *i. e.*, one joule per second. This unit is called a Watt in honour of James Watt the inventor of the steam engine. A bigger unit of power is the kilowatt *i. e.*, 1000 watts or 1000 joules per second.

The power of electric motors, dynamos, fans and other electric appliances is expressed in watts or kilowatts.

6. **Relation between the British and metric units of work.**

1 horse power (H. P.) = 746 watts or nearly $\frac{3}{4}$ Kilowatt (K.W.)

1 Kilowatt = 1·34 horse power.

Power multiplied by time gives work.

Thus watt-hour is the work done in one hour at the rate of one watt. Kilowatt hour (K. W. H.) is the amount of work done in one hour at the rate of one thousand watts or a kilowatt. One kilowatt hour is equal to 3,600,000 joules. The kilowatt hour is the unit according to which electric energy consumption is measured.

Example 1. Find the H. P. of an engine capable of lifting a body weighing two hundredweights (2×112 lbs.) to a height of 50 ft. in ten seconds.

$$\text{Work done} = 224 \times 50 = 11200 \text{ ft. lbs.}$$

$$\text{Time} = 10 \text{ sec.}$$

$$\text{Power} = \text{Rate of doing work} = \frac{11200}{10} = 1120 \text{ ft lbs/sec.}$$

$$= \frac{1120}{550} = 2{\cdot}04 \text{ H. P.}$$

Example 2. A man weighing 150 lbs. runs up a stair case, rising through a height of 20 feet in 10 seconds. Find out his power.

$$\text{Power} = \frac{\text{work done}}{\text{time}} = \frac{150 \times 20}{10} \text{ ft. lbs/sec.}$$

$$= 300 \text{ ft. lbs./sec.}$$

$$= \frac{300}{550} = .55 \text{ H. P.}$$

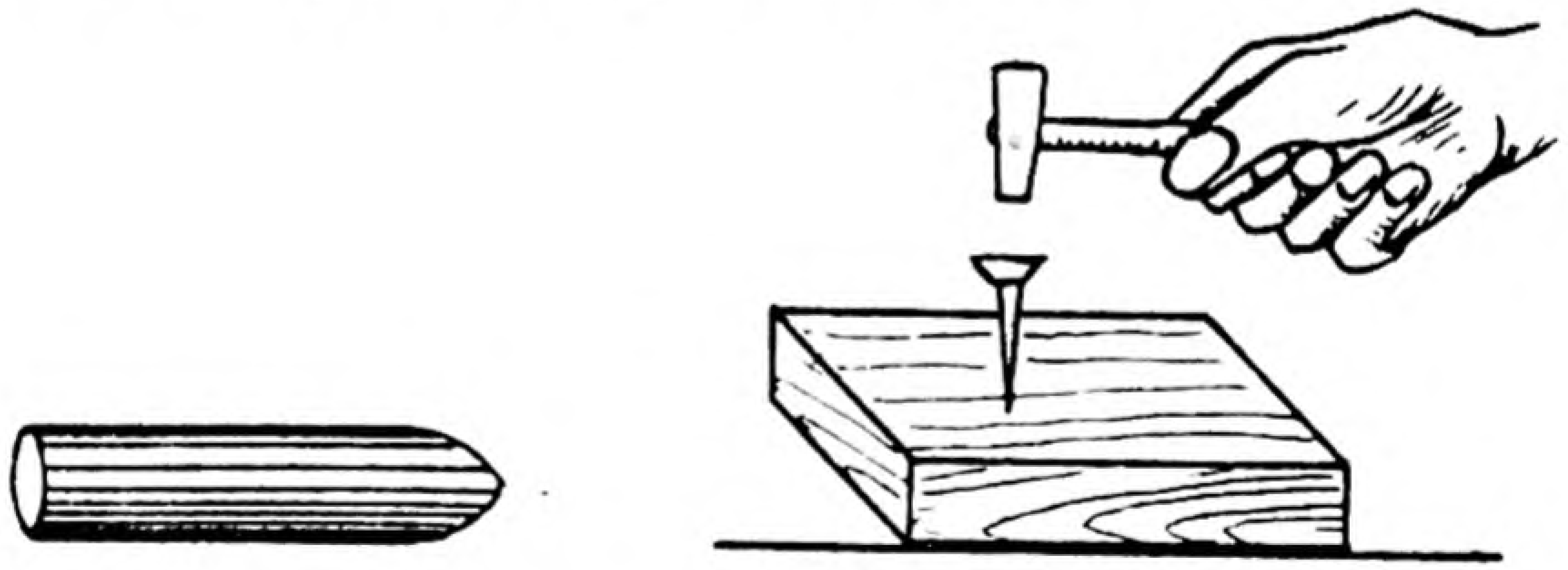
7. **Energy.** The word energy means the capacity or ability to do work. Steam is used in the steam engine to run machinery as railway trains. In the steam engine the heat energy in steam is partly converted into mechanical work. In the engine of the motor car the energy present in petrol is changed into work. Petrol has chemical energy and the engine of the motor car is only a suitable device for converting the chemical energy into motion. The food that human beings and animals eat, give them energy to do the everyday work. A water fall is used to drive a water wheel and work is obtained from the falling water stream. These are some examples of conversion of energy into

work. In order to obtain useful and controlled work from energy, suitable devices as engines, are employed.

8. Forms of energy.

The energy in nature exists in various forms and it is constantly changing from one form into another. In fact all the activity of nature is due to the existence of energy and its transformation from one form into another.

The energy possessed by a body by virtue of its motion is called the Kinetic energy (K. E.). The kinetic energy of wind has been used since ancient times to sail ships and to rotate wind mills. Thus wind is capable of doing work while stationary air is not, so that motion gives energy to the air. Similarly a bullet, when fired, is able to pierce through bodies because of its high velocity of motion, or its kinetic energy. Also when a nail is to be driven into wood the hammer must strike against it while simply placing the hammer on the head of the nail will not drive it. The striking hammer has kinetic energy which in this case does work in driving the nail against the resistance of the wood.



A flying bullet and a striking hammer have kinetic energy.

Fig. 7.4.

If a body is able to do work by virtue of its position or its condition or configuration, it is said to possess potential energy (P. E.). Thus after we wind a clock or a watch the wound spring can drive the mechanism of the clock or watch for a day or a week as the case may be. The wound spring has potential energy and when it becomes unwound it has lost its potential energy. In this process of losing potential energy, mechanical work is done in driving the clock. When an arrow is shot from the bow, the latter is pulled out, a process which gives it potential energy. As soon as the bow is loosened, it gives its potential energy to the arrow which is released with a high velocity, thus gaining kinetic energy from the potential energy of the bow. When we raise a body we have to do work against gravity, but the raised body gains in potential energy. A body situated at a greater height has more potential energy than one at a lower position. As a body falls down its potential energy decreases but an increase of kinetic energy takes place. In a water fall potential energy is converted into kinetic energy and the power for hydro-electric generating stations is obtained from the energy in the water fall.

Heat is a form of energy and the hotter a body is the more energy it possesses. Heat engines are used to obtain useful work from the heat energy of bodies. Electricity is another form of energy. In an electric motor or an electric fan the energy of electric current is changed into the kinetic energy of revolving bodies. In an electric lamp the energy of the electric current is converted into light and heat. Light is a form of an energy which is being given off in immense quantities by the heavenly bodies as the sun and the stars.

According to modern ideas, matter itself is a form of energy and under suitable conditions matter may be destroyed and changed into energy. Scientists have found that the energy of heat and light given off from stars is due to the annihilation or destruction of matter. The atomic energy used in the atom bomb is also produced by the loss of matter.

9. **Expressions for Potential and Kinetic Energy.** We have seen that if a body of mass m pounds is raised through a height of h feet, the work done on the body is $m \times h$ foot pounds. Since a weight of one pound is equivalent to a force of g or 32.2 poundals the work done on the body is also equal to $m \times g \times h$ foot poundals. This work becomes stored in the body as its potential energy. If the same body drops down by a height h feet, the

potential energy is released and converted into kinetic energy of the falling body. If the mass of the body is given in grams and the height h in centimeters, the potential energy gained by the body is $m \times h$ gram-centimeters, or $m \times g \times h$ ergs.

It is to be noticed that energy and work are interconvertible. They are both expressed in the same units. The absolute units of energy are therefore an erg and a foot poundal in the metric and the British units respectively, while the practical units are a joule, a gram-centimeter, a foot pound etc.

To find the amount of kinetic energy in a body of mass m and moving with a velocity v, we adopt the following reasoning. Suppose that a uniform opposing force F begins to act on the body which is moving with the velocity v. This opposing force has the effect of reducing the velocity of the body *i. e.* to produce retardation. The value of the retardation will be $\frac{F}{m}$. Now suppose that the body comes to rest after traversing a distance S from the time the retarding force acts upon it. When the body has come to rest, it has lost all its Kinetic energy which has been used in doing work against the force F. This work is F × S, and we therefore have from the acceleration formulae already studied,

$$v^2 - 0 = 2aS$$

where $$a = \frac{F}{m}$$

$$\therefore \quad v^2 = \frac{2F}{m} S$$

or $$F \times S = \frac{1}{2} m v^2$$

Thus the quantity of work which a body of mass m and moving with a velocity v is capable of doing is $\frac{1}{2} m v^2$ which expression therefore represents its kinetic energy.

If m is given in grams and v in centimeters per second the kinetic energy $\frac{1}{2}mv^2$ is given in ergs. If mass is given in pounds and v in feet per second the kinetic energy is given in foot-poundals.

Example 1. A 40 lb. stone is carried from the ground to the roof of a building 60 feet high. How much does its potential energy increase.

Increase in potential energy $= m \times h$ foot pounds

$= 40 \times 60 = 2400$ ft. lbs.

Example 2. A bullet of mass 50 grams and possessing a speed of 400 meters per second, strikes against a wooden target. If it penetrates a distance of 10 cms. into the wood, calculate the average stopping force against the bullet.

Suppose the average opposing force $=$ F dynes

Then work done against this force $= F \times S$

$= 10\,F$ ergs

This work is done because of the Kinetic energy of the bullet.

$$\text{K. E. of the bullet} = \tfrac{1}{2}mv^2$$
$$= \tfrac{1}{2} \times 50 \times (40000)^2 \text{ ergs}$$
$$= 4 \times 10^{10} \text{ ergs}$$

Since
$$F \times S = \tfrac{1}{2}mv^2$$
$$10F = 4 \times 10^{10}$$
$$F = 4 \times 10^9 \text{ ergs}$$
$$= \tfrac{4000}{980} \times 10^6 \text{ gm. wt.}$$
$$= 4.08 \times 10^6 \text{ gm. wt.}$$
$$= 4080 \text{ Kgm. wt.}$$

10. Transformation of Energy.

In the case of a falling body the potential energy is decreasing and the Kinetic energy increases. If the body is falling freely it can easily be shown that the gain of Kinetic energy is exactly equal to the loss of potential energy. Suppose a body of mass m is situated at a height, h above the ground. Then as shown previously its potential energy is mgh units. As it falls down through a distance equal to h, its initial velocity is zero and if there is no friction or resistance it falls with an acceleration equal to g. Its velocity after falling through a distance h is given by

$$v^2 - 0 = 2gh$$

$$\text{or } gh = \frac{v^2}{2}$$

and $$mgh = \tfrac{1}{2}\, mv^2.$$

This means that the Kinetic energy gained by the body ($\frac{1}{2}\, mv^2$), is exactly equal to the potential energy possessed by it (mgh), or that all the potential energy in the body has been transformed into Kinetic energy.

When the body has fallen through a height $\frac{h}{2}$, its potential energy is $\frac{1}{2}\ mgh$ and its velocity v' is given by the equation.

$$v'^2 - 0 = 2g \times \frac{h}{2} = gh.$$

Therefore its K. E. $= \frac{1}{2} mv'^2 = \frac{1}{2} mgh.$

In this position the total energy Kinetic plus potential is therefore

$$= \tfrac{1}{2} mgh + \tfrac{1}{2} mv'^2$$
$$= \tfrac{1}{2} mgh + \tfrac{1}{2} mgh = mgh.$$

Thus the total energy of the body is the same as it had before it began to fall. Half of its potential energy has been converted into Kinetic energy but the sum total of the two energies is constant. It can be shown by the same reasoning as above that whatever may be the position of the falling body, at every instant the total energy Kinetic plus potential is equal to mgh.

Suppose the body has fallen through a distance x, fig. 7·5. Then its height is now $(h-x)$ and its potential energy is $mg\ (h-x)$. If its velocity in this position is called v_1 then

$$v_1^2 - 0 = 2gx$$

or $$v_1^2 = 2gx$$

and $$\text{K.E.} = \tfrac{1}{2}mv_1^2 = \tfrac{1}{2}m \times 2gx = mgx$$

$$\begin{aligned}\text{Therefore total energy} &= \text{P.E.} + \text{K.E.}\\ &= mg\,(h-x) + \tfrac{1}{2}mv_1^2\\ &= mg(h-x) + mgx\\ &= mgh.\end{aligned}$$

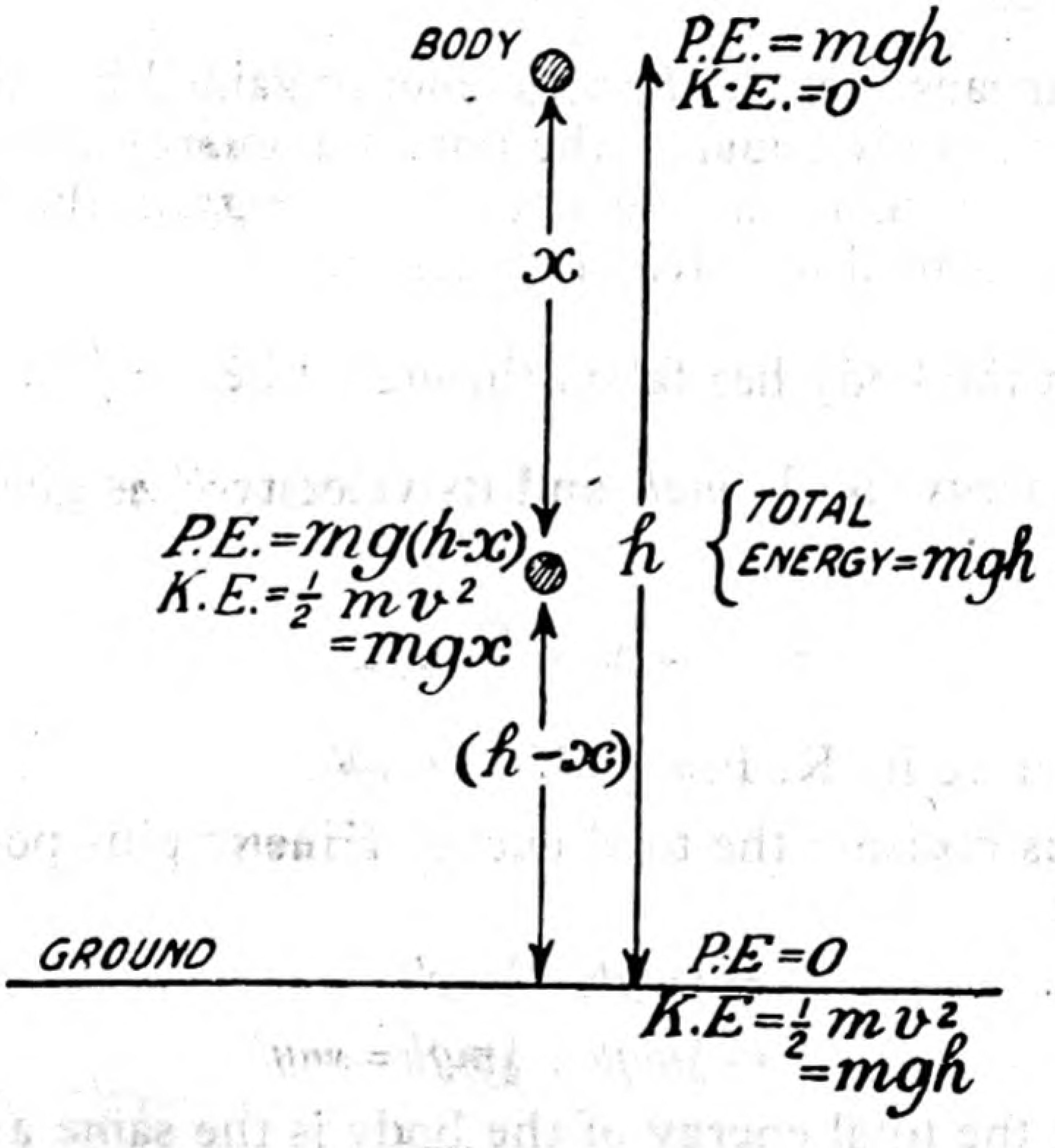


Fig. 7.5.

When the body strikes against the ground it comes to rest and both its potential and kinetic energies have been lost. A careful study shows that some heating of the body and the ground where it strikes has taken place and also sound has been produced at the time of striking. Now heat and sound are also forms of energy and in fact the whole Kinetic energy of the body has been converted into heat and sound.

A good example of the transformaton of energy from kinetic to potential and *vice versa*, is provided in the case

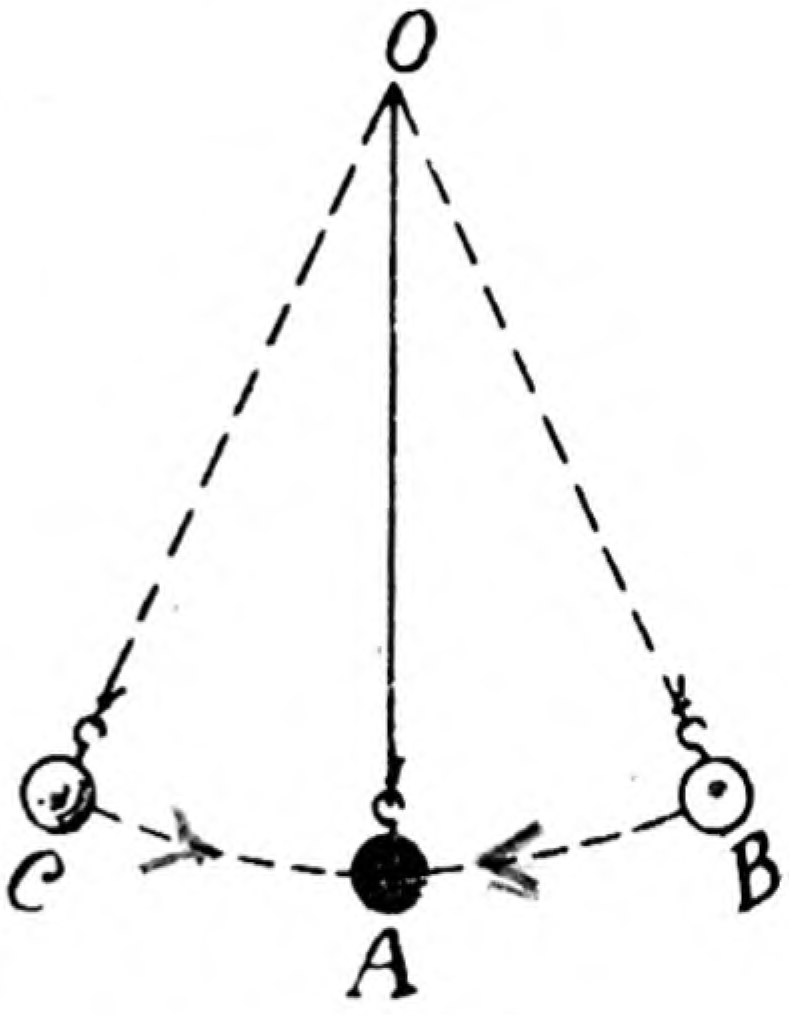


Fig. 7.6

of an oscillating pendulum. When the pendulum reaches the extreme position as at B, fig. 7.6, it momentarily comes to rest and its energy is potential. As it moves back from B to A its velocity increases and potential energy is being converted into kinetic energy. At the point A it has the maximum velocity and the energy is all kinetic. When it moves from A to C, its velocity decreases, but it is raised up in height and it is gaining in potential energy. At C the energy is all potential and as it returns to A energy is again converted from potential to kinetic. If there was no loss of energy this process would go on indefinitely, but because of air resistance and friction at the point of support, energy is lost gradually and after some time the pendulum comes to rest. The pendulum of a clock however continues to oscillate because any loss of energy is being supplied from the potential energy of the wound spring. The case of a boy using a swing is similar. The conversion of energy between potential and kinetic takes place, but by his effort, the boy supplies the loss of energy and keeps the swing going.

Similar is the case of a weight attached to the end of a coiled spring, fig. 7.7. When the weight is pulled down and then released it bobs up and down about its mean position. Its energy keeps on changing from potential to kinetic and *vice versa* until the whole energy is dissipated into heat energy due to resistance of air and friction between the molecules of the spring.

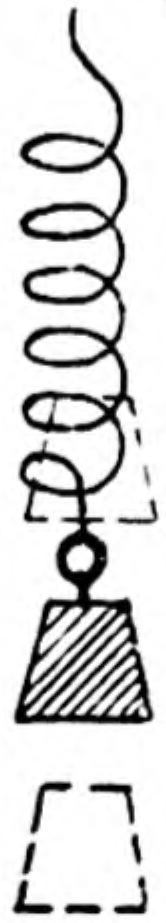


Fig. 7.7

11. **Conservation of Energy.** The above examples are special cases of a very general law called the *principle of conservation of energy.* According to this law whenever the transformation of energy from one form into other takes place, the gain of energy in some forms is exactly equal to the loss of the energy in other forms. This means that energy is neither created nor destroyed but it only changes from some form to another. This is one of the most fundamental or basic laws in Physics.

Certain forms of energy can be more easily used for obtaining useful work in machinery etc. For example the energy of a waterfall is used to drive electrical or mechanical machinery, while the immense quantity of water in the ocean may have much energy but it cannot be used to advantage. Similarly there is energy in hot bodies but to get useful work compressed steam is commonly employed. When energy passes from a form in which it can be usefully employed to other forms in which it cannot be put to use, the energy is said to be dissipated. The steam engine utilizes a small fraction of the total energy in the steam for driving machinery but a large part of the energy of steam is carried away by the exhaust steam given out from the engine.

Summary.

Work = Force × Distance travelled in the direction of the force.

$= F \times S.$

Erg is the amount of work done when a force of one dyne moves a body through one centimeter in the direction of the force.

Gram-centimeter is the amount of work done when a weight of one gram is raised vertically through one centimeter against the force of gravity.

Gram-centimeter = 1 gram wt. × 1 cm.

$= 981$ ergs.

1 joule $= 10^7$ ergs.

1 kilogram-meter $= 100{,}000$ gram-centimeters.

$= 98.1 \times 10^6$ ergs.

Foot-poundal (ft-lbl) is the amount of work done when a force of one poundal moves a body through on foot along its line of action.

Foot-pound (ft-lb) is the amount of work done when a mass of one pound is raised vertically through one foot against the force of gravity.

1 Foot-pound $=$ 1 Pound wt $\times$ 1 foot.

$= 32$ ft.-lbs.

Relation between the C.G.S. and F.P.S. system units.

1 Ft. lb $= 1.355$ joule $= 1.355 \times 10^7$ ergs.

1 Ft. lbl $= 42.1 \times 10^4$ ergs.

1 joule $= .738$ ft-lb.

Rate of doing work is called Power.

$$\text{Power} = \frac{\text{work}}{\text{time}} = \frac{F \times S}{t} = F \times \frac{s}{t}$$

$$= F \times v = \text{Force} \times \text{velocity}.$$

Watt is the power of doing work at the rate of 1 joule (10^7 ergs) per second.

1 Kilowatt $=$ 1000 watts $=$ 1000 joules per second.

Horse Power is the rate of doing 33,000 ft. lbs of work per minute or 550 ft-lbs per second.

1 Horse Power (H. P.) $=$ 746 watts $=$ ·746 Kilowatt

1 Kilowatt $= 1.34$ H. P.

Energy is the capacity to do work. The units of energy are the same as of work that is erg, joule, foot-poundal and foot pound.

Two practical units of energy $=$ a watt-hour and kilowatt hour.

Watt hour $=$ work done in one hour at the rate of one watt

$= 3600$ joules

Kilowatt hour=work done in one hour at the rate of 1000 watts

=3600,000 joules

The energy of a body by virtue of its position or condition is called its potential energy (P.E.).

Potential energy of a body of mass m, situated at a height h above the ground$=mgh$ ergs

or $=mh$ gram-centimeter

$\Big\}$ when m is in gms and h in cms.

$=mgh$ ft. lbs.

or $=mh$ ft. lbs

$\Big\}$ when m is in lbs. and h in ft.

The energy of a body by virtue of its motion is called its Kinetic energy (K.E.).

K.E. of a body of mass m and velocity v

$=\frac{1}{2}mv^2$ ergs in C.G.S. System.

$=\frac{1}{2}mv^2$ ft. lbls. in F.P.S. system.

Principle of Conservation of Energy. Energy can neither be created nor destroyed. It can, however, change from one form into another.

Energy is said to be dissipated when it passes into a form in which it cannot do any useful work.

Questions

1. An engine pulls a train on a level surface against a total resistance (frictional and wind) of 2000 pounds, with a uniform speed of 60 miles per hour. Find the H. P. exerted by the engine. 320 H.P.

2. A car travels at a uniform speed of 45 *m.p.h.*, against a frictional and wind resistance of 500 pounds. Find the H.P. used. 60 H. P.

3. What is the H.P. of an engine which pumps 2,000 gallons of water per minute from a well 50 feet deep to the surface of the ground. One gallon of water weighs 10 lbs. 30.3 H.P.

4. A man weighing 132 lbs. runs to the top of a building 50 feet high in 20 seconds. Calculate the work done by the man and his horsepower. 6600 ft. lbs ; $\frac{3}{5}$ H.P.

5. Explain the terms : Work, Power and Energy and give the *C.G.S.* units used in measuring their quantities.

What horsepower engine is required to hoist 100 tons of coal per hour from a mine 200 feet deep ?

(*P. U. 1945*) 22.63 H.P.

6. What do you understand by "Conservation of Energy ?" Show that the sum of Kinetic and Potential energies of a falling body is constant at all points.

(*P.U. 1950*).

7. A hammer weighs one Kilogram. If it strikes a nail with a velocity of 300 cms per second and drives it 5 cm, find the average force with which it strikes the nail.

Suppose average force of the hammer = F dynes

Then work done by the hammer $=F\times S=5$ F ergs.

$$=\text{K. E. of the hammer.}$$

$$=\tfrac{1}{2}mv^2$$

$$\therefore \qquad 5F=\frac{1000\times 300\times 300}{2}$$

or

$$F=9{,}000{,}000 \text{ dynes}$$

$$=\frac{9000000}{980\times 1000}$$

$$=9.18 \text{ Kilogram wt.}$$

8. Derive an expression for Kinetic energy of a body moving with a uniform velocity.

A lorry weighing one ton is moving at 30 *m.p.h.* Calculate its K. E. 67760 ft. lbs.

9. A man pulls a lawn roller on the ground through a distance of 60 ft. If he applies a force of 40 lbs. wt. in a

direction inclined at 30° to the ground find the work done by him.

Effective component of the force parallel to the ground. $\Big\} = 40 \cos 30^\circ$

$\therefore$ work done $= FS \cos\theta$ $= 60 \times 40 \cos 30$

$= 2078.4$ ft. lbs.

CHAPTER VIII

GRAVITATION AND GRAVITY

1. Forces of Gravitation.

Some very important types of forces in nature are what are called forces of gravitation or gravitational forces. In the universe material masses are found to exert attractive forces on each other. Although the real nature of these forces is very intricate yet their existence is universal. By understanding such forces the stability and motion of heavenly bodies is explained by astronomers and their study has been the object of investigation by some of the greatest scientists.

2. Law of Universal Gravitation.

This law was discovered by Newton and states that in nature there exists a force of attraction between any two material bodies. This force is directly proportional to the mass of each of the bodies and inversely proportional to the square of the distance between their centres. Expressed in the form of a formula, if m and m' are the masses and r is the distance between them, (fig. 8.1) then the force

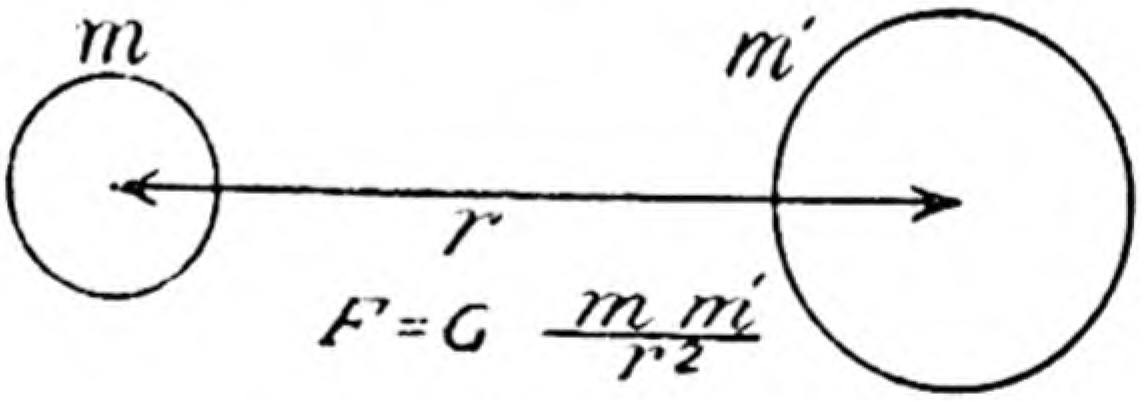


Fig. 8. 1

$$F \propto \frac{m\,m'}{r^2}$$

or $\frac{F}{m\,m'/r^2}$ is a constant which is called the constant of universal gravitation and generally denoted by the letter G. The law of gravitation is thus stated as

$$=G\frac{m\ m'}{r^2}$$

On the basis of this equation the constant of gravitation G may be defined as the force of attraction between two bodies each of which has a unit mass and which are placed at a unit distance apart.

The value of G has been found experimentally by determining the force between large lead spheres and a small metallic ball suspended from a fine fibre Fig. 8.2. The arrangement is very sensitive and capable of measuring very small force between the material masses. Working with such an experiment Cavendish in 1798 determined the value of G. Later experiments with more refined apparatus were done by Boys and others. The value of G is 6.65×10^{-8} in the C. G. S. or metric units.

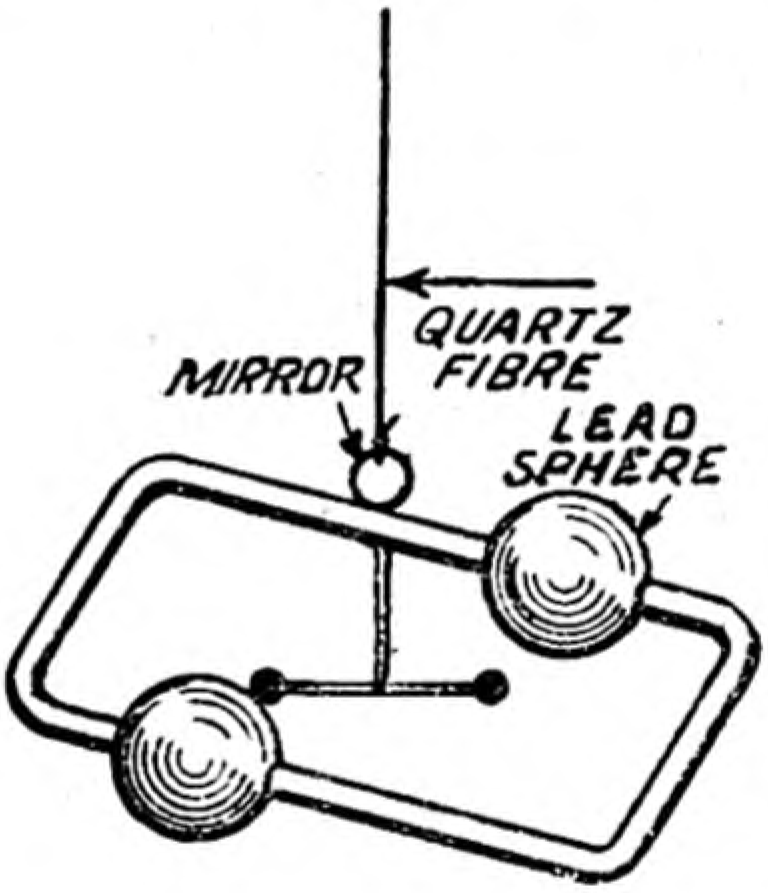
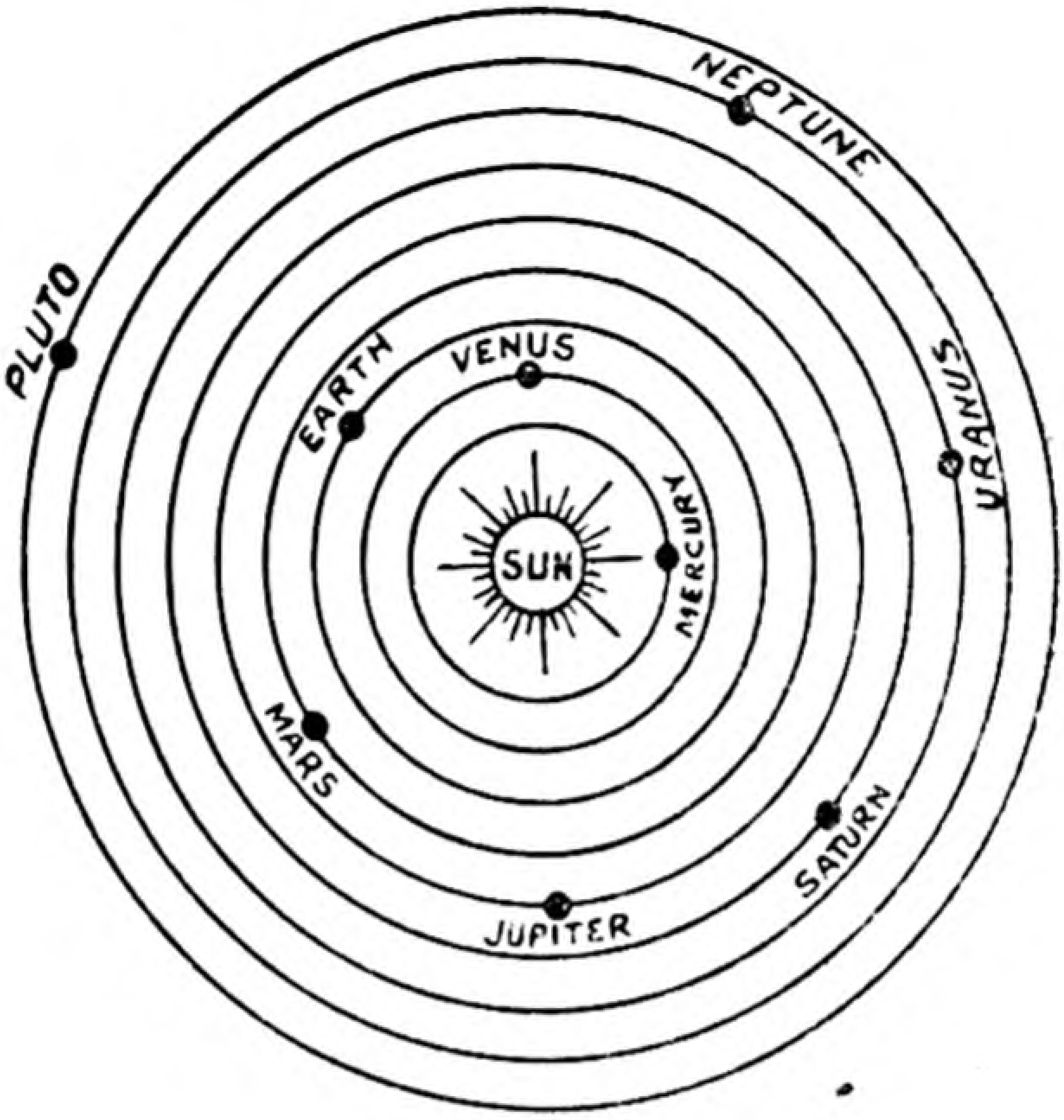


Fig. 8.2

3. Planetary motion.

There are a number of planets in the solar system. In the order of their increasing distances from the sun the planets are Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune and Pluto, fig. 8·3. It was shown by Kepler (1571—1630), that the planets revolve in somewhat elliptical orbits around the sun which is situated



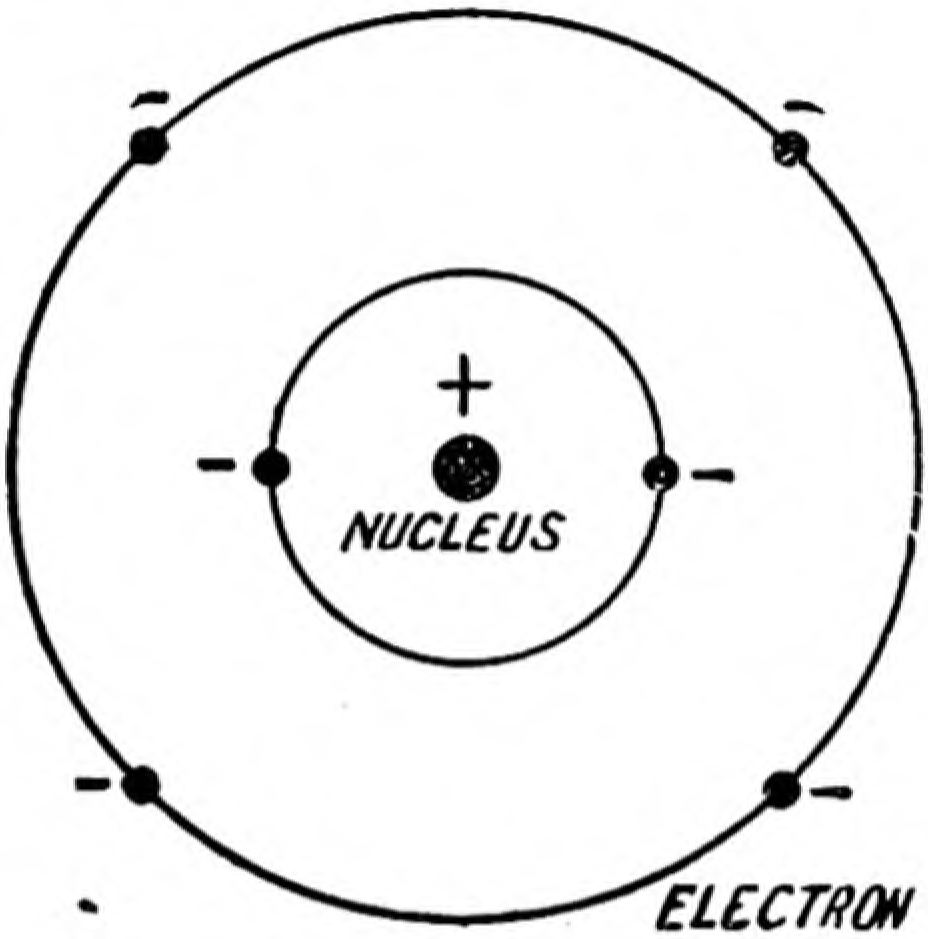
Solar system

Fig. 8.3.

at a focus of the ellipse. Newton gave the explanation of the dynamic equilibrium of the planets in their revolution. Each planet is attracted by the sun with a force given by the law of gravitation, while due to the velocity of revolution a centrifugal force also acts on the planet and these two forces are equal though acting in opposite directions. Thus if M is the mass of the sun and m the mass of the planet and R the distance between them, then if v is the velocity of revolution and w the angular velocity, we have

$$G\,\frac{Mm}{R^2}=\frac{mv^2}{R}=mRw^2.$$

Modern investigations have shown that the atoms of elements are also constituted on the same principle. The mass of the atom is mostly concentrated in a very small central part called the nucleus having a positive electric charge. A number of very tiny units of negative charge called electrons revolve around the nucleus. The centrifugal force of the revolving electron just equals the electrical attraction between the oppositely charged nucleus and an electron. The nucleus is thus like the sun and electrons like the planets so that the atom may be considered as a miniature solar system, fig. 8·4.



A carbon atom.

Fig. 8.4

4. Gravity—a special case of Gravitation.

The force which earth exerts on bodies is called the force of gravity. When a material body is released it falls downward towards the earth because of the pull or the force of gravity which the earth exerts upon it. A freely

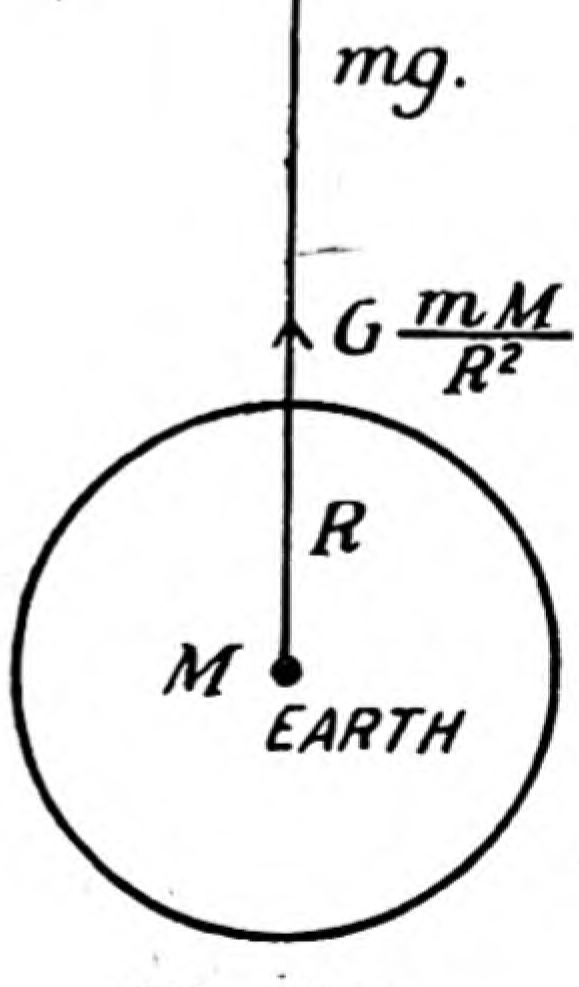


Fig. 8.5

falling body acquires an acceleration which is denoted by g. The force acting on the body is therefore $m \times g$, since, as seen previously, force equals product of mass and acceleration, fig. 8·5. Now according to the law of gravitation the force

$$mg = G\frac{mM}{R^2}$$

where M is the mass of the earth which is supposed to be located at its centre and R is the radius of the earth and G the constant of gravitation.

Since R is 4000 miles, any small extra height of the body above the surface of the earth makes no difference in the equation. This is the explanation given by Newton for the cause of a falling apple.

5. Weighing the Earth.

Cavendish used the equation $mg = G\frac{mM}{R^2}$ *or* $g = \frac{GM}{R^2}$ to determine the mass of the earth after he had found the value of G from his experiments. His experiment on measurement of G was therefore called the experiment for weighing the earth. g is the acceleration of a freely falling body and if we take its mean value as 981 cm./sec^2, the mass of the earth $M = \frac{gR^2}{G}$. Since R = 4000 miles and $G = 6.65 \times 10^{-8}$ by substituting these values we obtain

$$M = \frac{981 \times (4000 \times 5280 \times 30.48)^2}{6.65 \times 10^{-8}}$$

$$= 6.1 \times 10^{24} \text{ Kilograms}$$

$$= 13.4 \times 20^{24} \text{ lbs.}$$

Knowing the mass and radius of the earth its mean density can be easily calculated. Since

$$\text{mass} = \text{Volume} \times \text{density}$$

$$M = \tfrac{4}{3}\pi R^3 \times P,$$

where P stands for the mean density of the earth which is considered to be a sphere of radius R and volume $\frac{4}{3}\pi R^3$. Substituting the value of M and R in this equation, P is found to be equal to 5.46 grams per *c.c.*

6. Falling Bodies and Air Resistance.

When a body is dropped from the top of a building, it falls downward to the earth. If it is compact heavy material it falls faster, but if it is light and thin as a piece of paper or a feather, it falls slowly. This is because in the case of a body falling through air, there are two forces acting on it, i. e., the attraction or the pull of the earth and the resistance of the air through which it moves. If the surface area is large, the air resistance becomes considerable and the rate of fall becomes slow. A flat piece of paper falls slowly, but if the same paper is crumpled into a small ball, it falls faster. When a parachuter jumps from an aeroplane, he falls rapidly, but after he opens the parachute, his rate of fall becomes less. This is due to the large resistance offered by the air to the open parachute, fig. 8.6.

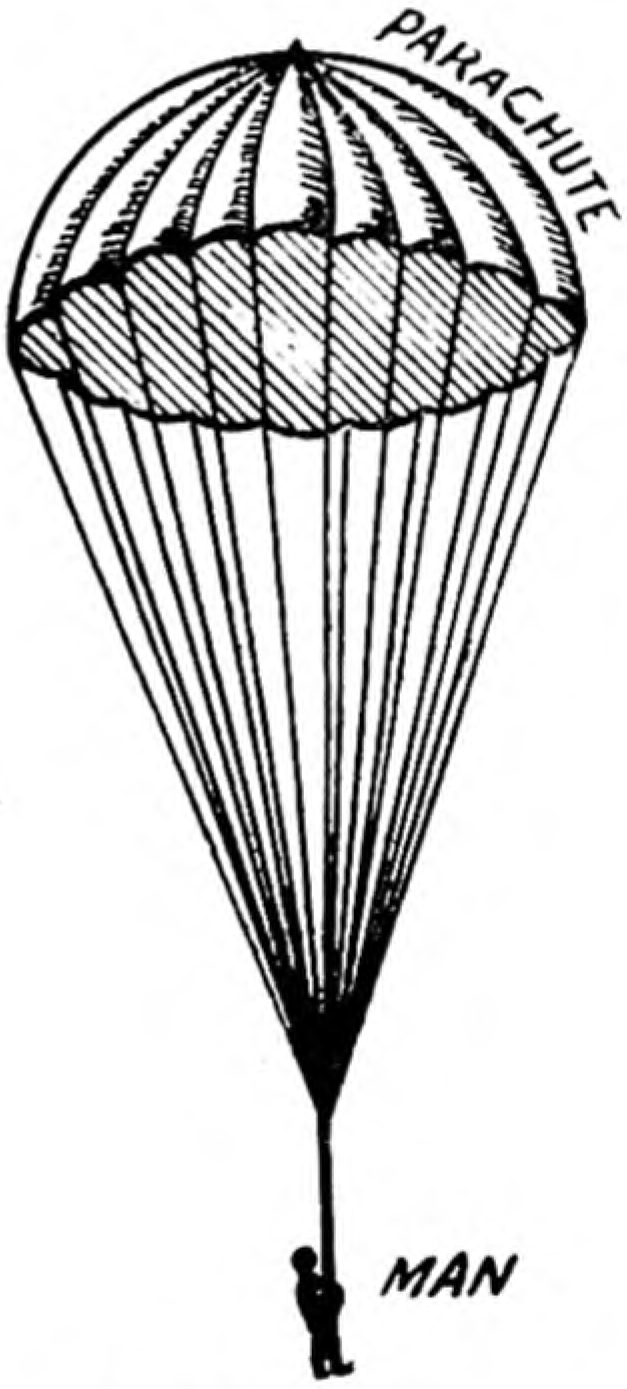


Fig. 8.6

7. Galileo and his experiments on falling bodies.

Falling bodies have been a subject of inquiry by various thinkers. Aristotle, an ancient Greek philosopher, taught more than two thousand years ago that bodies fall towards the earth, at a rate which is proportional to their masses. According to Aristotle's views if a weight of one pound and a weight of five pounds are released from the top of a building, the five pound weight should fall five times faster than the one pound weight.

The views of Aristotle remained unquestioned until Galileo (1564-1642), an Italian scientist, determined experimentally the time of fall of bodies. He conducted his experiments at the famous leaning tower in Pisa. Solid bodies of different metals and of different weights were placed inside spherical boxes of same shape and size and let fall at the same moment from the top of the tower. Galileo found that they all reached the ground at the same instant. He thus disproved experimentally the teachings of Aristotle. Galileo is often called the father of modern experimental science. He was a pioneer who placed his faith in the results of experiments rather than in pure speculation. Inspite of his practical proof that bodies of different weights fall at the same rate through air people refused to believe him and he was convicted for holding views contrary to those of a Christian Church. He was sentenced and sent to prison. But Galileo's faith in the truth of his experiments was unshaken and ultimately reasonable men bagan to believe his views, which have since been found to be correct.

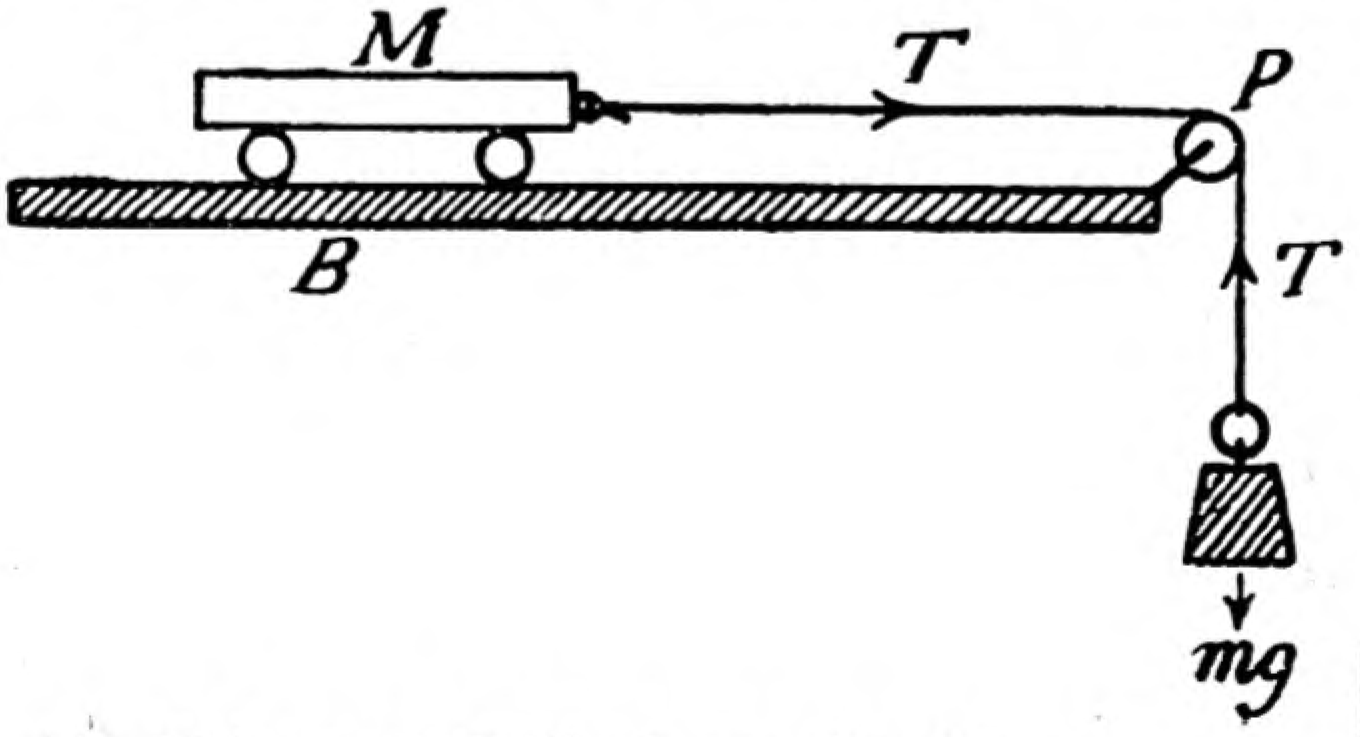
8. Measurement of g and the necessity for its dilution.

It has been mentioned previously that the acceleration of gravity or g is one of the important physical quantities at any place on the earth. Experimental determination of g cannot be easily made by simply taking the time of fall of a body through a known distance. Theoretically if a body falls through a height h in a time t, we have, on the basis of the formulae of acceleration already given, that $h=\frac{1}{2}gt^2$. But since the value of g is near about 32 ft. per second a body falls through a distance of about 16 ft. in one second and 64 ft. in two seconds. This rate of fall is so rapid that to measure the time for any ordinary distance is a difficult experimental problem. We, therefore, vary the experiment in such a way that the rate of fall is reduced by a known calculable amount. This is expressed in other words by saying that g or the acceleration of the falling body is diluted. The falling body has a smaller acceleration and it takes longer time to fall through measurable distances. The time now is more easily and accurately found out and g determined. Examples of reducing the rate of fall or diluting g are given in the following sections.

9. Motion of connected Bodies.

The acceleration of a falling body may be reduced if the downward force of the earth acting upon it is opposed by some other force. This opposite force may be produced if the falling body has to move another mass in addition to its own. This means that another mass is connected by an inextensible string with the falling body, the string being made to pass over a smooth and frictionless pulley. Two cases are of importance for such motion of connected bodies. In the first case, the falling body drags a second mass on a horizontal frictionless surface. In the second case, while the falling body moves downwards it pulls in the upward direction another mass connected to it with a string passing over a pulley.

Case 1. *Horizontal motion of the connected body.* Suppose in figure 8.7, m is the mass which falls in the



B is smooth horizontal table.

Fig. 8.7

downward direction. It is connected by an inextensible string, which passes over a frictionless pulley P. The other end of the string is attached to the mass M which can move over a horizontal table B without friction. The forces acting on the falling body m are, the pull of the earth mg in the vertically downward direction and the tension T of the string in the upward direction. Suppose that the body falls with an acceleration equal to a. Then we may write.

$$mg - T = ma \qquad (i)$$

The dragged mass M also moves with the same acceleration a and the pull acting on it is the tension of the string, and therefore

$$Ma = T \qquad (ii)$$

Substituting this value of T in equation (i) we get

$$mg - Ma = ma$$

or

$$(m+M)a = mg$$

$$\therefore \quad a = \frac{m}{(m+M)} g \ldots \text{(iii)}$$

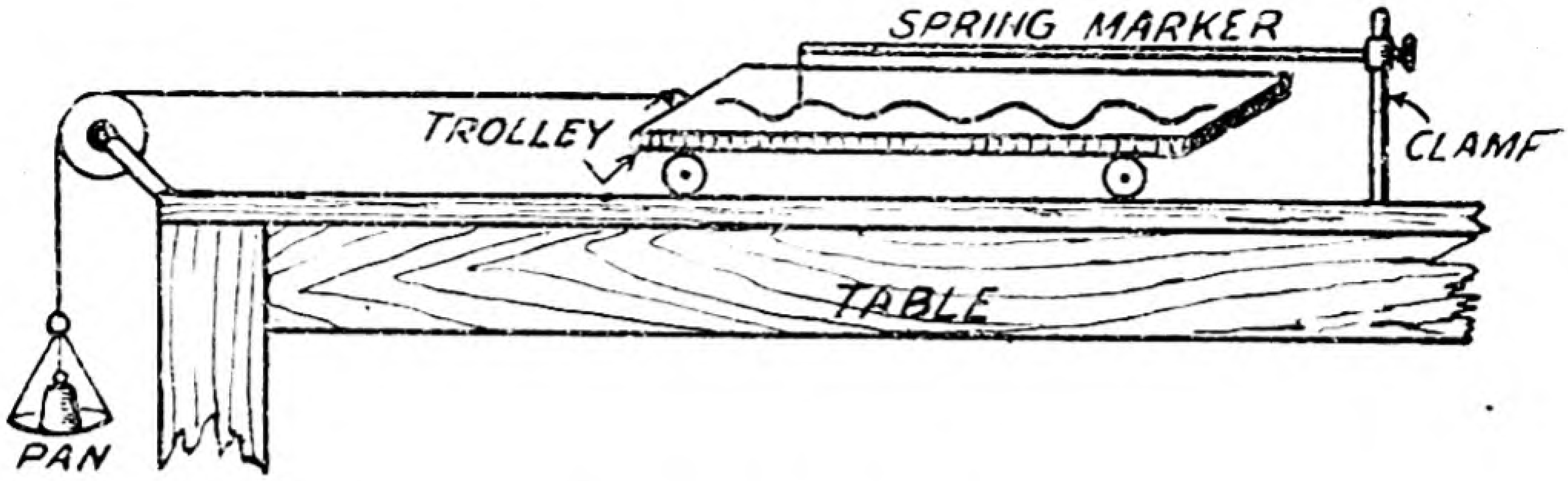
The same equation may be derived if the tension in the string is not taken into consideration. The total mass which is moved is $(m+M)$ and it acquires an acceleration a. The total force acting is mg.

$$\therefore \quad mg = (m+M)a$$

or

$$a = \frac{m}{(m+M)} g$$

Fletcher's trolley. A practical form of apparatus based on the above principle, which can be used for measuring the acceleration, a, and thereby deducing the value of g, is called Fletcher's trolley. It consists of a long rectangular wooden or light metallic bar, called the trolley which can move smoothly on a horizontal table. A string attached to the trolley, passes over a smooth pulley and is fixed to a scale pan suspended vertically, fig. 8·8. By placing weights in the pan or on the trolley, m and M


SPRING MARKER
TROLLEY
CLAMP
TABLE
PAN

Fletcher's trolley.

Fig. 8.8

can be varied. The distance moved in known small intervals of time is measured from a wavy trace marked on the upper surface of the moving trolley by a vibrating metallic strip. A small pencil or an inkmarker attached to the vibrating strip, leaves a trace on a paper fixed to the surface of the trolley.

Case 2. Suppose that two bodies of masses M and m are suspended from the ends of an inextensible string

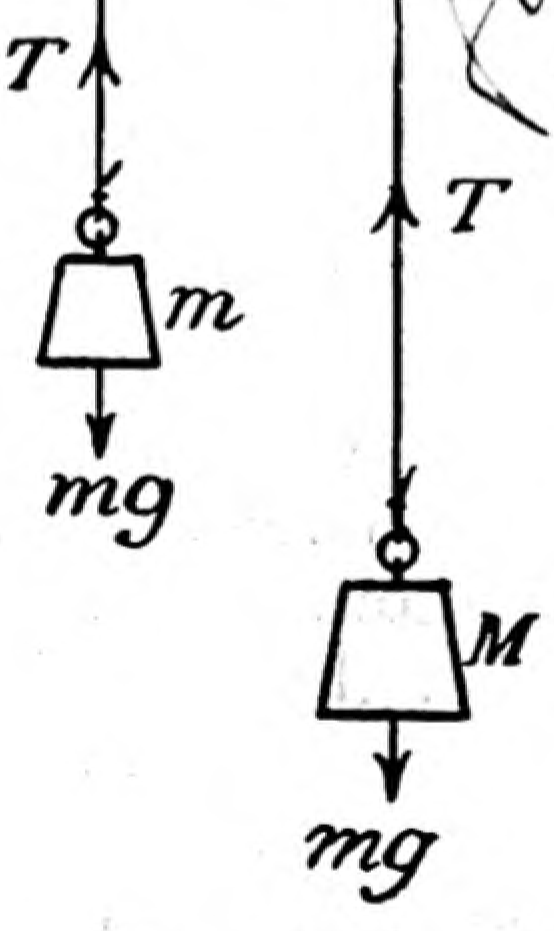

FRICTIONLESS PULLEY
T
T
m
M
mg
mg

Fig. 8.9

of negligible weight which passes over a smooth and frictionless pulley, as in fig. 8.9. If the mass M is greater than m, then the system when free, will move so that M descends with a certain acceleration while m moves upwards with the same acceleration. The forces acting on M are Mg downwards and the tension T of the string upwards. If a is the acceleration with which M falls then

$$Mg - T = Ma \ldots\ldots(\text{iv}).$$

The forces acting on m are tension of the string T upwards and mg downward. Since T is greater than mg this mass rises up but with the same acceleration as that with which M descends. Therefore we may write

$$T - mg = ma \ldots\ldots(\text{v})$$

Adding equations (iv) and (v) we get

$$Mg - mg = Ma + ma$$

or $$g(M - m) = a(M + m)$$

$$\text{Whence } a = \frac{M - m}{M + m} \cdot g \ldots\ldots(\text{vi})$$

To calculate the force of tension T in the string we divide equation (iv) by equation (v)

$$\frac{Mg - T}{T - mg} = \frac{M}{m}$$

or $$MT - mMg = mMg - mT$$

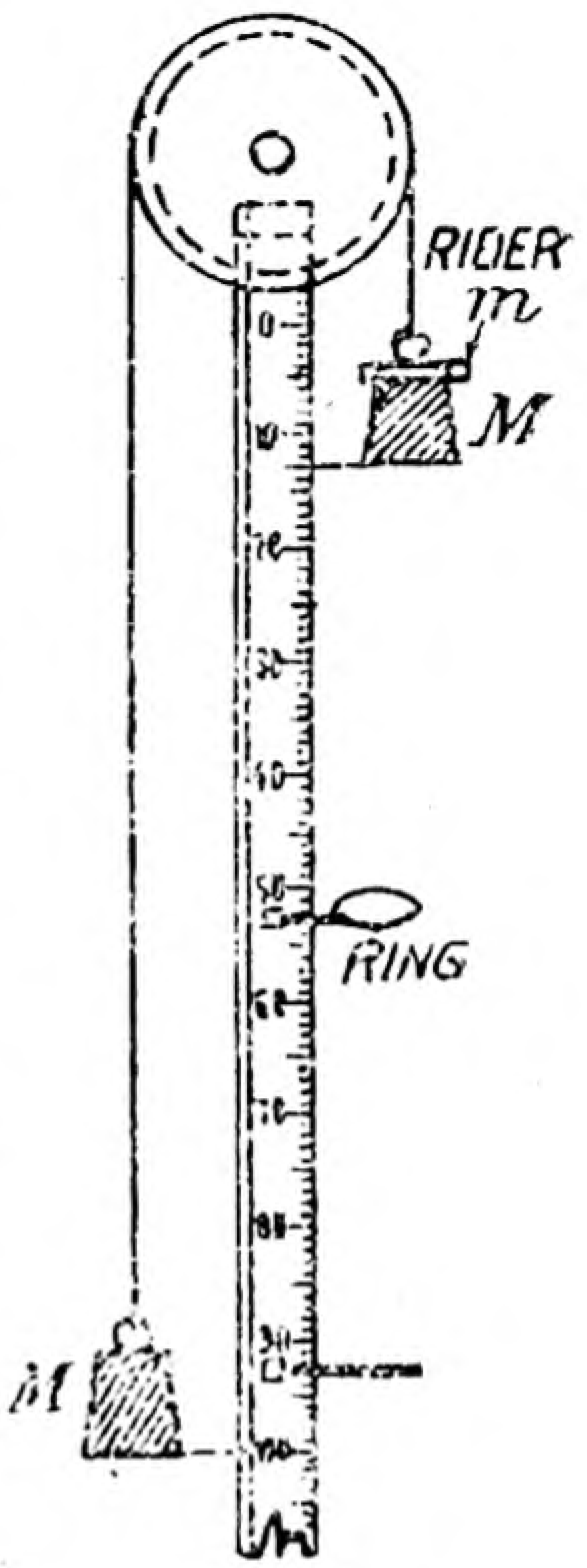
or $$T(m + M) = 2mMg$$

$$\therefore \quad T = \frac{2mM}{(m + M)} \cdot g \ldots\ldots(\text{vii})$$

The force acting on the axle of the pulley is given by 2T.

10. Atwood's machine.

This is an apparatus which utilizes the motion of two masses connected with a string which passes over a pulley, one of the masses moving downward and the other in the upward direction. A rough estimate of g or acceleration of gravity can be made with the Atwood machine. It consists of a vertical wooden pillar about 6 to 7 feet high and carrying a smooth pulley ball bearings in order that its friction may be very small. The vertical pillar carries a scale marked in centimeters or inches. A light string passes over the pulley and has two equal masses M suspended from it fig. 8·10. Ordinarily the masses are at rest but if a small rider of mass m is placed on one of the bigger masses it becomes heavier and begins to descend with a certain small acceleration. The force moving the system is mg since the two heavy weights are equal. The mass moved is $(2M+m)$ if we, neglect the small mass of the string and the friction of the pulley. If the acceleration of the moving system is a then



Atwood machine.

Fig. 8.10

$$a\,(2M+m)=mg.$$

$$\text{or}\qquad a=\frac{\cancel{M}\,m}{(2M+m)}\cdot g$$

To determine the acceleration the time taken for the mass to descend through known height is found out with a stop watch. Since $h=\frac{1}{2}\,at^2$, if the initial velocity is zero, we can find a from measurement of h and t.

The acceleration a can also be found out if we use the formula

$$v_2^2-v_1^2=2\,a\,h.$$

Since v_1 is zero the formula used is

$$v_2^2=2\,a\,h.$$

v_2 is found by letting the mass M pass through a ring which stops the rider and the system then moves with a uniform velocity v_2. Measurement of the time taken to move a known further distance gives the velocity v_2.

In an improved form of the Atwood machine fig. 8.11 a light paper tape is used instead of the string and it passes over the pulley and carries the two equal masses at its ends. When the rider is placed on one of the masses, it begins to descend. A thin metal strip having an ink marker attached to it, vibrates with a definite frequency and makes a mark on the moving strip. Since the paper strip moves with an acceleration and the spring marker vibrates perpendicular to the direction in which the paper tape moves, a wavy curve is produced on the paper tape. From this curve the distance moved by the tape in known small intervals of time is found out and the acceleration a calculated. By substituting the value of a in the equation

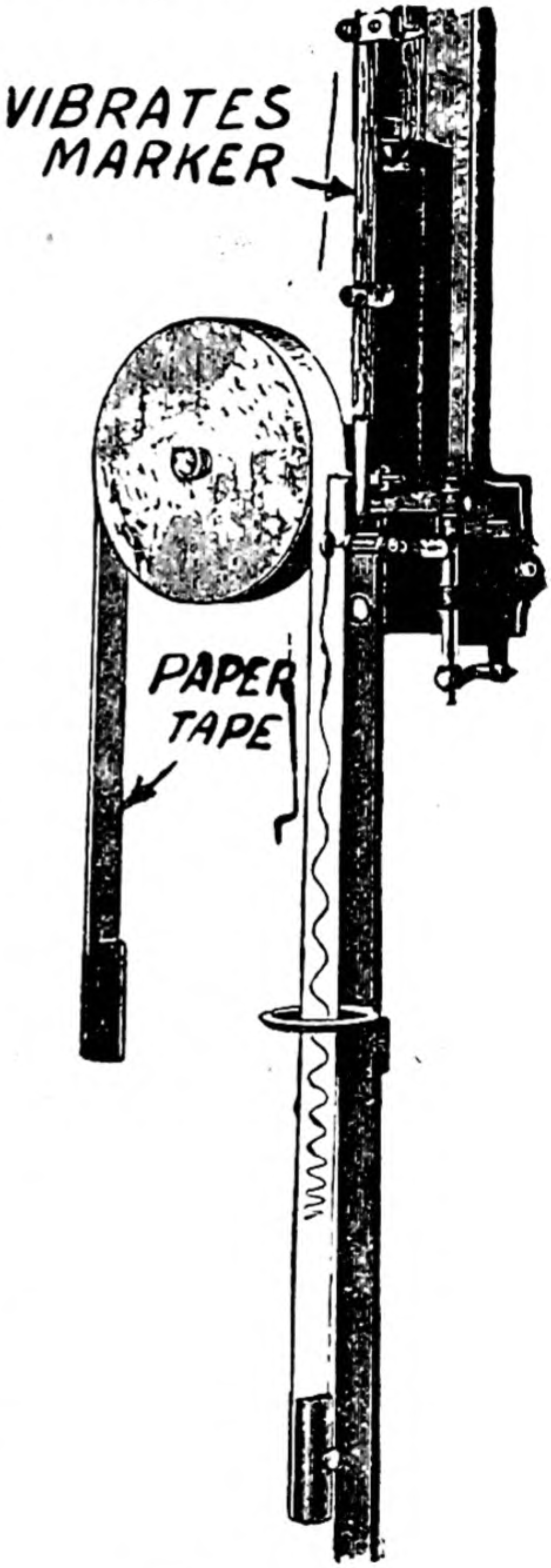


Fig. 8.11

$$a = \frac{m}{2M+m} \cdot g,$$

the value of g, the acceleration of gravity can be found out. If necessary a correction for the mass of the paper tape and the friction can be applied.

11. Acceleration of bodies moving down along an inclined plane.

Acceleration of a falling body is also reduced if a body is made to move down an inclined plane instead of falling vertically downward. Suppose, in fig. 8.12, AB is a smooth frictionless plane inclined at an angle θ to the horizontal direction BC, so that $\sin\theta = \frac{AC}{AB}$. A body of mass m resting on the plane can only slide down along the surface of the plane AB. The forces acting on the body are (1) pull of the earth equal to mg acting downward and (2) the normal reaction R of the plane acting perpendicular to the plane. The pull of the gravity mg can be resolved into two components $mg \cos\theta$ acting perpendicular to the plane and in a direction opposite to R, and $mg \sin\theta$ in the downward direction of the inclined plane. Since there is no motion perpendicular to the plane

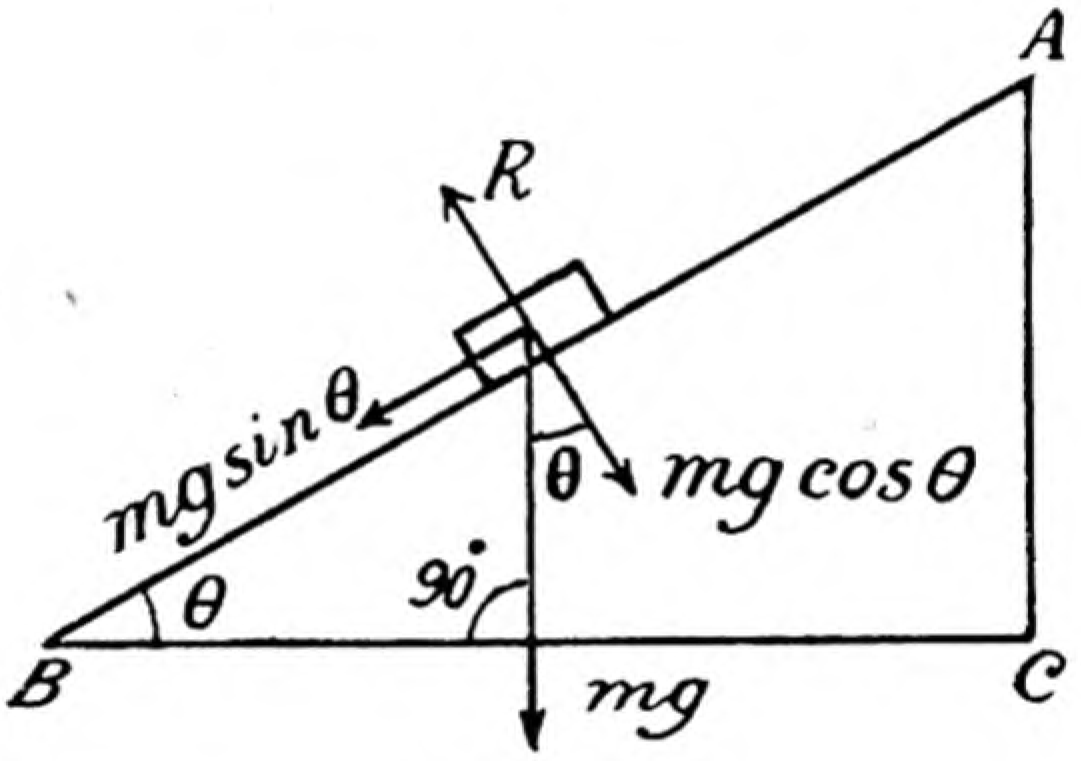


Fig. 8.12

$$R = mg \cos\theta$$

The body moves down the plane under the action of the component $mg \sin \theta$. If it moves with an acceleration a then

$$ma = mg \sin \theta$$

or
$$a = g \sin \theta$$

Since the value of $\sin \theta$ lies between zero and one, the acceleration a can be decreased to any desired amount between zero and full value of g by giving a suitable value to θ

It is obvious that if the body m is to be moved up the inclined plane a force at least equal to $mg \sin \theta$ must be applied to it in the upward direction parallel to the inclined plane.

Suppose the body starts from rest from the point A and slides down the plane to the bottom B. The velocity v acquired by it on reaching B is given by the equation.

$$v^2 = 2a \times AB = 2g \sin \theta \times AB$$
$$= 2g \times \frac{AC}{AB} \times AB = 2g \times AC.$$

If the body falls vertically under the action of gravity from A to C, the velocity acquired by it will also be

$$v^2 = 2g \times AC.$$

An inclined plane is often said to rise h feet in l feet. This means that

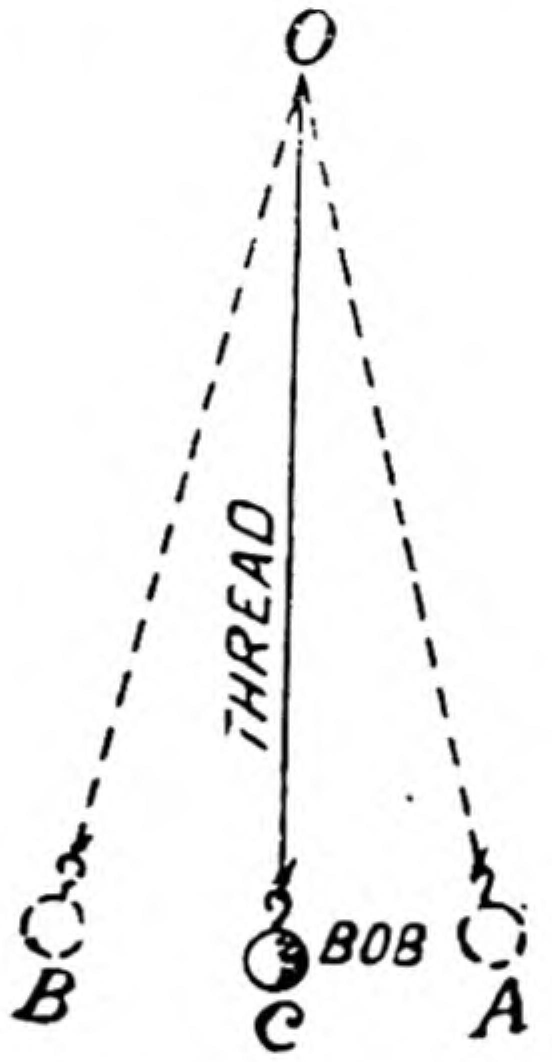
$$\text{Sin } \theta = \frac{h}{l}$$

Thus an incline of 1 in 50 means that the inclination θ is such that

$$\text{Sin } \theta = \frac{1}{50}.$$

12. The Simple Pendulum.

A simple pendulum provides an easy and quite accurate method of measuring g at a place. It consists of a small heavy bob tied to a light, thin and inextensible string suspended from a point O, fig. 8.13. The distance between the point of suspension O and the centre of gravity G of the bob is called the length of the pendulum. If the bob is displaced from its position of equilibrium C to a point A and released, it oscillates between the positions A and B.



OG=l

l=Length of thread plus radius of Bob

Fig. 8.13

The maximum displacement from the mean position C to A or B is called the amptitude of oscillation of the pendulum. Thus in the figure the amptitude equals CA or CB. The motion of the bob from C to A, back to C and then to B and again to C makes one complete oscillation of the pendulum and the time taken for performing one oscillation is called the time period or simply the period of the pendulum. As time passes the amptitude of the vibration goes on decreasing due to air resistance and the pendulum finally comes to rest.

Experiments show that if the amptitude is small the time period remains constant even though the amptitude may vary due to air resistance. This is also expressed by saying that the oscillations are isochronous. The time period is found to depend on the length of the pendulum and the value of g. At any place the time period t of a simple pendulum is expressed by the formula

$$t=2\pi\sqrt{\frac{l}{g}},$$

where l is the length of the pendulum and g the acceleration due to gravity. This relation can also be written as

$$t^2=\frac{4\pi^2 l}{g}$$

$$\text{or }\ g=\frac{4\pi^2 l}{t^2}.$$

Thus the value of g can be found out by measuring the length and periodic time of the simple pendulum. The actual determination of g by this method is one of the experiments for the Intermediate students.

A pendulum which has its time period equal to two seconds is called a seconds pendulum. Such a pendulum takes one second for half a vibration.

If we take a pendulum to different places on the surface of the earth and keeping its length constant determine its time period it is found that the time period is different at different places. This clearly shows that the value of g is not constant at all places.

$$\text{Since } t=2\pi\sqrt{\frac{l}{g}},$$

time period will increase with decrease in g and *vice versa.*

13. Variations in the value of g.

Since the acceleration of a falling body is due to the force which the earth exerts on it *i. e.*, the gravitational force of the earth, it follows that the value of g will decrease, when we go higher and higher up, because the distance from the centre of the earth increases. Suppose that a body is at a distance h above the surface of the earth. Then if R denotes the radius of the earth, the distance of the body from the centre of the earth is $(R+h)$.

Let m represent the mass of the body and M the earth, which may be supposed to be located at its centre. According to the law of gravitation the force of the earth acting on the body is $=G\frac{mM}{(R+h)^2}$, and if the body is free to fall it will have an acceleration, $g=\frac{GM}{(R+h)^2}$, since acceleration is the ratio of force divided by mass. It is thus seen that the higher is a body taken up from the earth, the less becomes the value of g, and when $h=R$ *i. e.*, at a height above the earth's surface equalling its own radius (4000 miles) g becomes one fourth of what it is at the surface of the earth or only about 8 ft/sec.^{2}

Since the polar diameter of the earth is about 13½ miles less than the equatorial diameter, the value of g at the poles is greater than at the equator. The rotation of the earth also produces the same effect as has been described previously. It is nearly 983 cm/sec^2 at poles and 978 cm/sec^2 at equator. For normal purposes an average or mean value of 980 cm/sec^2 is used. Further as the weight of a body is due to the pull of the earth, the weight is greater at the poles than at the equator, even though the mass of a body remains constant. Similarly when a body is taken higher and higher up its weight decreases, although its mass is constant. The weight of a body depends on the gravitational force and therefore if the body of the same mass is taken to the moon, since moon is smaller than the earth, the weight of the body becomes only one sixth on the moon. Similarly the weight of a body would become 27 times greater on the sun.

As we descend down into the earth as in mines etc., the value of g decreases. If we descend a distance d, then g is proportional to $(R-d)$, where R is the radius of the earth.

Very fine and accurate measurements of g often show the effect of neighbouring mountains or mineral deposits.

Such measurements are used for geophysical prospecting, that is to say, finding out the presence of underground oil deposits and other minerals by physical experiments.

Summary.

Force of gravitational attraction between two bodies is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

$$F = G\frac{m\,m'}{r^2}$$

G is the constant of gravitation and equals $6.65 \times 10,^{-8}$ in C. G. S. system.

The gravitational force between the sun and the planets revolving round it, balances the centrifugal force acting on the planets due to their revolution round the sun.

The force of gravity is only a special case of gravitation.

Acceleration due to gravity, $g = G\frac{M}{R^2}$, when G is the gravitational constant and M and R are the mass and radius of the earth respectively. This acceleration is the same for all bodies in the absence of air resistance.

Mass of the earth $= 6.1 \times 10^{24}$ kilograms

$= 13.4 \times 10^{24}$ lbs.

Mean density of the earth $= 5.46$ grams per c. c.

When a mass m, in falling vertically downward drags another mass M on a horizontal plane, its acceleration

$$a = \frac{m}{(M+m)} \cdot g.$$

In an Atwood machine arrangement, if a mass M while falling freely, pulls up a lighter mass m, then the acceleration of the system

$$a = \frac{(M-m)}{(M+m)} \cdot g$$

and tension of the string

$$T=\frac{2m\,M}{(m+M)}\cdot g$$

Acceleration of a body moving down a smooth plane inclined at an angle θ to the horizontal is $g \sin \theta$

The time period t of a simple pendulum of length l is given by the formula

$$t=2\pi\sqrt{\frac{l}{g}}$$

The value of g slightly changes at different places on the surface of the earth. This is because the earth is not a perfect sphere and secondly because the centrifugal force due to the rotation of the earth is greatest at the equator and decreases as the latitude increases. Large land masses and deposits of minerals in the locality also affect the value of g.

The value of g also changes with the distance from the surface of the earth.

Above the surface of the earth

$$\frac{\text{gravity at height } h_1 \text{ from ground surface}}{\text{gravity at height } h_2 \text{ from ground surface}}=\frac{g_1}{g_2}$$

$$=\frac{(R+h_2)^2}{(R+h_1)^2}$$

and inside the earth,

$$\frac{\text{gravity at depth } d_1 \text{ from ground surface}}{\text{gravity at depth } d_2 \text{ from ground surface}}=\frac{g_1}{g_2}=\frac{(R-d_1)}{(R-d_2)}\cdot$$

R is the radius of the earth.

Exercises

1. State Newton's law of universal Gravitation. How can the law be used to weigh the earth ?

2. A man weighs 120 lbs on the surface of the earth. What will be his weight at a height of (*a*) 1000 miles, (*b*) 4000 miles and (*c*) 16000 miles above the earth's surface ?

Take radius of earth equal to 4000 miles.

(*a*) 76·8 lbs. (*b*) 30 lbs. (*c*) 4·8 lbs.

3. Why is acceleration due to gravity slightly different at different places on the earth ? Is it greater at the equator or at the poles ?

4. Show with the help of Newton's formula for gravitational attraction between two bodies, that acceleration due to the pull of the earth is the same for all bodies at a place.

$$\left.\begin{array}{l}\text{Force of attraction between a}\\ \text{body of mass } m \text{ and the earth}\end{array}\right\} = G\frac{mM}{R^2}$$

$$= m \times \text{acceleration}$$

$$\therefore \qquad \text{acceleration} = \frac{GM}{R^2}$$

Since mass of earth M and radius of earth R are constant at a place, the acceleration $\frac{GM}{R^2}$ must be constant.

5. Explain why it is necessary to dilute gravity for determining the acceleration of a freely falling body. What are the various methods of diluting gravity ?

6. Find the length of a seconds pendulum at a place where $g = 980$ cm/sec.^{2}

The period of a seconds pendulum = 2 sec.

$$\therefore \qquad 2 = 2\pi\sqrt{\frac{l}{g}} \quad \text{or} \quad 1 = \pi^2\frac{l}{g}.$$

$$\therefore \qquad l = \frac{g}{\pi^2} = \frac{980}{3{\cdot}14 \times 3{\cdot}14} = 99{\cdot}4 \text{ cms.}$$

7. A smooth inclined plane rises one in ten. Find the acceleration of a body sliding down this plane. If the body weighs 5 lbs what force will drive it down the plane and what will be the reaction of the smooth inclined plane ?

$$\text{Sin } \theta = \tfrac{1}{10}, \qquad \cos \theta = {\cdot}995.$$

$\therefore$ acceleration down the plane $= g \sin \theta = 3{\cdot}2$ ft/sec^2

Force down the plane $= \text{W} \sin \theta = \frac{5}{10}$

$= \frac{1}{2}$ lb. wt. $= 16$ lbls.

Reaction of the plane $= \text{W} \cos \theta = 5 \times {\cdot}995$

$= 4{\cdot}975$ lb. wt.

$= 159{\cdot}2$ lbls.

8. The value of g at the poles is 983 and at the equator 978 cm/sec^2. How will the time period of a pendulum vary when it is taken from the equator to the north pole ?

$$\text{Since } t = 2\pi \sqrt{\frac{l}{}}$$

$$\frac{\text{Time period at the equator}}{\text{Time period at the pole}} = \sqrt{\frac{g \text{ at the pole}}{g \text{ at the equator}}}$$

$$= \sqrt{\frac{983}{978}} = 1{\cdot}00256$$

9. In an Atwood machine the weights are 250 grams each. If the weight of the rider be 20 grams, calculate the acceleration of the system. 37.7 cm/sec^2.

19. Which is greater, the attraction of the earth for a 5 lb. mass or the attraction of the 5 lb. mass for the earth ?

11. Two masses of $33\frac{1}{2}$ lb. and $30\frac{1}{2}$ lb. are tied to the ends of a string, that passes over a smooth frictionless pulley. If the two masses are released, calculate (*a*) the acceleration of the system (*b*) the tension in the string and (*c*) the downward pull acting on the axle of the pulley. Suppose that the string is weightless.

(*a*) 1·5 ft. /sec.^{2}
(*b*) 1021.75 lbls.
(*c*) 2043 5 lbls.

12. The length of smooth inclined plane is l and its height is h. Show that a body which slides down the length l will acquire the same velocity as a body that falls through the vertical height h.

acceleration down the plane $= g \sin \theta$

$\therefore$ velocity after sliding distance $l = \sqrt{2g \sin \theta \times l}$

$$= \sqrt{2g \frac{h}{l} \times l} \quad \left[\sin \theta = \frac{h}{l}\right.$$

$$= \sqrt{2gh}$$

$=$ velocity after falling vertical height h.

13. A mass of 60 lbs. is resting on a smooth horizontal table. A string passing over a smooth frictionless pulley connects it to a 4 lb. mass which is free to fall vertically downward. Calculate the acceleration of the body and the tension of the string. (*a*) 2 ft./sec^2, (*b*) 3·75 lbs. wt.

CHAPTER IX

FORCES AND TORQUES.

1. Rotative action of forces.

We have previously seen that when a force acts on a body it produces or tends to produce motion of the body. There are two types of motion which commonly occur in a rigid body. Either the body is displaced as a whole unit in the direction of the force and this type of motion is called linear motion or translation ; or a force may however produce or tend to produce rotation of the body around some axis. As an example the force exerted on the pedals by a person driving a bicycle, goes to produce rotation of the chain wheel of the bicycle, fig. 9.1. Similarly when we pull at the spoke of a heavy wheel, the effect of the force applied is to cause the rotation of the wheel around the axle. As a result of experience we know that the effectiveness of a force to produce rotation depends on the magnitude of the force and also on the distance of its line of action from the point or the axis around which rotation is produced. In fig. 9.2 (*a*), force F is shown acting at a point A on the

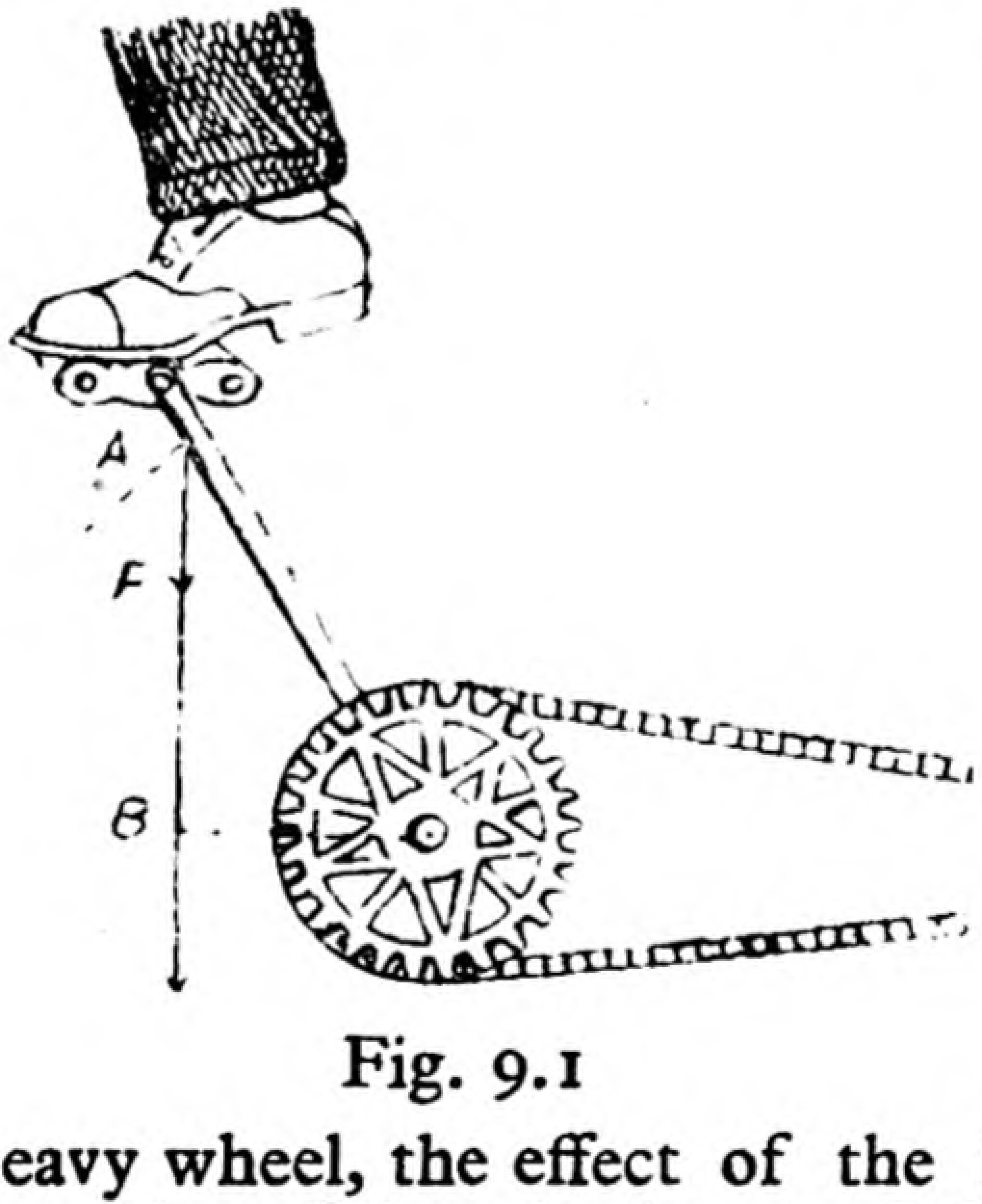


Fig. 9.1

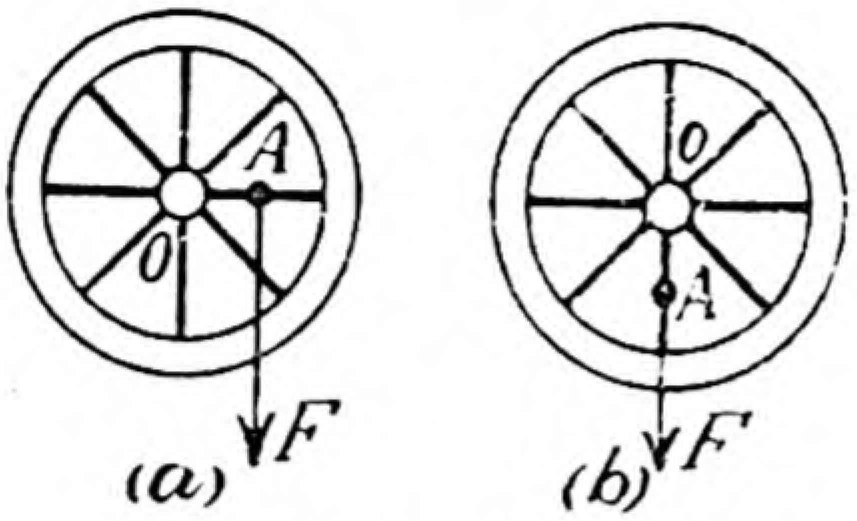


Fig. 9.2

spoke of a wheel. The rotative action of this force depends not only on the magnitude of F, but also on the distance between O and the line of action of F. While in fig. 9.2 (*a*), the line of action of the force is at a perpendicular distance OA from the axis of rotation O, in the case of fig. 9.2 (*b*), the line of action of the force passes through the axis of rotation. In the case (*b*) the force has no rotative action around O.

2. Moment of a force.

Moment of a force around any point is its tendency to produce rotation about the point, and it is measured by the product of the force and the perpendicular distance of the line of action of the force from the point. Consider a rigid body capable of rotation around a point P, and a force F acting at the point A as in the fig. 9.3. Then the moment of the force F around P equals F × PN, where PN is the perpendicular from P on the line of action of the force.

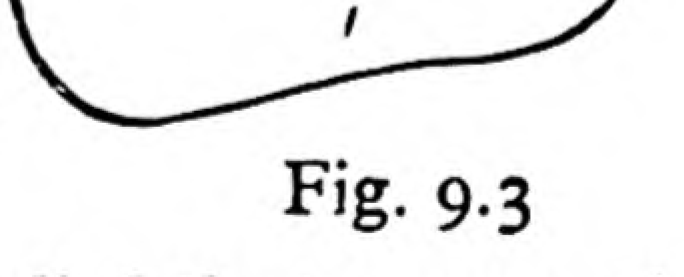
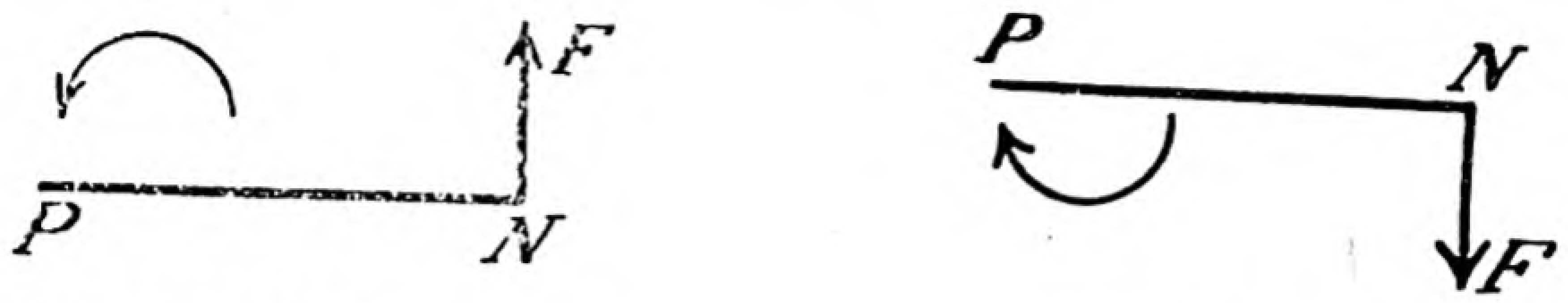


Fig. 9.3

The movement of a force is also called the torque and is taken as positive if it tends to cause the rotation in the counterclockwise direction, and it is considered as negative if it tends to produce rotation in the clockwise direction. In fig 9.3, the movement of F around P is positive. The direction of rotation and the sign of the torque are more clearly shown in fig 9.4, in which (*a*) represents a counter-



Counterclockwise or positive torque=F×PN
(a)

Clockwise or Negative torque=F×PN
(b)

Fig. 9.4

clockwise or positive torque and (*b*) a clockwise or negative torque, the magnitude of which in both cases is F×PN.

3. Parallel Forces.

Forces which have their lines of action parallel to one another are commonly called parallel forces. If the parallel forces are acting in the same direction, they are called like parallel forces but if their directions are opposite they are termed unlike parallel forces.

4. Two parallel forces and their resultant.

We can learn about the effect of parallel forces by performing the following experiment. Let us take a uniform meter rod and find its weight W. Take a known weight W

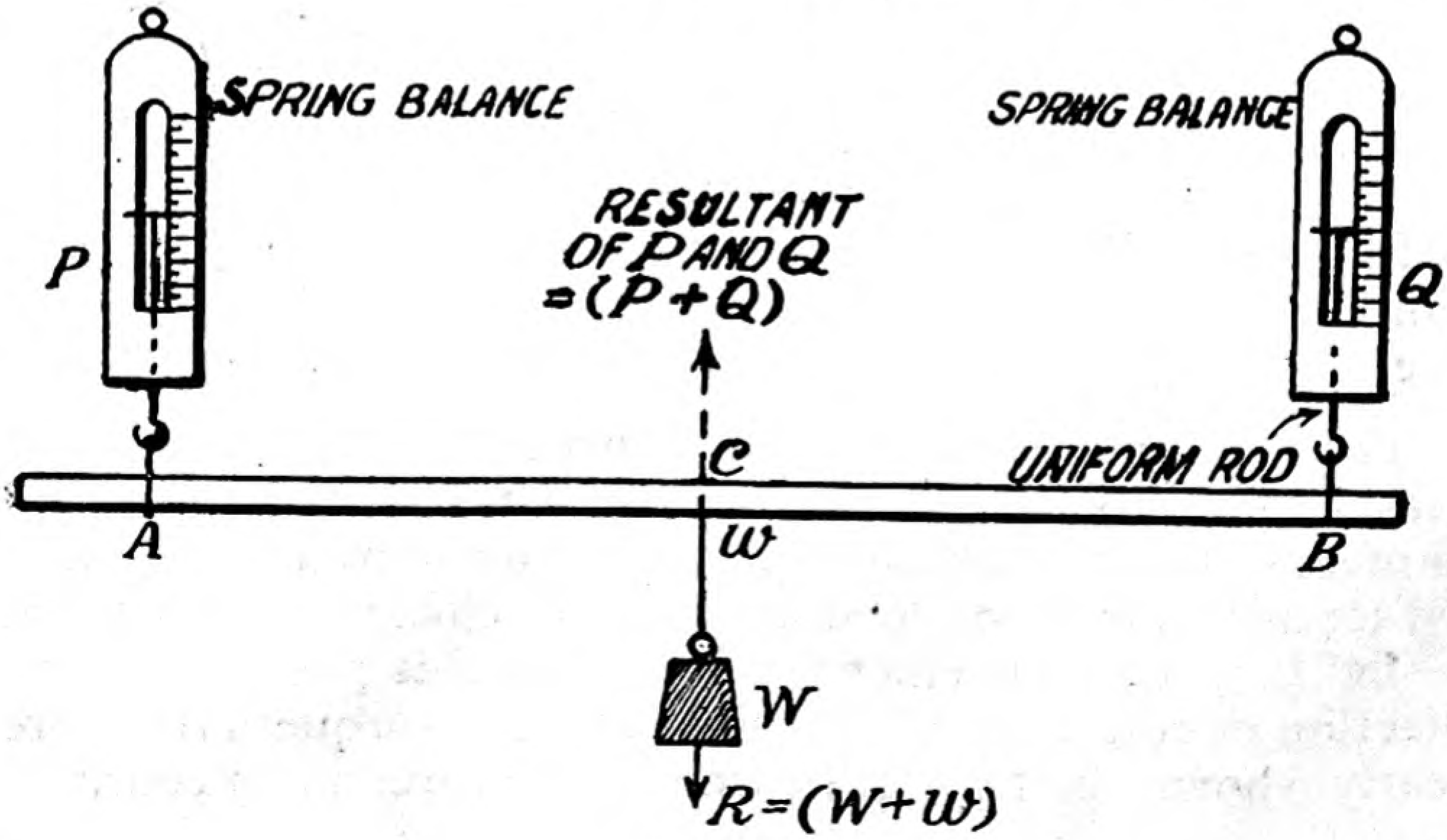

SPRING BALANCE
SPRING BALANCE
P
RESULTANT
OF P AND Q
=(P+Q)
Q
C
UNIFORM ROD
A
w
B
W
R=(W+w)

Fig. 9.5

and suspend it by a thread from the middlepoint C of the uniform rod. We now take two spring balances and suspend the rod from their hooks so that, as shown in fig. 9.5, the vertical balances support the rod at the points A and B, the rod being horizontal. The total weight acting at the point C is $(W+w)=R$. Suppose that the readings of the spring balances are P and Q. It will be noticed that the sum of the readings P and Q is equal to the weight $(W+w)=R$. It is seen that in this case three parallel forces are acting on the rod of which P and Q are in the vertically upward direction and the force R is directed vertically downward. Since the rod is in equilibrium, the force R must be balancing the combined effect of the upward forces P and Q. A single force which has the same effect on a body as a number of given parallel forces, is called the resultant of those parallel forces. Thus the resultant of P and Q is equal in magnitude to R and acts through the point C in the upward direction as shown dotted in fig. 9.5. By actual measurement we have seen that

$$R=P+Q.$$

Now we measure the distances AC and BC. Then it will be found that the moments of P and Q about C are equal, *i. e.*

$$P\times AC=Q\times BC.$$

or

$$\frac{AC}{BC}=\frac{Q}{P}.$$

The moment $P\times AC$ tends to rotate the rod clock-wise, while the moment $Q\times BC$ is anticlockwise.

This shows that the point C divides the distance AB internally in the inverse ratio of P and Q.

Thus the resultant of two like parallel forces is equal in magnitude to their sum and acts at a point so as to divide internally the distance between them in the inverse ratio of the values of the forces. This is only equivalent to saying that the moments of the individual force about the point of application of the resultant are equal and opposite.

We can look upon the above experiment in a slightly different way. Since the three forces P, Q and R keep the rod in equilibrium, any one of them can be considered to be equal and opposite to the resultant of the other two. If we consider the force Q to balance the resultant of P and R, then the resultant of two unlike parallel forces P and R is equal in magnitude to the force Q and acts at the point B in the downward direction as shown in fig. 9·6.

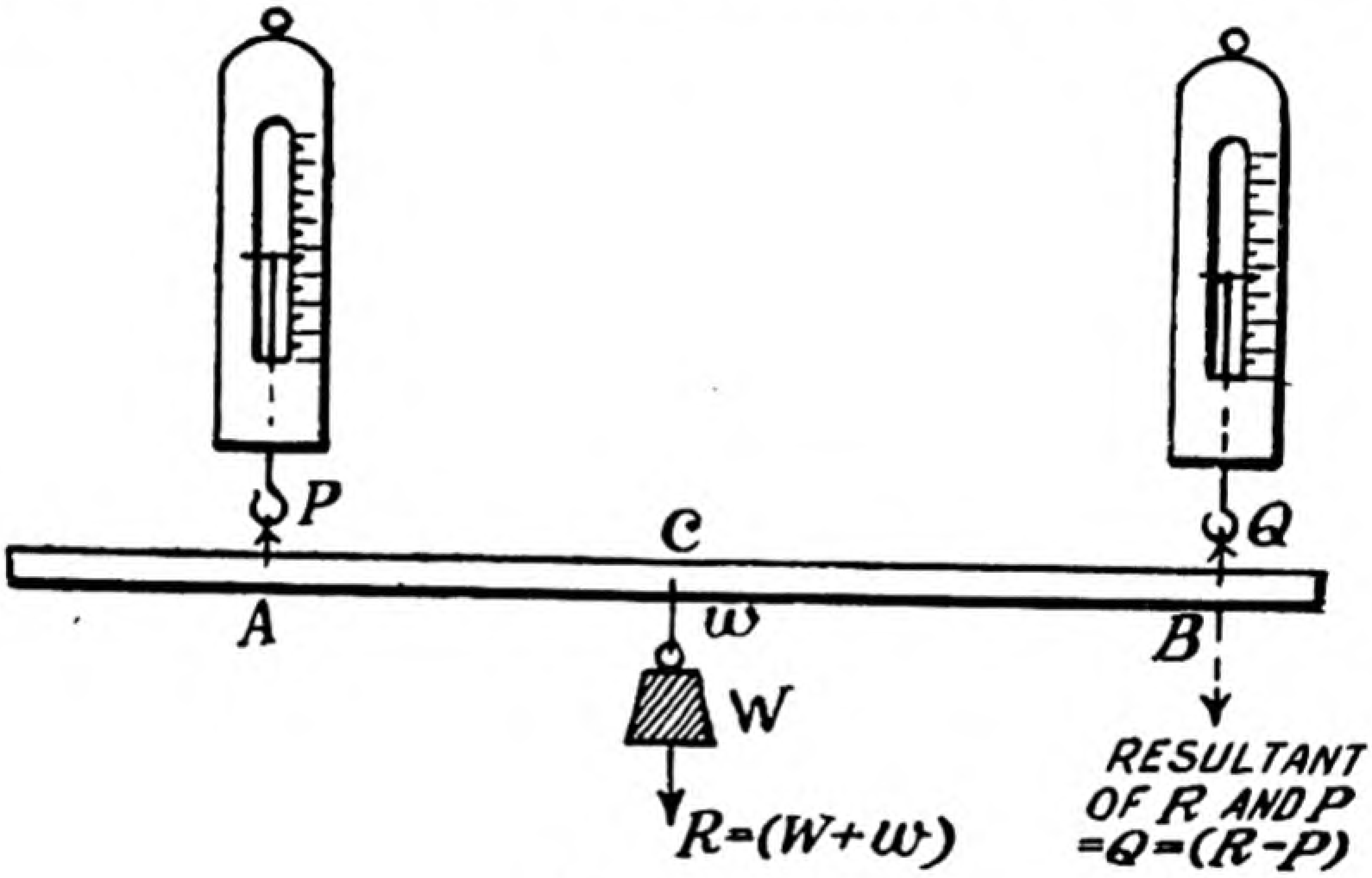


Fig. 9.6

We have already seen that

$$P+Q=R.$$

$$\therefore \quad Q=(R-P).$$

Thus the resultant of P and R = Q = (R − P),

and acts in the direction of the bigger component R that is downward.

If we determine the moments of P and R about the point B we shall find that

$$P\times AB=R\times CB$$

or

$$\frac{AB}{CB}=\frac{R}{P}.$$

Further the moment P × AB is clockwise and the moment R × CB is anticlockwise.

The point B, therefore, is said to divide the distance AC externally in the inverse ratio of P and R.

Thus the resultant of two unlike parallel forces is equal to their difference and acts in the direction of the bigger force, being nearer to it. It divides the distance between the unlike forces externally in their inverse ratio or the moments of the two component forces about the point of application of the resultant, are equal and opposite.

5. Resultant of any number of parallel forces.

In general if there are a number of parallel forces acting on a rigid body, the magnitude of their resultant is given by the algebraic sum of the forces. To find the point of application of the resultant we apply the principle of moments. This principle states that the moment of the resultant of a number of parallel forces, about any point in the body, is equal to the algebraic sum of the moment of the individual forces about the same point.

In the fig. 9.7, a number of parallel forces P_1, P_2, P_3, P_4, and P_5 are shown acting at the points A, B, C, D, and E respectively. Their resultant R is given by the equation

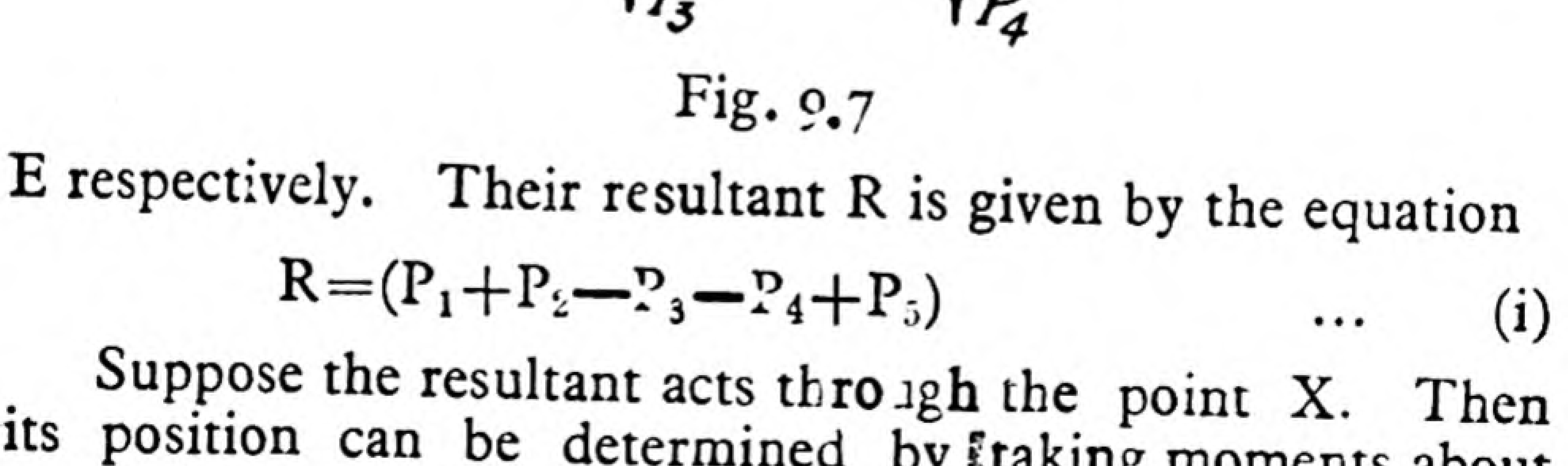


Fig. 9.7

$$R=(P_1+P_2-P_3-P_4+P_5) \qquad \dots \qquad (i)$$

Suppose the resultant acts through the point X. Then its position can be determined by taking moments about any point O. We therefore have

$$R \times OX = P_1 \times OA + P_2 \times OB - P_3 \times OC - P_4 \times OD + P_5 \times OE$$

$$\text{or}\quad OX=\frac{P_1\times OA+P_2\times OB-P_3\times OC-P_4\times OD+P_5\times OE}{R}$$

Substituting for R from equation (i)

$$OX=\frac{P_1\times OA+P_2\times OB-P_3\times OC-P_4\times OD+P_5\times OE}{(P_1+P_2-P_3-P_4+P_5)}\quad \ldots \quad (ii)$$

Thus OX, the distance of the point of application of the resultant from the point O can be determined.

Example. 1. Two parallel forces of 500 gram wt. and 300 gram wt. act on a body in the same direction from two points A and B 40 cms. apart. Find the magnitude and the point of application of the resultant.

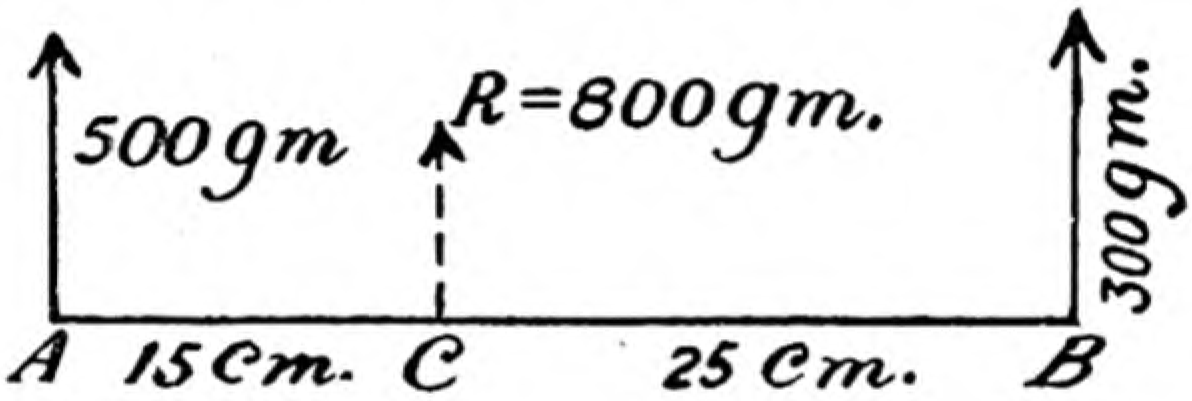


Fig. 9.8

$$AB=40 \text{ cm.}$$

$$\text{Resultant}=300+500=800 \text{ gram wt.}$$

Suppose the resulant acts through the point C such that $AC=x$, and $BC=40-x$.

Since the point C divides AB internally in the inverse ratio of 500 and 300, we have

$$\frac{AC}{BC}=\frac{300}{500}$$

$$\text{or}\quad 500\times AC=300\times BC$$

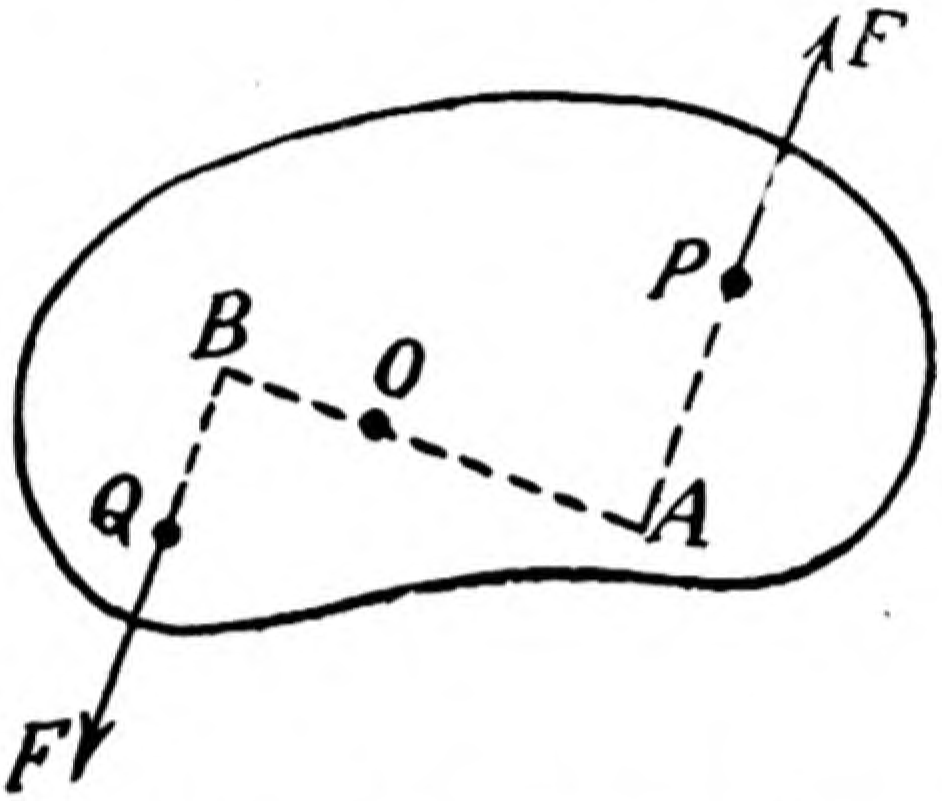
$$\text{or}\quad 500x=300\times(40-x)$$

$$\text{or}\quad x=15 \text{ cm.}$$

So the resultant is a like parallel force of 800 gm. wt. acting from the point C so that AC=15 cm.

6. Couple.

A special case of parallel forces is a system of two equal, parallel and oppositely directed forces acting at different points in a body. Such a pair of forces is called a mechanical couple. To find the moment or the turning effect of a couple consider fig. 9.9 in which two equal parallel forces F and F act at points P and Q of the body, which is capable of rotating about O. The perpendicular distance of one of the forces from O is OA and of the other is OB. Their moments about O are F × OA and F × OB and since both the moments have the turning effect in the same direction (counter clockwise in this case), the total moment of the two forces is



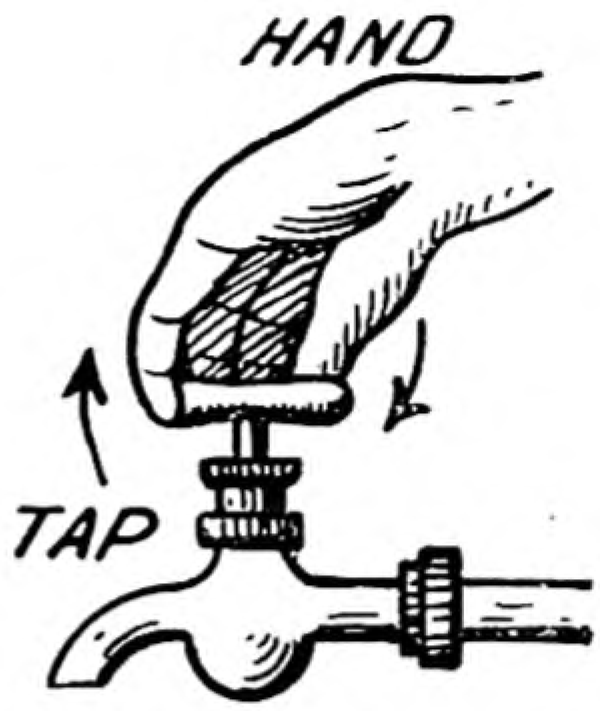
Parallel forces F and F form a couple.

Fig. 9.9

$$F\times OA+F\times OB=F\times(AB)$$
$$=F\times \text{distance betwen the two forces.}$$

This is called the moment of a couple. Thus the moment of a couple is the total rotating effect of the couple and equals the product of one of the forces into the distance between them.

The effect of a couple is one of pure rotation without any translatory effect by the forces. Some common examples of couples are (1) the forces applied by the hand when turning a key in the lock, (2) forces applied when turning a cork-screw, and (3) forces used when opening or closing a water tap.



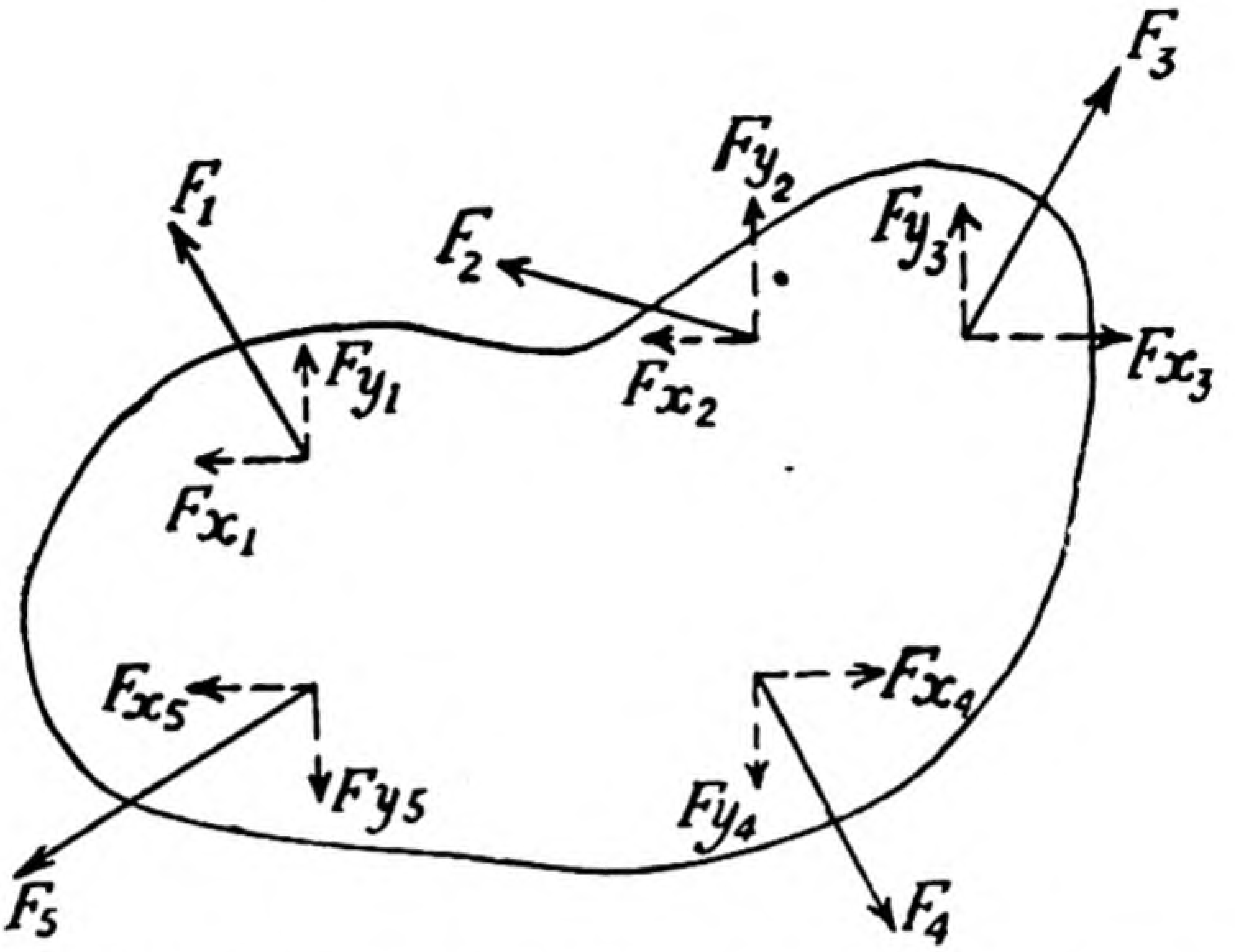
The hand applies a couple when opening or closing a tap.
Fig. 9.10

7. Equilibrium of a rigid body acted on by a number of forces.

In previous chapters we have seen that if a number of forces are acting on a body so that their lines of action pass through the same point, then these forces are in equilibrium if they can be completely represented by the sides of a closed polygon taken in order. Forces, the lines of action of which pass through the same point are called concurrent forces and their condition of equilibrium is given by the above law.

However, a number of forces may be acting on a body, with their lines of action not necessarily passing through the same point. In such a case the total effect of the forces will not only be to tend to cause linear motion or translation of the body, but also to produce rotating motion in it. Therefore complete equilibrium in such a case requires that either the body should remain at rest, or if it is moving or rotating its linear velocity and angular velocity should not change. This means that there should be no change in the translatory or the rotatory motion of the body.

To simplify matters we consider a number of forces, which are in the same plane, to be acting on a body as in fig. 9.11. The translatory equilibrium of such forces is best studied by resolving each of the forces along two chosen directions which are perpendicular to each other.



A number of forces acting on a rigid body in one plane.

Fig. 9.11

Let us call these directions as X and Y and the resolved parts of the forces in these directions as Fx_1, Fx_2......etc., and Fy_1, Fy_2......etc. Then if the sum of all the x-components is zero, and the sum of all the y-components is zero there will be no change of linear motion of the body in any direction in the plane. This condition may be expressed by writing that

$$\Sigma Fx = 0$$

$$\text{and } \Sigma Fy = 0,$$

where sigma (Σ) means the sum and Fx and Fy are the resolved components along x and y.

For rotational equilibrium we choose any point in the body and find the moment of each force about that point. If now we calculate the sum of the moments of all the forces about the chosen point, then their sum should be equal to zero, if there is to be no change in the rotatory motion of the body. We may express this condition as

$$\Sigma L = 0,$$

where L stands for the moment or the torque of the force about any point in the body. We thus see that there are two conditions to be satisfied for complete equilibrium, one for translatory equilibrium and the other for rotational equilibrium.

Example. A uniform ladder of length 20 ft. and weight 30 lbs. rests against a smooth wall making an angle of 60° with the floor. A man weighing 120 lb. is standing on the ladder 15 feet from its bottom. Find the reactions of the wall and the floor.

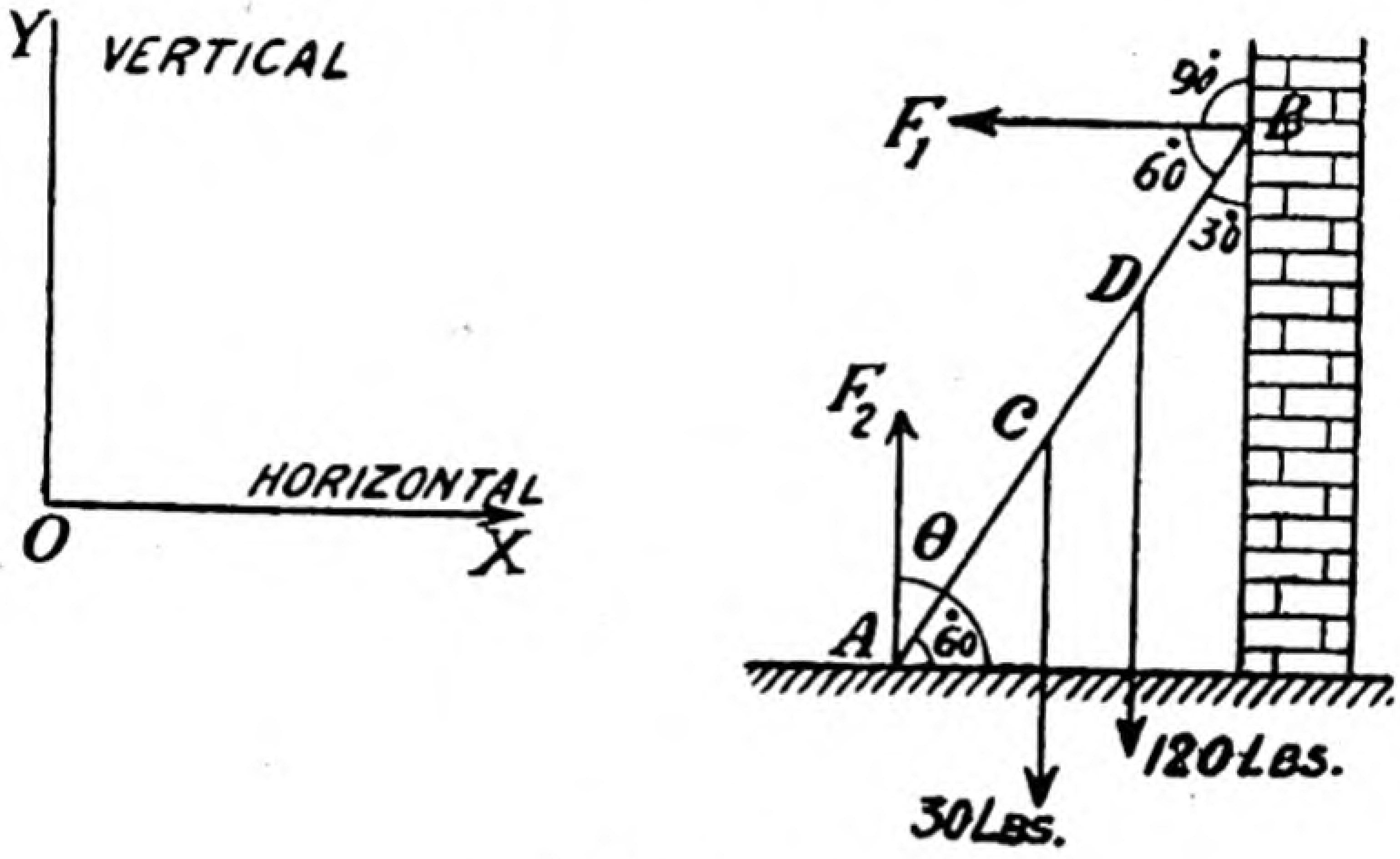


Fig. 9.12

AB is the ladder resting against the smooth wall BE. Four forces are acting on the ladder.

1. Weight of ladder 30 lbs. acting vertically downward at C where AC=10 ft.

2. Weight of the man 120 lbs. acting vertically downward at D where AD=15 ft.

3. Reaction of the smooth wall, F_1, acting perpendicular to the wall at B.

4. Reaction of the floor, F_2, acting at some unknown angle θ to the floor at A.

Since the ladder is in equilibrium the algebraic sum of the moments of all the forces about any point must be zero. In order that F_2 may not come into the calculation, we take moments about the point A.

$$F_1 \times AB \sin 60° - 30 \times AC \cos 60° - 120 \times AD \cos 60° = 0$$

or $$F_1 \times 20 \times \frac{\sqrt{3}}{2} - \frac{30 \times 10}{2} - \frac{120 \times 15}{2} = 0.$$

or $$F_1 = \frac{1050}{10\sqrt{3}} = \frac{105\sqrt{3}}{3} = 60.6 \text{ lb wt.}$$

To find the reaction F_2, we apply the first condition of equilibrium and resolve all the forces along the horizontal direction OX and vertical direction OY. The algebraic sum of the components along OX and along OY must be zero,

Force	X-Component	Y-Component
F_2	$F_2 \cos \theta$	$F_2 \sin \theta$
30 lb	0	-30
120 lb	0	-120
F_1	$-F_1$	0

Taking the x-componets

$$F_2 \cos \theta - F_1 = 0, \quad \textit{or } F_2 \cos \theta = F_1 = 60.6$$

Taking the y-components

$$F_2 \sin \theta - 150 = 0$$

or $$F_2 \sin \theta = 150$$

and $$F_2 \cos \theta = 60.6.$$

Squaring and adding

$$F_2^2 \sin^2 \theta + F_2^2 \cos^2 \theta = (150)^2 + (60.6)^2$$

or $$F_2^2 = 26172$$

or $$F_2 = 161.8 \text{ lb.}$$

8. Centre of gravity.

We have seen that every particle of matter near the earth is affected by it because of the force of gravity. In any large piece of matter, every particle of it is attracted downwards by the earth, and we therefore say that a very large number of parallel forces directed vertically downwards act on the various particles of the piece of matter, as in fig, 9.13. Since all these parallel forces are acting in the same direction, that is vertically downwards, their

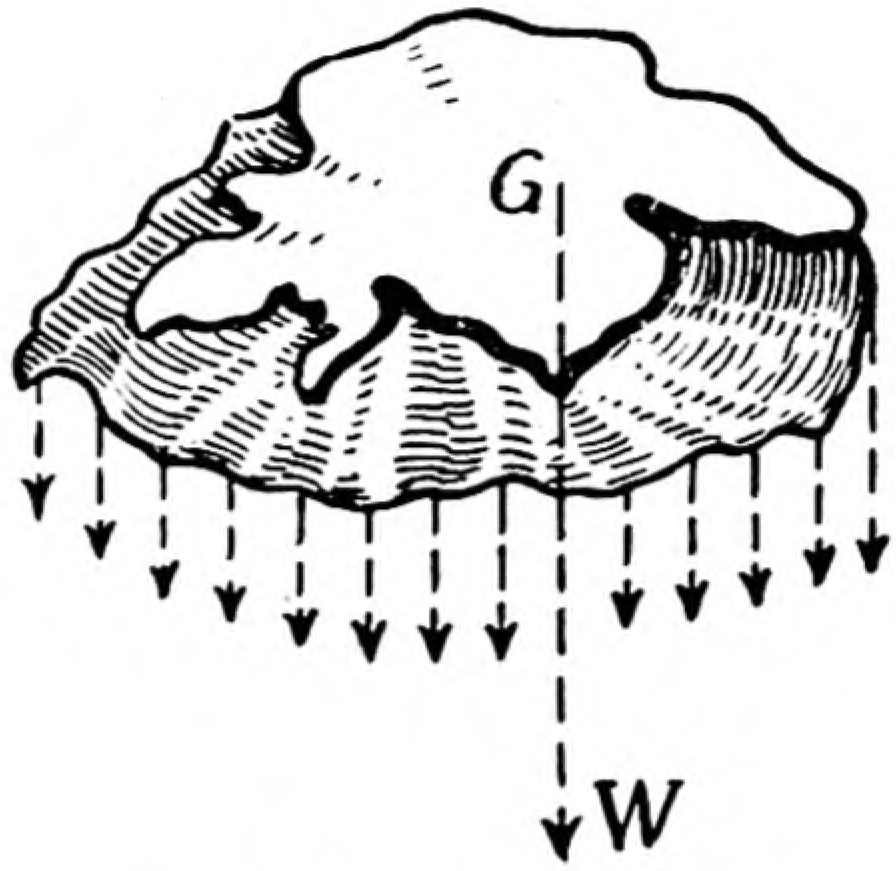


Fig. 9.13

resultant is equal to the sum of all of them. If the respective forces are called mg, $m_2g+\ldots\ldots m_ng$, the magnitude of the resultant is $(m_1+m_2+\ldots\ldots\ldots m_n)g=Mg=W$. This is equal to the total weight of the body. Further the resultant may be considered to act at a point in the body such that the sum of the moments of the individual forces on the particles around that point is zero. Such a point is called the centre of gravity of the body. Its position in the case of the irregular solid body shown in fig. 9.13, is at G where the entire weight W of the body is supposed to be acting. The centre of gravity (C. G.) of a body may thus be defined as a point at which the resultant of the weights of all the particles of the body may be supposed to act.

9. Shape of a body and the position of its centre of gravity.

The centre of gravity of a body having a geometrical shape is easily determinable from conditions of symmetry. In the following table the shape of the body and the position of its centre of gravity are described. It has to be observed that the centre of gravity does not always lie in the solid portion of a body. In the case of a ring, the C.G. is situated at the centre of the circle that is in the hollow region inside the ring. Similarly in the case of a tube of a cylindrical shape, the C.G. is situated in the hollow portion inside, and at the middle point of the length of the tube.

	Shape of Body	Position of C.G.
1.	Uniform rod	Middle point of the rod.
2.	Circular disc	Centre of the disc.
3.	A plate of the shape of a rectangle parallelogram or square	Point of intersection of the two diagonals.
4.	Triangular plate	Point of intersection of the medians of triangle.
5.	Rectangular or cube block	Point of intersection of diagonals.
6.	Sphere	Centre.
7.	Cylinder	Middle point of the axis.
8.	Cone or pyramid	Lies on the line joining the apex to the centre of the base at a distance equal to $\frac{1}{4}$ of the length of this line from the base.

10. Finding the C.G. of an irregular lamina, i. e. a flat sheet having no geometrical shape.

A small hole A is made near some edge of the irregular sheet, and putting a thread through the hole, the body is suspended from the thread, fig. 9.14 (*a*). In such a state

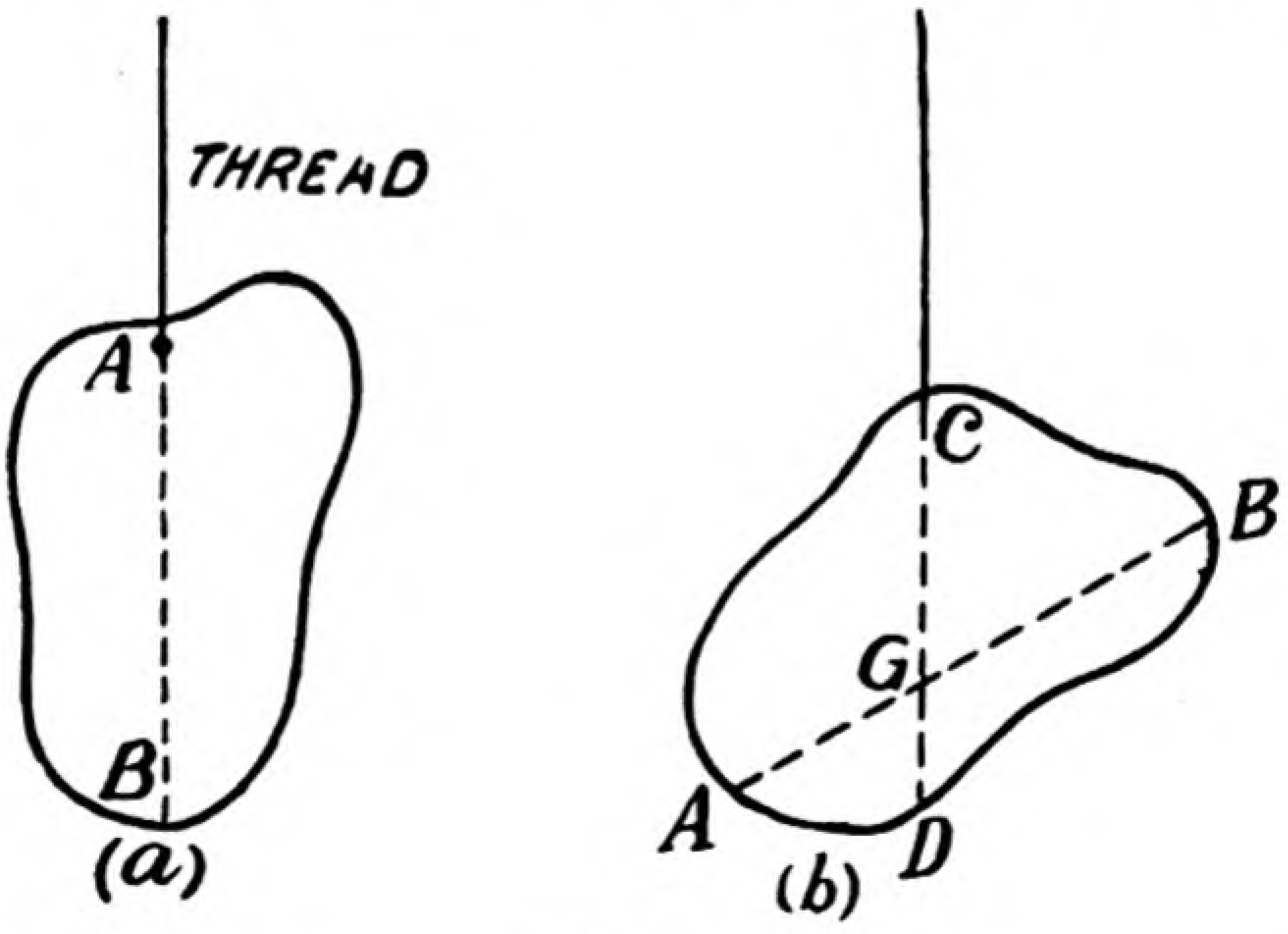


Fig. 9.14

of suspension, the C.G. of the sheet must adjust itself so as to be vertically underneath the point of suspension. If we draw a vertical line AB on the sheet, by prolonging the direction of the suspension thread, then the C.G. must be situated some where on the line AB.

Next the Lamina is suspended by a thread from another hole as at C, fig. 9.14 (*b*), the line CD drawn in prolongation of the thread. The centre of gravity must also be situated on this line. Therefore the point G, which is the point of intersection of AB and CD, gives us the position of the centre of gravity of the lamina.

Another method of finding the centre of gravity of a lamina is to support it from below on a sharp point, fig. 9.15, so that the sheet becomes balanced in the horizontal position. When this takes place the resultant moment of all the forces due to gravity acting on the particles, about the point of support is zero. This point is thus the centre of gravity, where the whole weight of the body can be supposed to be acting.

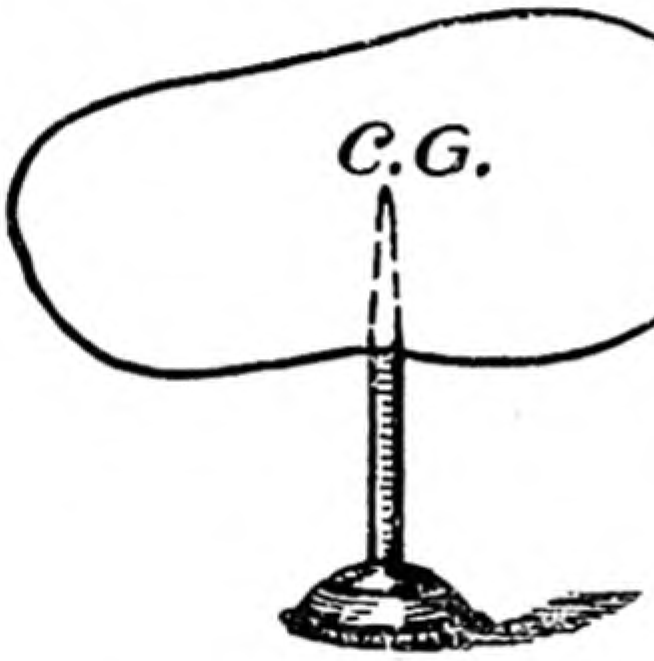
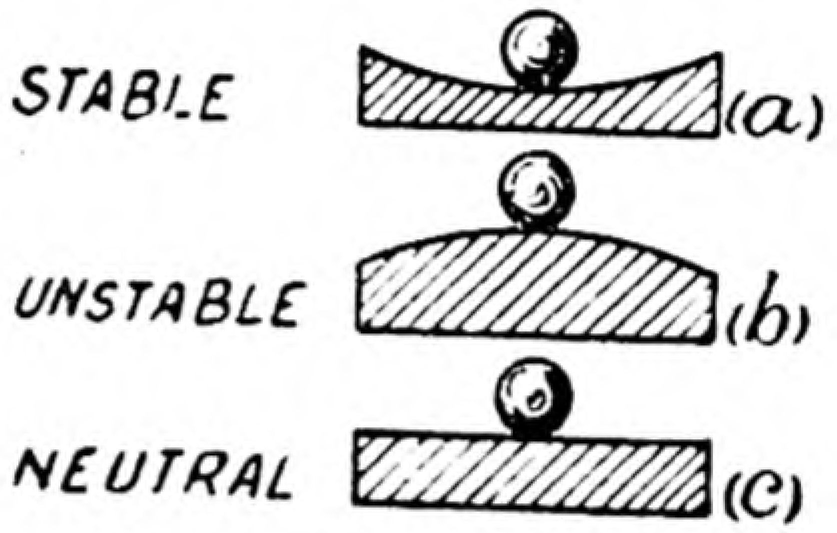


Fig. 9.15

11. Equilibrium of bodies.

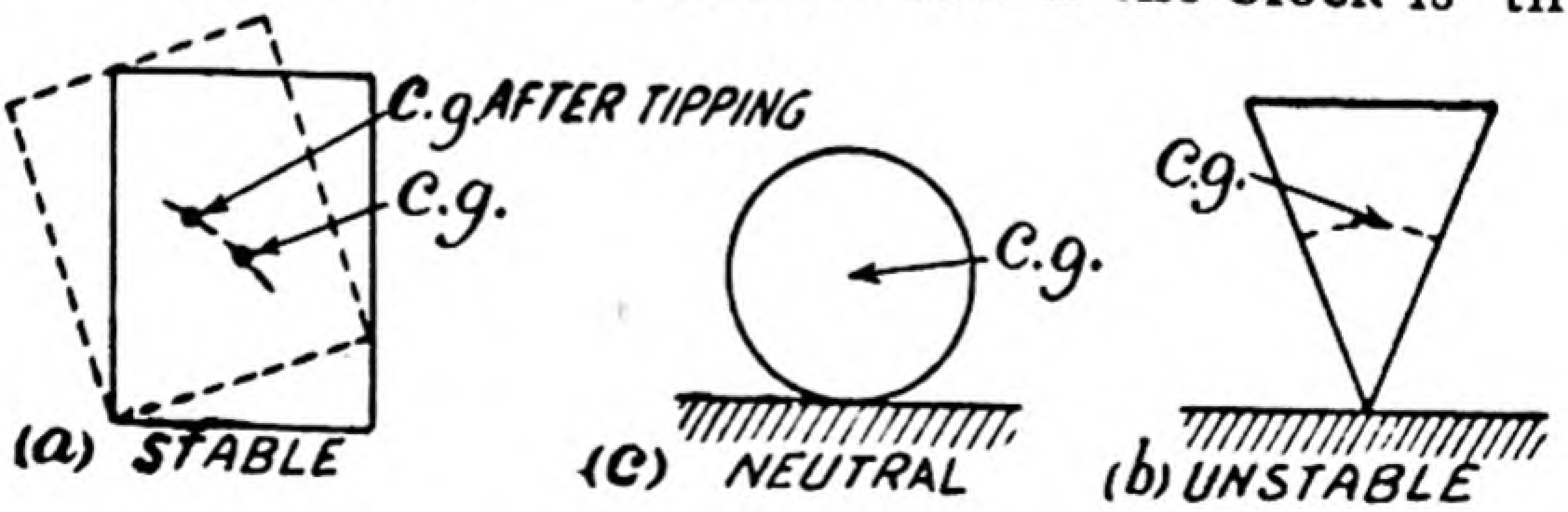
The situation of the centre of gravity is intimately connected with the equilibrium of solid bodies. There are three states of equilibrium commonly recognized and called the stable, the unstable and the neutral equilibrium. A body is said to be in a stable equilibrium, if its centre of gravity returns to its original position, after it has been displaced from it. If, however, a slight displacement given to the body tends to move the centre of gravity away from its original position, the equilibrium is called unstable. In unstable equilibrium the centre of gravity of the body does not return to its original position after a slight displacement, but the equilibrium is destroyed and the body topples over. In neutral equilibrium any small displacement of the body leaves it in the new position without any tendency to overturn or to return to its original position. In fig 9.16, a ball is shown resting on a surface. In (*a*) the surface is concave and if any displacement is given to the ball, it has a tendency to return to its original position and the equilibrium is therefore stable. In (*b*) the ball is resting on a convex surface and if the body is slightly displaced it will roll away, and not return back. The equilibrium in this case is unstable. In (*c*) the ball is resting on a plane surface and displacement given to the ball simply leaves it in the new position without returning or rolling away. In this case the ball is in neutral equilibrium.



Types of Equilibrium

Fig 9.16

The three states of equilibrium are also described in fig. 9.17. In (*a*) a rectangular block is shown with one face resting on a table. It is clear that if the block is tilted through a small angle and then released, it returns to its original position. If the angle of tilt is such that the

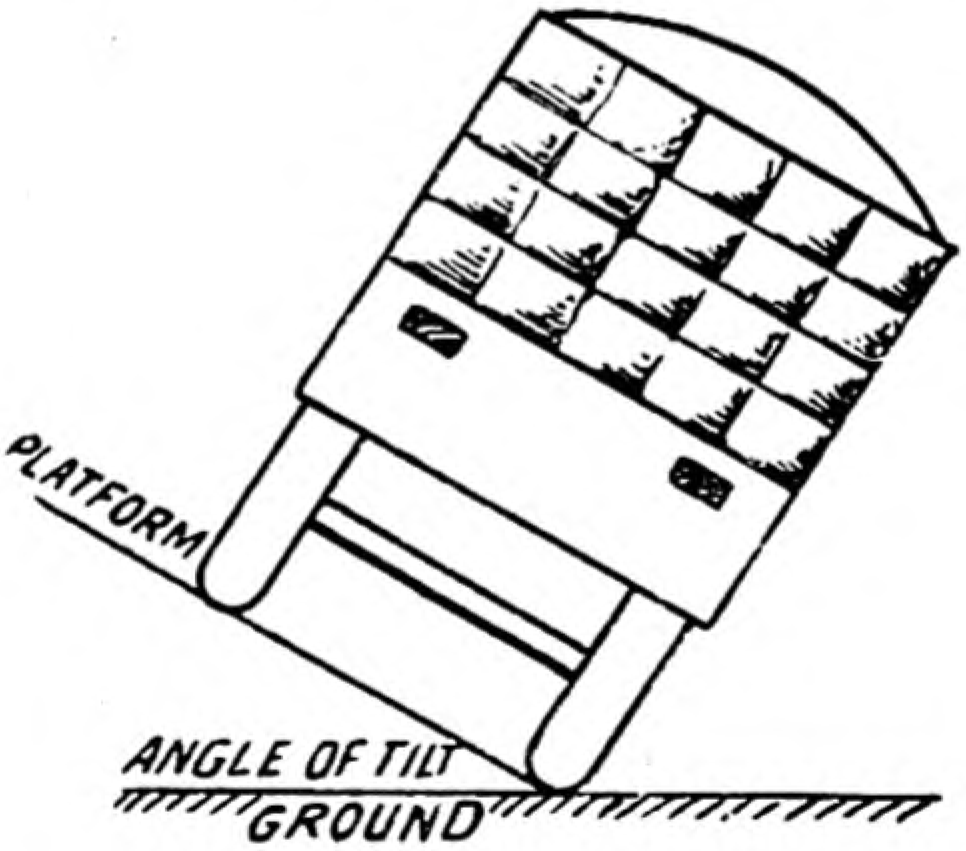


Equilibrium

Fig. 9.17

perpendicular from the centre of gravity does not fall outside the base, the equilibrium is stable. If, however, the tilt is increased to such an amount that the perpendicular from the centre of gravity falls outside the base, the equilibrium becomes unstable and the block topples over. In (*b*) a wedge is supported on one of its edges. The equilibrium in this case is unstable as a small tilt will tumble the wedge. In (*c*) the case of a ball resting on a plane surface represents neutral equilibrium.

In general if a small displacement, given to a body, raises its centre of gravity, the equilibrium is stable. Bodies with a broad base and low centre of gravity are in a state of more stable equilibrium. If a small displacement lowers the centre of gravity the equilibrium is unstable while if the height of the centre of gravity is unchanged by displacement, the equilibrium is neutral. For this reason it is very desirable that motor buses and other vehicles should have a broad base and low centre of gravity. The motor bases are tested by standing them on a platform and tilting the platform (fig. 9.18) to find out the maximum angle of tilt for which they will not overturn. This test indicates the road stability of the bus.



Stability Test of a Bus.

Fig. 9.18.

Exercises

1. What is meant by the moment of a force about an axis? On what factors does it depend? What will be

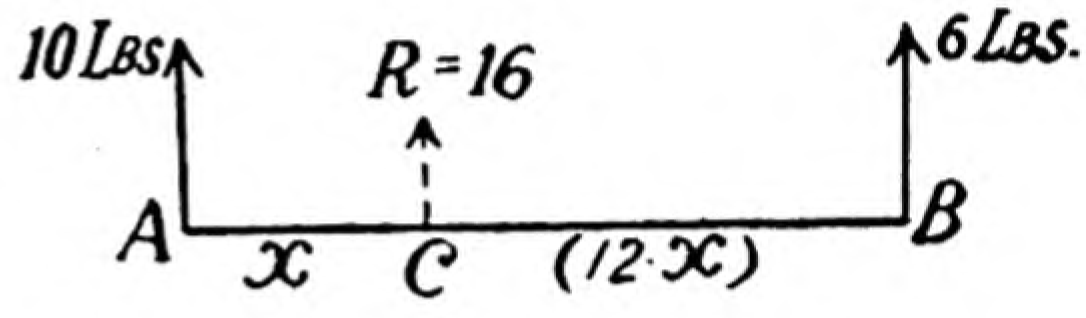
the moment of a force about a point that lies on its line of action ?

2. Two parallel forces equal to 6 lbs and 10 lbs. are acting on a body in the same direction. If the perpendicular distance between the forces be 12″, find their resultant and its point of application.

Resultant $= 10+6=16$ lbs.

$AB = 12''$

Suppose the point of application of resultant is C such that $AC=x''$



9.19

Take moments about C

$$10x=6(12-x)$$

or $$x=AC=4\tfrac{1}{2}''$$

3. Find the resultant and its point of application if the forces in question 2 are acting in opposite directions.

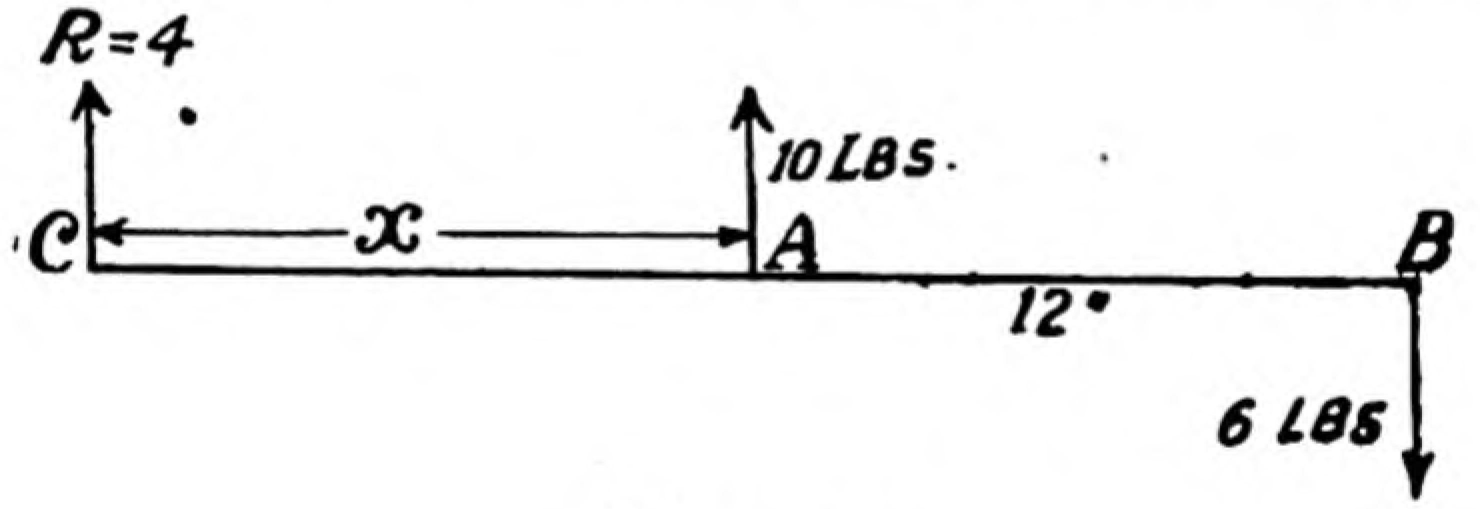


Fig. 9.20

Resultant $=10-6=4$lb. acting in the same direction as 10 lb. force.

If the point of application of the resultant be C such that $AC=x$, then taking moments about C we have

$$10x=6(12+x)$$

or $$x=AC=18''$$

4. A uniform rod 2 meters long weighs 200 grams. Weights of 40, 80, 120 and 160 grams are suspended from it at distances of 40, 80, 120 and 160 cms. respectively from one end. Find the position of the point where the

rod can be balanced on a wedge. What will be the reaction of the wedge?

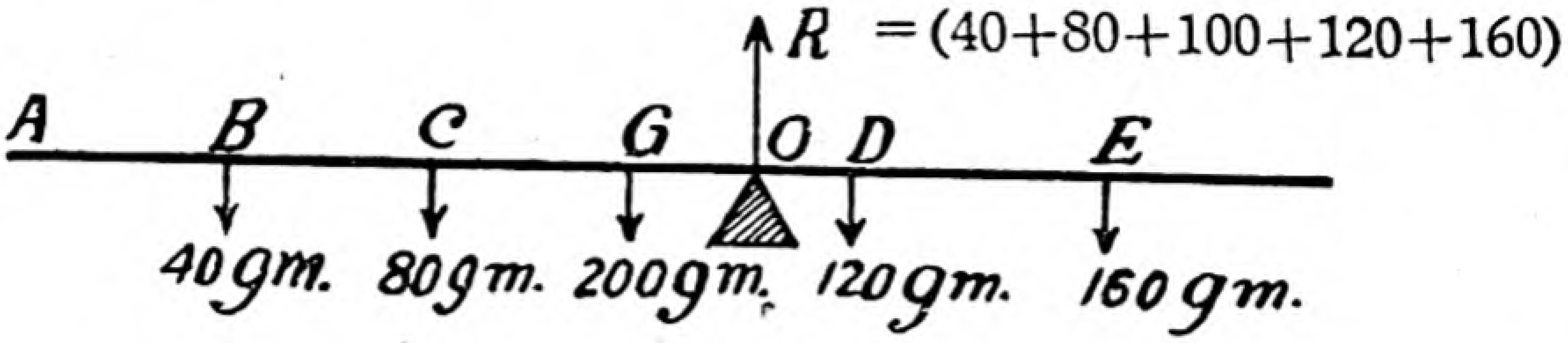


Fig. 9.21

AB = 40 cm., AC = 80 cm, AG = 100 cm.,
AD = 120 cm, and AE = 160 cm,

Suppose the point of balance is O such that $AO = x$.

Reaction of wedge $= R = 40+80+200+120+160$

$= 600$ grams

Take moments about A

$$600 \times x = 40 \times 40 + 80 \times 80 + 200 \times 100 + 120 \times 120 + 160 \times 160$$

$$= 68{,}000$$

$$\therefore \quad x = AO = 113{\cdot}33 \text{ cm.}$$

5. Two men carry a load of 100 lbs. with a uniform beam 10 feet long and weighing 40 lbs, from its ends A and B. If the load is supported 4 ft. from A, find the weight that each man is carrying.

AD = 4 ft, AC = CB = 5 ft.

If A and B carry weights R_1 and R_2 respectively then

$R_1 + R_2 = 100 + 40 = 140$ lbs

Take moments about A, then

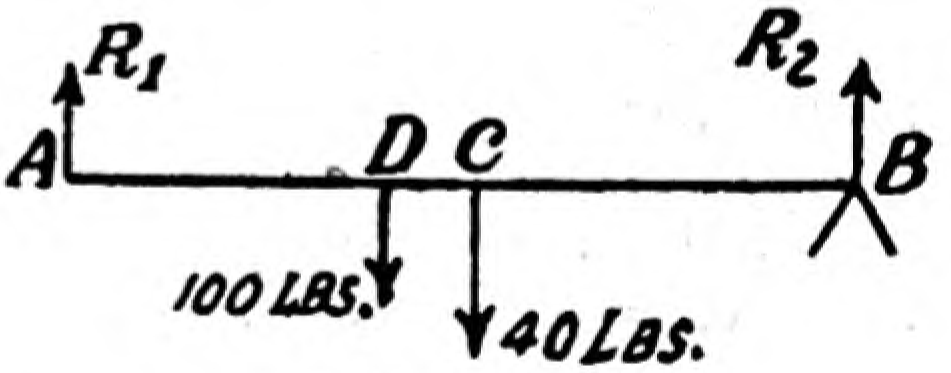


Fig. 9.22

$$10R_2 = 100 \times 4 + 40 \times 5 = 600$$

or $\quad R_2 = 60$

$\therefore \quad R_1 = 140 - 60 = 80$ lbs.

6. A uniform meter rod is supported from its 75 cm. graduation. If a 200 gram weight is suspended from

the 90 cm. mark the rod becomes horizontal. Find the weight of the rod.

7. What forces constitute a couple ? Prove that the moment of a couple about any point is constant. When are two couples said to be equal ?

8. A meter rod can be balanced at 52 cm. mark on a knife-edge. The same rod can be balanced at 70 cm. mark if it is loaded with 60 grams at 100 cm. graduation. Find the weight of the rod. 100 grams.

9. A uniform bridge weighing 100 tons is 80 feet long. If an engine weighing 50 tons is standing 32 feet from one end, find the thrusts acting on the two supports of the bridge. 80 tons, 70 tons.

10. A uniform beam 12 ft. long is supported in a horizontal position with two vertical ropes from its ends A and B. The beam weighs 50 lbs. and carries two weights of 100 and 200 lbs. at distances of 2 ft. and 8 ft. from end A. Find the tension in each rope. 175 lbs., 175 lbs.

11. A uniform rod 8 ft. long weighs 10 lbs, and is hinged at one end. A string tied to the other end of the rod keeps it horizontal by pulling in a direction making 60° with the horizontal. Find the tension in the string.

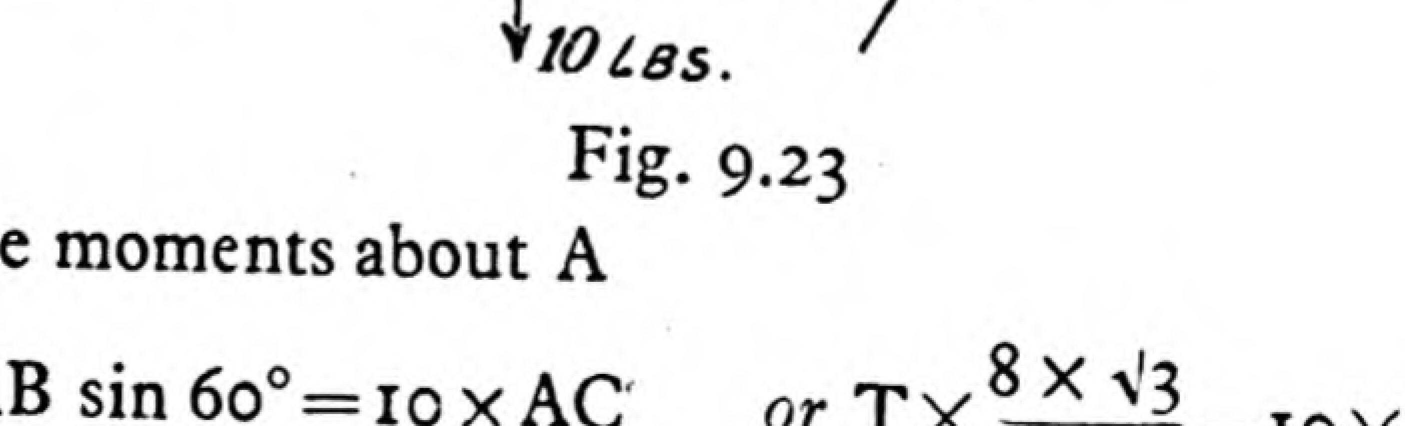


Fig. 9.23

Take moments about A

$$T.\,AB \sin 60^\circ = 10 \times AC \qquad \text{or } T \times \frac{8 \times \sqrt{3}}{2} = 10 \times 4$$

$$T = \frac{10\sqrt{3}}{3} = 5{\cdot}77 \text{ lb. wt.}$$

12. You are required to weigh an object whose weight is near about 1500 grams with a spring balance that can weigh up to 500 grams only. How will you use a uniform metre rod to do the weighing with the spring balance?

13. A man and a boy carry a uniform log of wood 12 ft. long and weighing 100 lbs. If the boy supports one end of the log, from where should the man support the log so that the boy may carry only one fourth of the total weight? 8 feet from the boy.

14. Find the common centre of gravity of two spheres whose masses are 500 gms. and 2000 gms. respectively and whose centres are 100 cms. apart.

(on the line joining their centres and 80 cm. from the 500 gm. sphere.)

15. Show that a couple can only be balanced by an equal and opposite couple.

16. A uniform rod 20 ft. long weighs 150 lbs. and is hinged at its upper end so that it is free to rotate round the hinge. A rope is tied to the lower end and is pulled aside so that the rod makes 30° with the vertical. If the rope be perpendicular to the rod find the tension in the rope.

37·5 lb. wt.

17. Describe the conditions of equilibrium of a body under the action of three forces whether parallel or not.

A uniform meter rod weighing 50 gm. is supported at its ends and two masses of 20 and 30 gm. are suspended at 20 and 90 cm. marks respectively. Find the reaction of the two supports.

Reaction at 0 mark 44 gm.

Reaction at 100 cm. mark 56 gm.

CHAPTER X

FRICTION

1. Friction is a force of resistance.

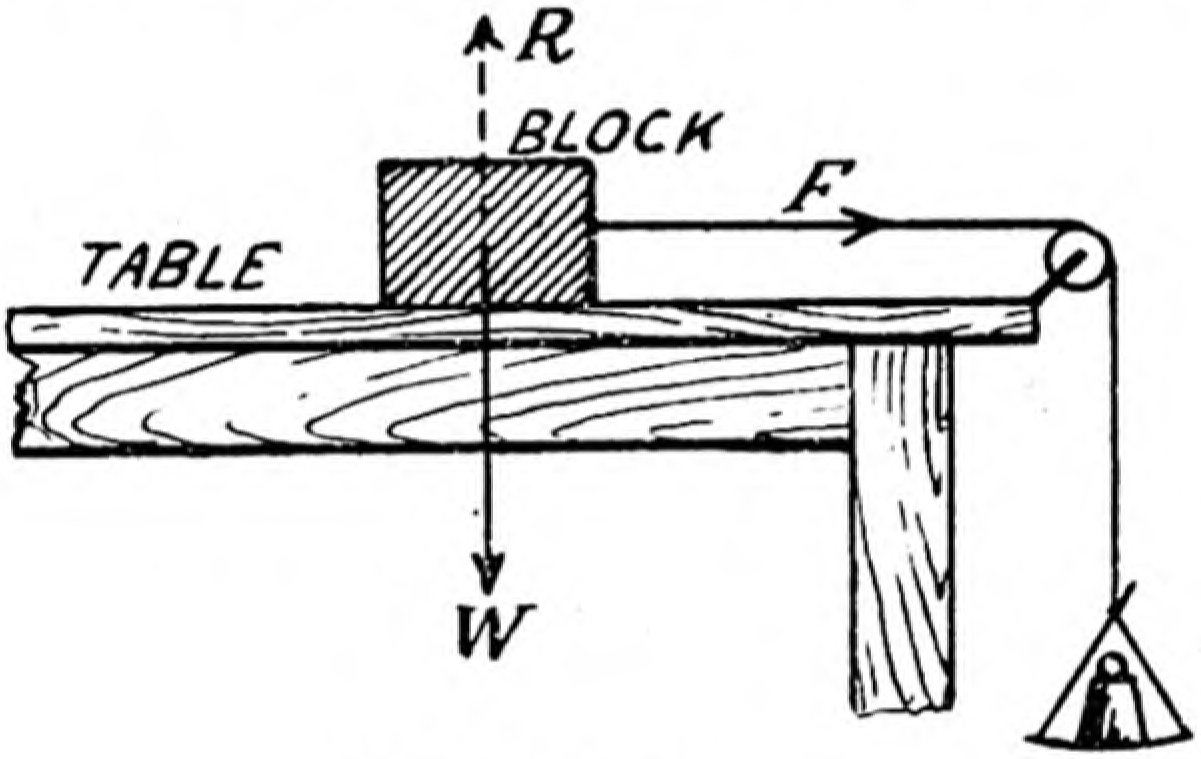
Friction is one of the commonest forces we come across in every day life. When an object moves in contact with another object frictional forces are brought into play and they tend to oppose the relative motion. These forces are due to roughness of the two surfaces and also to a property called adhesion, which is a sort of attractive force between two objects in contact with each other.

As the parts of machinery move one on the other, frictional forces are present between them and power has to be used in overcoming the friction. The moving parts of machinery also become worn out as a result of friction. Heat is also generated wherever friction is present between moving surfaces. It is, therefore, one of the attempts of designers and engineers to reduce friction between the parts of machinery.

However, friction is very necessary and desirable in many cases. We would not be able to walk if there was no friction between the feet and the ground. Any one who has tried to walk on a slippery floor must have experienced the difficulty. If there was no friction we would not be able to drive nails into wood. In fact the nail would come out of the hole the more we tried to drive it in. The screws would unwind themselves in the absence of friction. The belt, which is used to transfer power from an engine to the machinery, can do its work because of the friction which is present between the surface of the belt and the wheels of the machine which it rotates.

2. Co-efficient of friction.

To study the nature and the laws of friction, we may perform the following experiment. Take a block of wood, which has well planed surfaces, and find out what is the minimum force required to slide the block, when one of its faces is resting on a wooden table. The force is found by tying a string. fig. 10.1, to a hook in the block, and after passing the string over a pulley, a scale pan is attached to it, in which suitable weights from a weight box can be placed. As the weight in the pan is slowly increased, the motion of the block does not take place, until a certain value of the weight has been reached. This shows that the force of friction, which acts in a direction opposite to the pull of the string, correspondingly increases always adjusting itself to be equal and opposite to the pull of the string. The force of friction between the surfaces of the block and the table resists the pulling force of the string upto a certain limit, but if the weights in the pan are increased beyond that limit, the wooden block begins to slide on the table.



Sliding friction

Fig. 10.1

Suppose the weight suspended from the string (weight of pan plus the weight in the pan), when sliding just takes place is F grams, and that the weight of the block is W grams, then the force with which the block is pressing on the table is W grams, and the reaction of the table R is also equal to W. We may now find the ratio F/W = F/R, where F is, what is called, the limiting friction, and R the normal reaction. This ratio is called the Co-efficient of sliding friction between the two surfaces and denoted by the Greek letter μ ($m\mu$).

$$\therefore \frac{\text{Limiting}}{\text{Normal}} - \frac{\text{Friction}}{\text{Reaction}} = \text{Co-efficient of friction}$$

$$\text{or} \quad F/R=\mu,$$

If now a certain weight is placed in the wooden block, so that W or R increases, it will be found from experiment that F also increases, so that the ratio μ remains nearly constant for the two surfaces. It is also found that μ does not depend on the area of the surfaces in contact, if they are well planed, but only on the nature of the materials. By experiments of this type, certain general facts called Laws of sliding friction have been found out.

3. Laws of sliding-friction.

1. The force of friction between any two surfaces depends upon the nature of the surfaces in contact.

2. It is proportional to the normal force pushing the surfaces together.

3. It does not depend on the area in contact.

4. It is independent of the speed of motion.

5. Starting friction is greater than moving friction. This means that a somewhat greater force is required to start an object from rest to overcome friction, than is needed between the same surfaces when they are moving.

4. Angle of Friction or Angle of Repose.

The co-efficient of sliding friction can also be determined by inclining the surface on which a body rests. As has already been explained in the case of an inclined plane, when a body is lying on an inclined surface the weight of the body (W) which is acting vertically downward can be resolved into two components. One of these components is acting parallel to the surface of the inclined plane in the downward direction, while the other component is pressing normal to the surface of the plane. Suppose θ is some value of the angle of inclination of the surface, then the magnitude of the component acting parallel to the surface down the plane is W sin θ and the component pressing normally on the plane is W cos θ, fig. 10.2. If θ is less than a certain value, the force of friction between the surfaces, which acts in a direction opposite to the force W sin θ, will prevent the block from sliding downwards. If we slowly increase the angle of inclination, then the force W sin θ correspondingly increases and for a particular value of θ, the block begins to slide down the surface of the plane. When this takes place W sin θ is equal to the force of limiting friction. The normal force and its reaction are both equal to W cos θ. Since

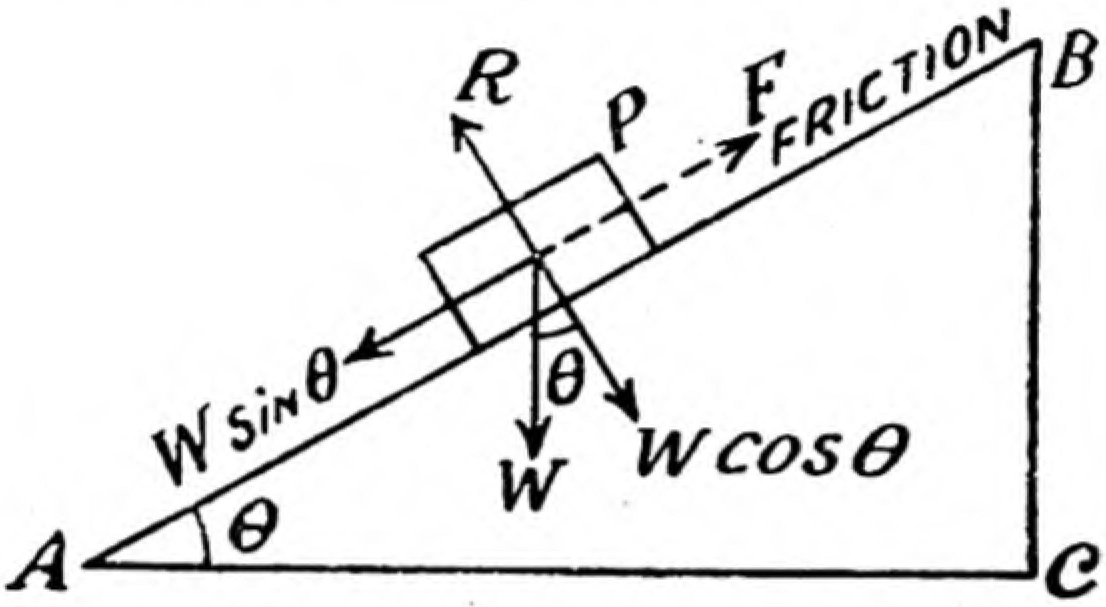


Fig. 10.2

$$\text{co-efficient of sliding friction} = \frac{\text{Limiting Friction}}{\text{Normal reaction}} = \frac{\text{F}}{\text{R}}.$$

$$= \frac{\text{W} \sin \theta}{\text{W} \cos \theta} = \tan \theta$$

The angle θ, defined by the above equation, the tangent of which equals the coefficient of sliding friction between the two surfaces, is called the angle of friction or the angle of repose.

When a body is moving up an inclined plane, work has to be done in raising the body and also in overcoming friction between the surfaces in contact. If the body is moving up with a uniform velocity, then these are the only two forces, i. e. friction and gravity against which work is done. If, however, the body gains speed when moving up the inclined surface, power is also used in producing acceleration in the body.

5. Value of the coefficients of sliding friction for some common materials.

In the following table some values of the coefficients of sliding friction are given

Materials	Coefficient of Friction
Steel on Steel	0·18
Rubber on Concrete	0·76
Leather on Metals	0·56
Surfaces Lubricated with grease	0 05
Iron on Concrete	0·30
Wood upon Wood	0·25 to 0·50
Metals on Wood	0·5 to 0·6

Example 1. A horizontal force of 15 lbs. wt. can just make a box slide on a floor. If the weight of box be 60 lbs. find the co-efficient of sliding friction between the box and the floor.

$$\text{Co-efficient of friction} = \frac{\text{Force of limiting friction}}{\text{Normal force or reaction}}$$

$$= \frac{15}{60} = \cdot 25.$$

Example 2. A block weighing 100 lbs is resting on a surface whose inclination to the horizontal can be varied. The block just begins to slide down when the surface is inclined at 30° to the horizontal. Find the co-efficient of friction.

Limiting friction = component of the weight down the plane

$$= W \sin \theta = 100 \sin 30.$$

Normal reaction $= W \cos \theta = 100 \cos 30.$

$$\therefore \text{Co-efficient of friction } \mu = \frac{100 \sin 30}{100 \cos 30} = \tan 30 = 0{\cdot}58$$

6. Rolling friction.

If we pull a heavy box, so as to slide it against the floor, we have to apply a greater force, than we will have to do, if a few small wheels are fixed to the bottom of the box. For this reason heavy furniture has small wheels fixed beneath the legs, in order that things may be moved easily on the floor. Wheels are used on carts and carriages because the resistance due to friction is much less in the case of rolling wheels, than if the same carriage was to be pulled against the surface of the ground without its wheels. We thus see that rolling friction is much less than sliding friction between the same surfaces. The co-efficient of rolling friction is defined in the same way as the co-efficient of sliding friction but its value is much less. While the co-efficient of sliding friction between steel and steel is 0·18 the co-efficient of rolling friction for cast iron wheels or iron rails, as in the case of railways is only 0·004. The rolling friction is due to slight deformation of the track, or of the wheel or both, producing a temporary unevenness of the surfaces. In fact in the case of iron wheels, the track becomes somewhat depressed, forming a little mound in front of the wheel as in fig. 10·3, and the wheel has to be pulled out resulting in performance of work.

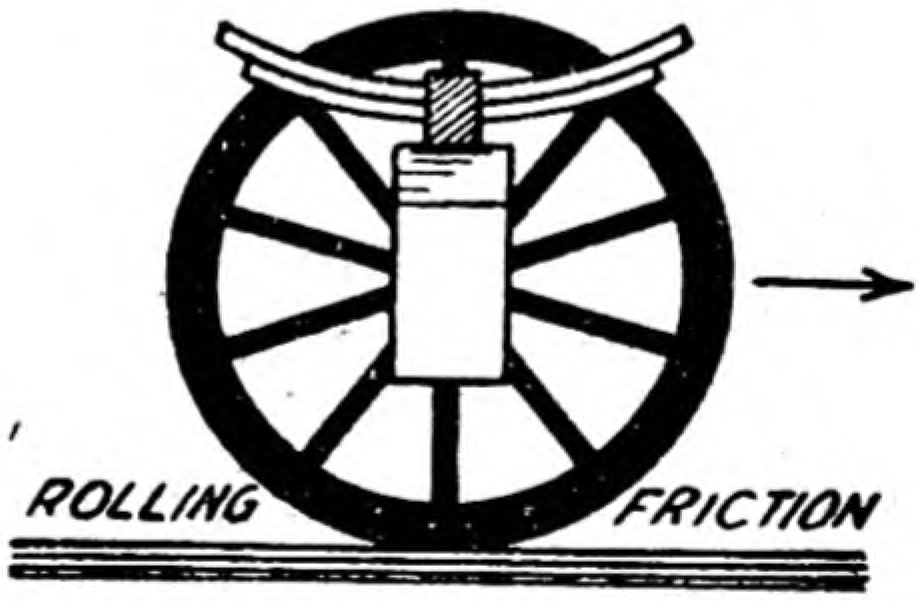
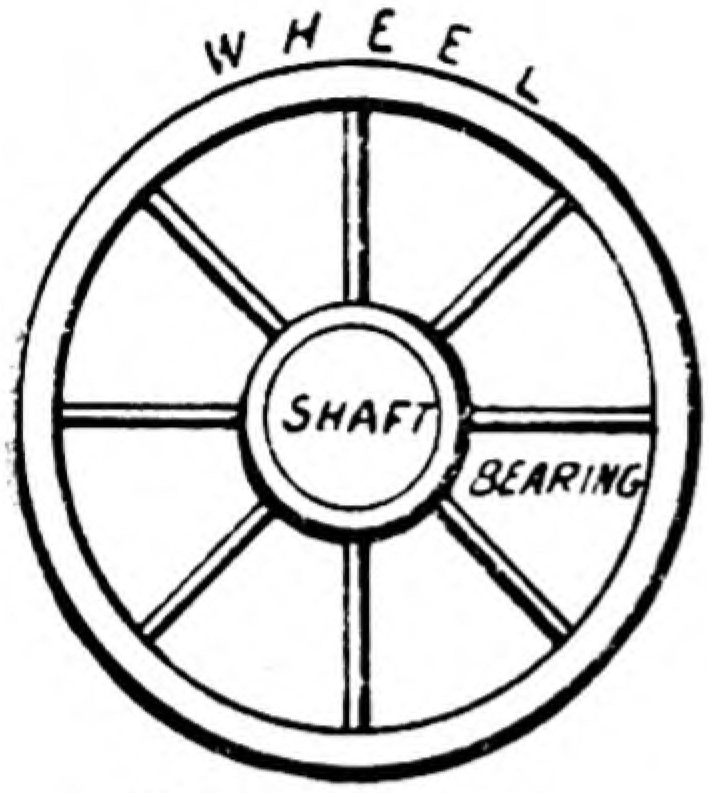


Fig. 10·3

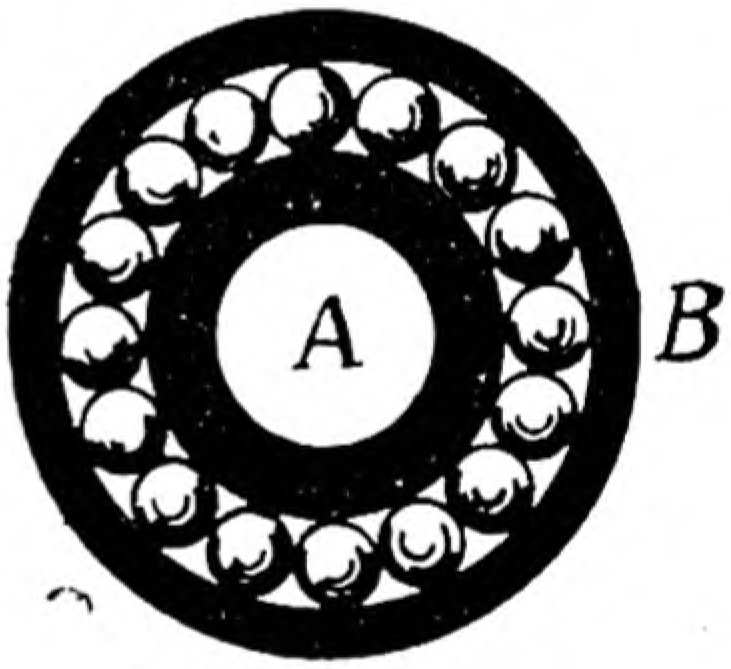
Since rolling friction is so much less than sliding friction, when parts of machinery have to rotate or move



Sleeve bearing
Fig. 10·4

on one another, roller bearings or ball bearings are often substituted in place of the sleeve bearing. In sleeve bearing, a shaft or axle rotates within a cylinderical cavity or bearing and sliding friction is present between them as shown in Fig. 10·4. In the case of a ball bearing, a number of hard steel balls are placed between the surfaces of the axle and the Outer Cavity or bearing. The axle A in fig. 10·5, which shows a ball bearing, rolls around the steel balls, which roll in a groove in bearing B. Thus rolling friction is substituted for sliding friction between parts of machinery and much saving of power is produced.

While wheels are used when a body is to move on roads or rails, but they are not suitable when the body has to move on sand or snow or mud. For this reason sledges are used in polar or arctic countries, which are snow covered. The sledge is a light wooden carriage with a flat bottom and it is pulled on snow, sometimes by teams of dogs : This method of motion was used by many explorers, who reached the poles of the earth.



A Ball Bearing
10.5.

7. Lubrication.

One of the effective means of reducing friction is to introduce some grease or oil between the moving surfaces. This is called lubrication. The lubricant forms a thin layer between the moving surfaces and has the effect of smoothing the roughness of the surface. Graphite is also often used as a solid lubricant if the surfaces become heated by friction, for heat may have injurious effects on oils. The laws of friction for lubricated surfaces are found to be different from the ordinary case. Thus the force of friction is found to depend on the nature of the oil, and to be independent of the normal pressure between the surfaces, and it is proportional to the relative speed and also to the area of cotact.

Summary.

Friction is a force of resistance which opposes relative motion between two surfaces.

Laws of sliding friction

1. Force of friction depends on the nature of the two surfaces in contact.

2. It is proportional to the normal force between the surfaces.

3. It does not depend on the area in contact.

4. It is independent of the speed of motion.

5. Starting friction is greater than moving friction.

$$\text{coefficient of friction} = \frac{\text{Limiting friction}}{\text{Normal reaction}}$$

$$\mu = \frac{F}{R}.$$

The angle of friction or the angle of repose for two surfaces is the angle, the tangent of which is equal to the coefficient of friction between the two surfaces. If this be θ, then

$$\mu = \tan\theta = \frac{F}{R}.$$

Rolling friction is much less than sliding friction and on this account ball bearings are used in machines to substitute rolling friction in place of sliding friction.

Lubrication reduces friction between two moving surfaces. Force of friction between two lubricated surfaces depends on the nature of the lubricant. It is independent of the normal pressure between the two surfaces. It depends on the relative speed and the area of contact.

Exercises

1. A block weighing 100 lbs. is resting on a floor. If a horizontal force of 30 lbs. just slides the box, find the coefficient of friction between the box and the floor.

$$\mu = \frac{F}{R} = \frac{30}{100} = .3$$

2. A force of 300 lbs can keep a car weighing 2500 lbs running with constant speed on a level road. Find the coefficient of friction between the wheels and the road. (Resistance due to air is neglected).

$$\mu = \frac{F}{R} = \frac{300}{2500} = .12$$

3. Brakes of a bus are applied so that the wheels stop rolling and skid. Calculate the force of friction if the weight of the bus is 3200 lbs. and the coefficient of friction $= 6$.

$$F = \mu R = .6 \times 3200 = 1920 \text{ lbs.}$$

4. Explain fully the terms friction and limiting friction. Justify the statement that friction is a necessity and an evil. Give some familiar methods for reducing and increasing friction. *(P. U.)*

5. Define coefficient of friction and angle of friction.

How do you account that a horse has to pull a cart harder during the first few feet of its motion than later on?

A block rests on an inclined plane which is tilted gradually until the block begins to slide down. Show that the coefficient of friction between the block and the plane is equal to the tangent of the angle made by the plane with the horizontal.

6. A force of 25 lb. weight is required to pull a slid weighing 250 lbs. over ice. What is the coefficient of friction. *(P. U.)*

7. How will you experimentally determine the coefficient of friction and angle of friction between two surfaces?

Find the horse-power of an engine capable of taking a train of mass 600 tons with a velocity of 40 miles/hr. on a level line, the forces due to friction being equivalent to 10 lb. wt. per ton. (*P. U. 1949*)

distance covered per second $= \frac{22}{15} \times 40 = \frac{176}{3}$ ft.

$$\therefore \left.\begin{array}{l}\text{Work done by engine}\\ \text{against friction per second.}\end{array}\right\} = \begin{array}{l}\text{Total frictional force} \times\\ \text{distance}\end{array}$$

$$= 600 \times 10 \times \frac{176}{3} \text{ ft. lbs.}$$

$$\therefore \quad \text{H. P. of engine} = \frac{600 \times 10 \times 176}{550 \times 3}$$

$$= 640 \text{ H. P.}$$

8. An engine is driving a train weighing 100 tons on a level track at a constant speed of 20 *m.p.h.* If the coefficient of friction be ·01 find the horse-power of the engine. If the steam is shut off, what distance will the train cover before coming to a stop ?

Force of friction $F = \mu R$.

Since the track is level, R = weight of train = 100 ton wt.

$$\therefore \quad F = \cdot 01 \times 100 = 1 \text{ ton wt.}$$

$$= 2240 \text{ lb wt.}$$

Velocity of the train = 44 ft./sec.

The engine is driving the train against the frictional force of 2240 lbs. wt. at the rate of 44 ft. per second.

$\therefore$ Work done per second = 2240 × 44 ft. lbs.

$$\text{Power} = \frac{2240 \times 44}{550}$$

$$= 179.2 \text{ H. P.}$$

When the steam is shut off, friction produces negative acceleration (retardation).

Applying acceleration formula

$$v^2_2 - v^2_1 = 2\,as$$

$$v_2 = 0, \quad v_1 = 44 \text{ ft.}$$

$$a = \frac{\text{Frictional force in lbls.}}{\text{mass in lbs.}}$$

$$=\frac{-2240\times 32}{100\times 2240}=-.32\text{ ft./sec.}^2$$

$$\therefore\qquad 0-44^2=-2\times .32\times S$$

or

$$S=\frac{44\times 44}{2\times .32}=3025\text{ ft.}$$

9. A lorry weighing one ton goes up an incline of one in ten at a constant speed of 15 m. p. h. If the forces due to friction be equal to 80 lbs wt, calculate the H. P. of the engine.

The engine works against two forces, friction and gravity.

Force of gravity opposing the engine $=\frac{2240\times 1}{10}=224$ lb wt

Force of friction $=80$ lbs.

$\therefore$ Total force exerted by engine $=224+80=304$ lbs wt.

speed $=22$ ft/sec.

$\therefore$ Power of engine $=\frac{304\times 22}{550}=12{\cdot}16$ H.P.

10. If the lorry in question 9, moves down the inclined plane under the action of gravity calculate its acceleration down the plane. $2{\cdot}06\text{ ft/sec.}^2$

CHAPTER XI

MACHINES.

1. Machinery and simple machines.

Modern age has been called the age of machinery. We find that so many things which would be difficult or impossible to do by unaided man, can be easily done with the help of machinery. Lifting of very heavy weights, application of intense pressures, and development of high speeds are all possible only by the use of machinery. A machine may be defined as a device by which forces acting at one point may be made available at another point and in a more convenient form, or else weaker forces acting over a longer distance may be converted into stronger forces acting over a shorter distance or vice-versa. All complicated machinery is made from a few simple machines combined in various ways. The simple machines are the lever, pulleys, inclined plane, wedges and screws. A machine may be worked by human power, electric motor, or a heat engine or any other motive power.

2. Efficiency, mechanical advantage, and velocity ratio.

It should be clearly grasped that one performs a certain work upon a machine by the applied force or the effort, and the machine does work in lifting a weight or overcoming a resistance. Suppose that a force P is applied by an external agency on a machine, and that this force P, which is called the effort, or sometimes as power, acts through a distance a, then the work done by the effort is $P \times a$, because work is the product of the force acting and the distance moved by its point of application in the direction of the force. This work $P \times a$ has been supplied to the machine and is therefore called the work input or just the input. Now the machine moves some weight or overcomes some resistance W through a distance say b. Then the work done by the machine is $W \times b$, and this is called the work output or the output of the machine. As the machine is not a source of energy, it is obvious that the work obtained from a machine can never be greater than the work done on it. It therefore follows that the output can never be greater than the input, for otherwise the law of conservation of energy is violated. This statement is known as the principle of work.

In any actual machine the output is always less than the input, because friction is present between the parts of a machine, and some work is used up in overcoming this friction. The ratio output/input is called the efficiency of a machine. We thus see that the efficiency is less than one in any actual machine, and for an ideal or perfect machine, in which there is no friction at all, the efficiency would be unity. If the efficiency is multiplied by 100, the product is called the percentage efficiency.

Thus

$$\text{efficiency} = \frac{\text{Output}}{\text{Input}}$$

or

$$E = \frac{W \times b}{P \times a} = \frac{W/P}{a/b}$$

The ratio of the resistance overcome to the effort applied in W/P is called the mechanical advantage of a machine, and it is a ratio of two forces. Again the ratio of the distance through which the effort force moves, a, to the distance through which the weight is moved or resistance overcome, b, is named the velocity ratio of a machine. The velocity ratio is thus a/b. The above equation for efficiency can therefore be written as.

$$\text{efficiency} = \frac{W/P}{a/b} = \frac{\text{mechanical advantage}}{\text{velocity ratio}}$$

In an ideal or perfect machine in which there is no loss of work in the machine itself, the efficiency is equal to unity and therefore,

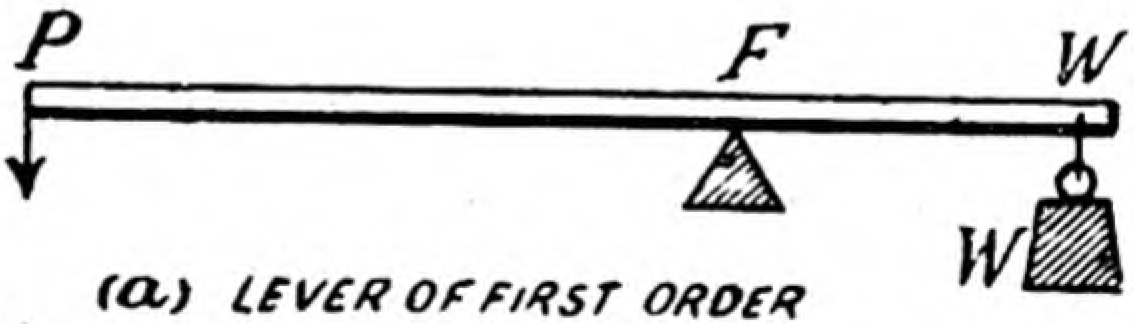
$$\text{Mechanical advantage} = \text{velocity ratio.}$$

3. Lever.

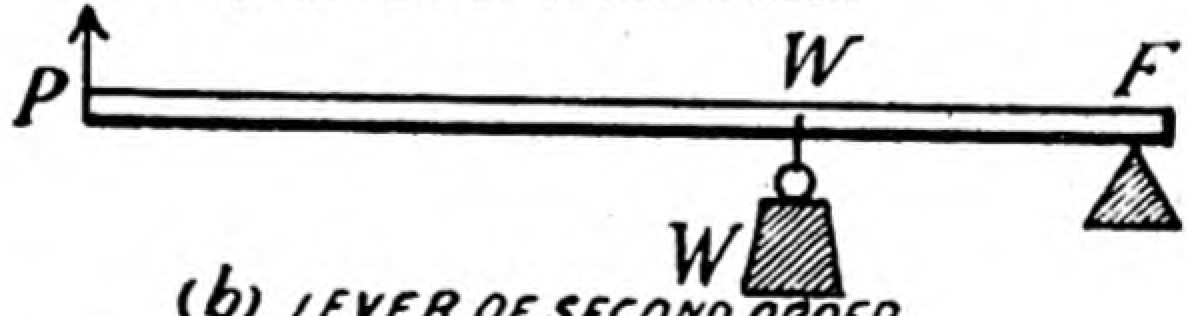
A lever is a rigid bar, which may be straight or bent, and by which a force or effort applied at one point is used to move a weight or overcome a resistance at some other point. There is always a certain point in a lever, called the fulcrum which remains fixed, but around it other parts can rotate. The perpendicular distance of line of action of the effort or power from the fulcrum is called the power arm of the lever while the distance, from the fulcrum, of the line of action of the weight or resistance is called the weight arm. Levers are of very wide use in machinery and in everyday life.

There are three classes of levers depending on the relative positions of the Fulcrum (F), the point of application of the Weight (W) and the point of application of the Effort or Power.

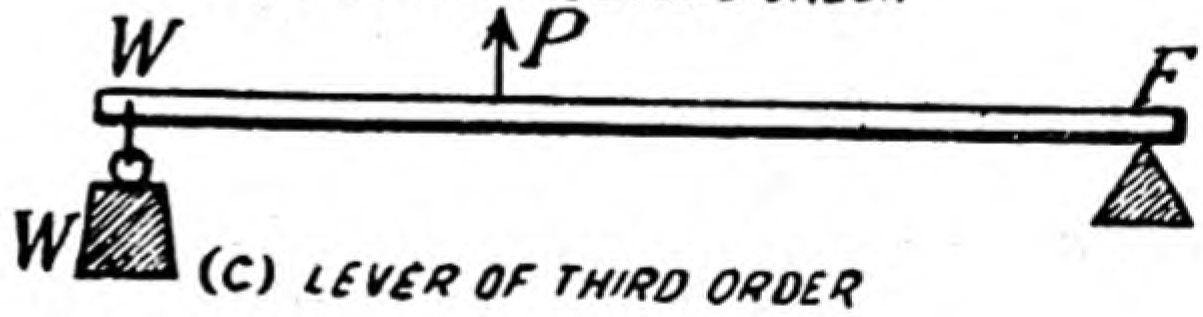
In a lever of the first order, F lies between W and P while in a lever of the second order W lies between F and P, and for that of the third order P lies between F and W. The three cases are diagrammatically represented in fig. 11·1.



(a) LEVER OF FIRST ORDER



(b) LEVER OF SECOND ORDER



(c) LEVER OF THIRD ORDER

Fig. 11.1

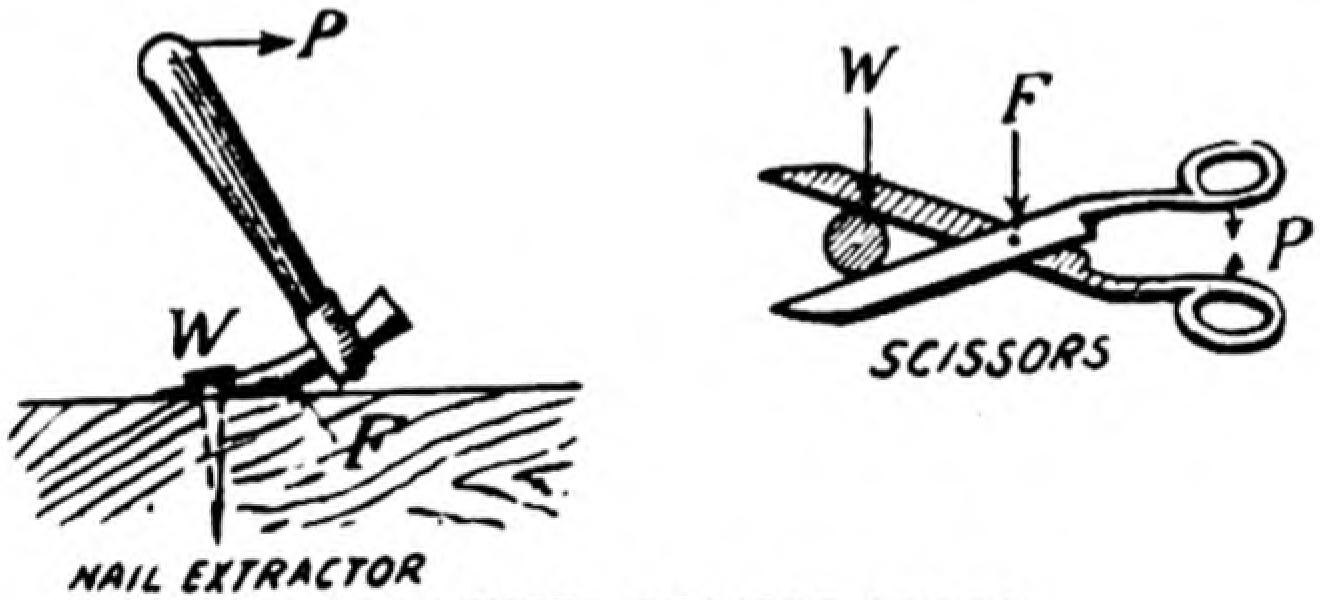
In order to find out the mechanical advantage in the case of a lever, we notice that when it is in equilibrium the moment of the effort or power around the fulcrum is equal and opposite to the moment of the weight around the fulcrum. If a is the power arm, then the moment of power is $P \times a$, and if b is the length of the weight arm then the moment of the weight is $W \times b$. These two must be equal or

$$P \times a = W \times b$$

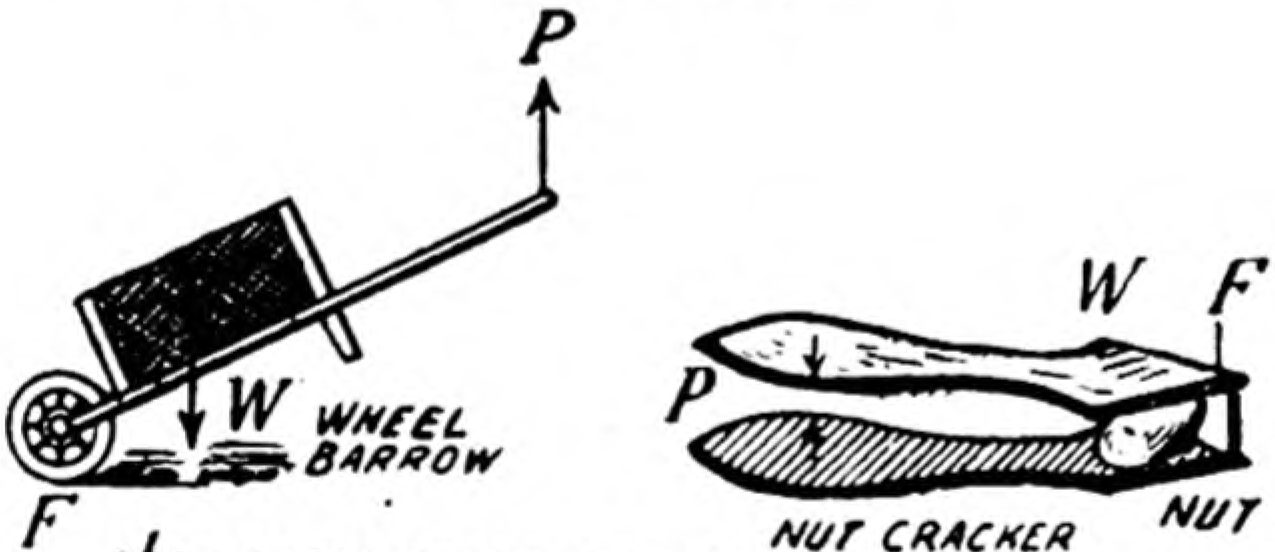
$$\therefore \quad \frac{W}{P} = \frac{a}{b}$$

$$\text{or Mechanical advantage of a lever} = \frac{\text{Length of the Power arm}}{\text{Length of the Weight arm or resistance arm}}$$

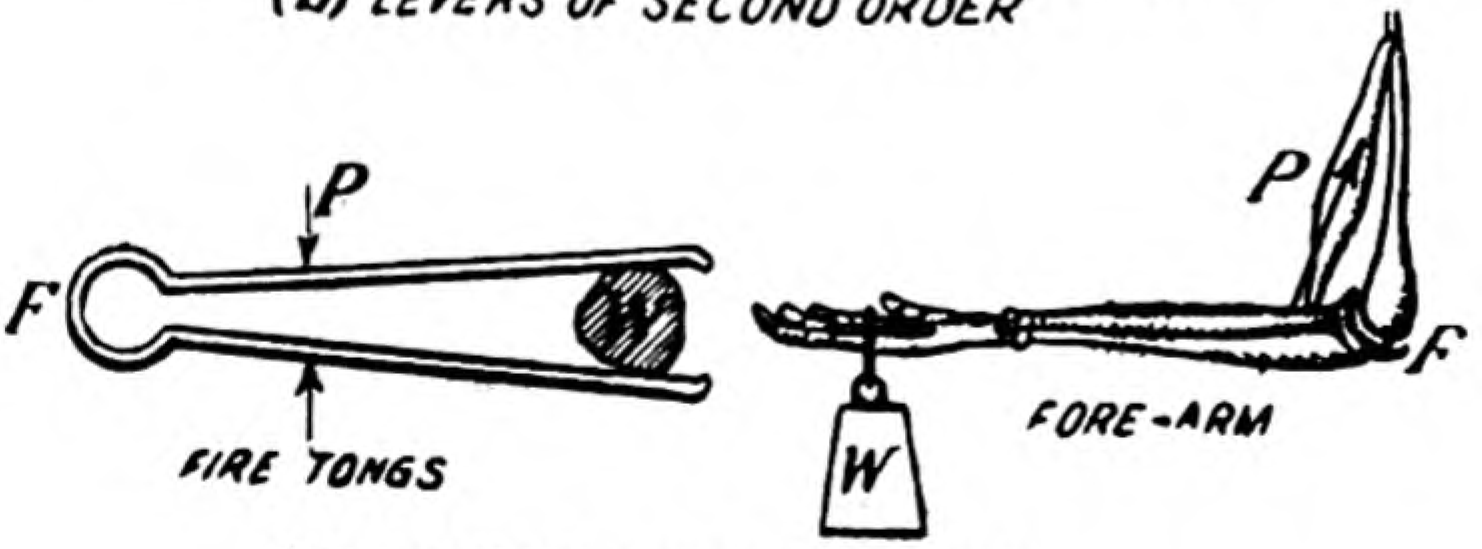
Following figures (11.2) show some levers of different orders.


P
W
F
NAIL EXTRACTOR
W
F
P
SCISSORS

(a) LEVERS OF FIRST ORDER


P
W
WHEEL BARROW
F
W
F
P
NUT CRACKER
NUT

(b) LEVERS OF SECOND ORDER


P
F
W
FIRE TONGS
P
F
FORE-ARM
W

(C) LEVERS OF THIRD ORDER

Fig. 11.2

As examples of levers of the first order we have a see-saw, a balance, a pair of scissors or pliers and a nail extractor etc. If the length a is greater than b, P will be less than W and the mechanical advantage greater than one.

Levers of the second order are a cork presser, a lemon squeezer, a nut cracker, a wheel barrow and an oar used for rowing the boat. Since the power arm is always greater than the weight arm, the mechanical advantage is always greater than one.

Some levers of third order are a firetongs, forceps and fore arm etc. In this case the power arm is always smaller than the weight arm and therefore P is greater than W. Mechanical advantage is less than one. We say that a lever of the third order has a mechanical disadvantage.

4. Wheel and axle.

This is a mechanical device commonly used for drawing water out of a well, or lifting ores out of a mine. It consists of a wooden or metallic drum, part of which is of a smaller radius termed axle and part of a bigger radius called the wheel. A rope is wound on the axle of which one end is fixed to a peg and the other end is attached to the body to be raised W. Similarly a rope is wound on the wheel, one of its ends being fixed to the wheel, the effort P being applied to the other end, as shown in Fig. 11.3. When one pulls at P the weight W is raised.

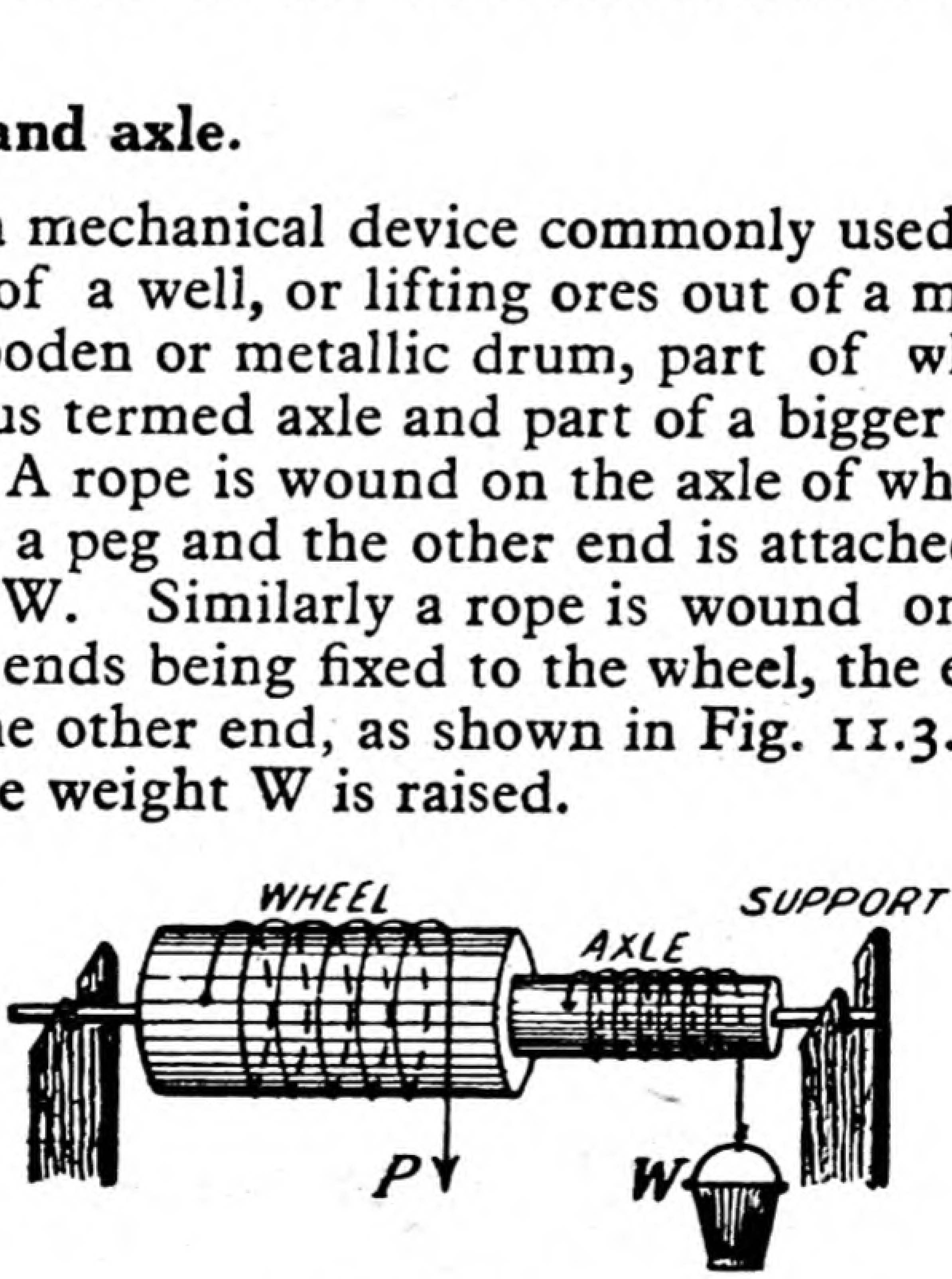


Fig. 11.3

In order to calculate the mechanical advantage, suppose that R is the radius of the wheel and γ that of the axle.

ڈر منظور احمد

In one complete revolution, a length $2\pi R$ of the rope on the wheel is pulled off by the force P acting on it. Therefore the work done on the machine is $P \times 2\pi R$. Also in one revolution, a length $2\pi r$ of the rope on the axle becomes wound, and therefore the weight W is raised by the height $2\pi r$. Thus the work done by the machine is $W \times 2\pi r$. If we suppose that there is no loss of work, then by the Principle of work applied to machines.

$$W \times 2\pi r = P \times 2\pi R$$

or

$$\frac{W}{P} = \frac{R}{r}$$

The mechanical advantage therefore equals the ratio of the radius of the wheel to the radius of the axle. This rotation is only true in the ideal case, when there is no friction or work done on parts of the machine. Actually the mechanical advantage is less than $\frac{R}{r}$.

We can also calculate the mechanical advantage $\frac{W}{P}$ by

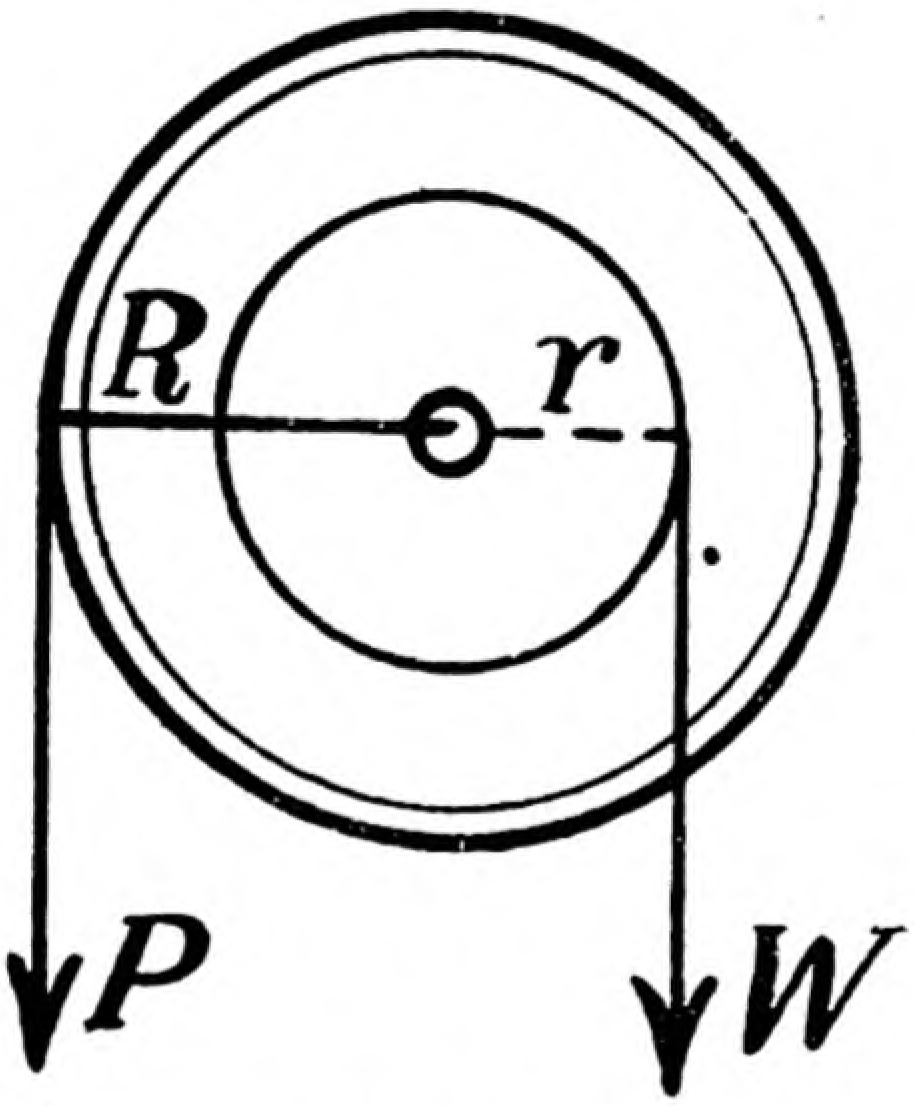


Fig. 11.4

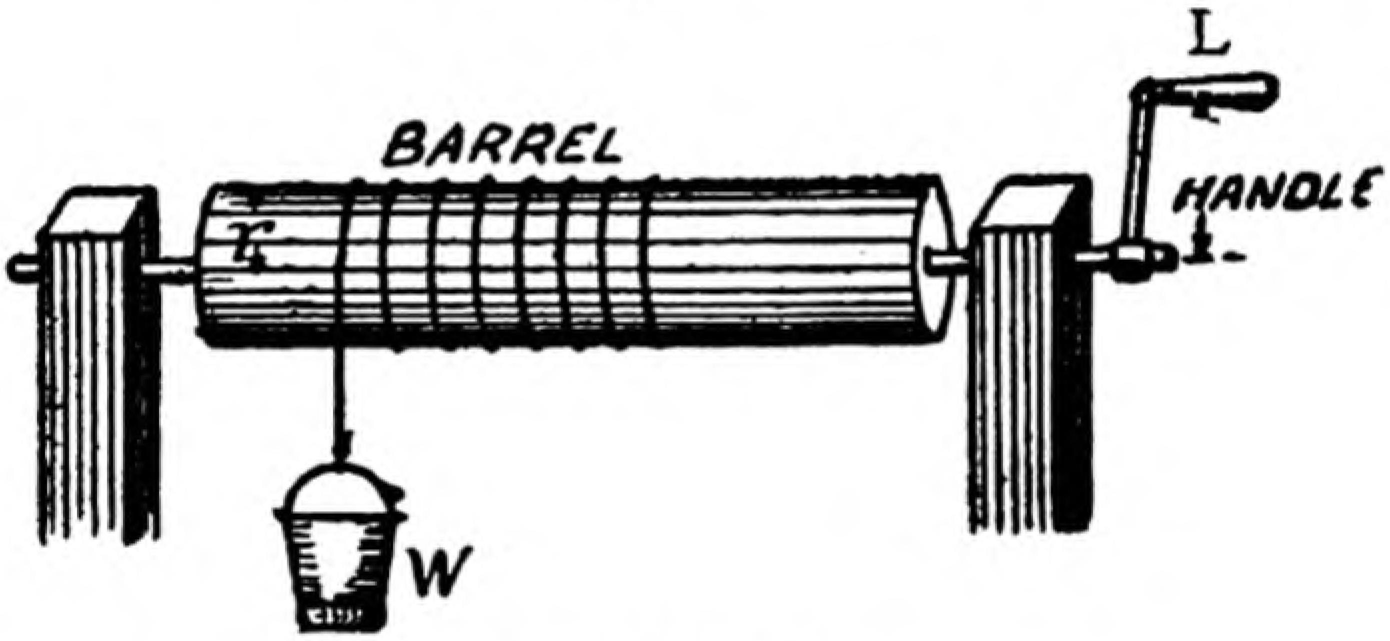
finding the moments of the two forces P and W around the central axis as in Fig. 11.4 The two moments are $P \times R$

and $W \times r$ and these are in opposite sense. For equilibrium the two should be equal or

$$W \times r = P \times R$$

i. e.
$$\frac{W}{P} = \frac{R}{r}.$$

In a modified form of this machine, called the **Winch** or the **Windlass**, Fig. 11.5, a long handle is substituted for the wheel. By rotating the handle, the rope on the axle or the barrel is wound or unwound and the weight is raised or lowered. If L is the length of the handle, or its distance from the axis of the barrel, and r the radius of the barrel, the mechanical advantage is $\frac{L}{r}$.



The Windlass
Fig. 11.5

The Winch is used on a crane and also as Capstan on ships to haul and lower the heavy anchor.

5. Pulley.

Pulleys are very widely used for lifting heavy weights. A pulley fig. 11.6 consists of a circular wheel or disc called the sheave S, which has a groove G around the rim, and which can revolve around an axle held on a frame called the block B.

When a single fixed pulley is used for raising weights, the block is fixed to some rigid support as in fig. 11.7

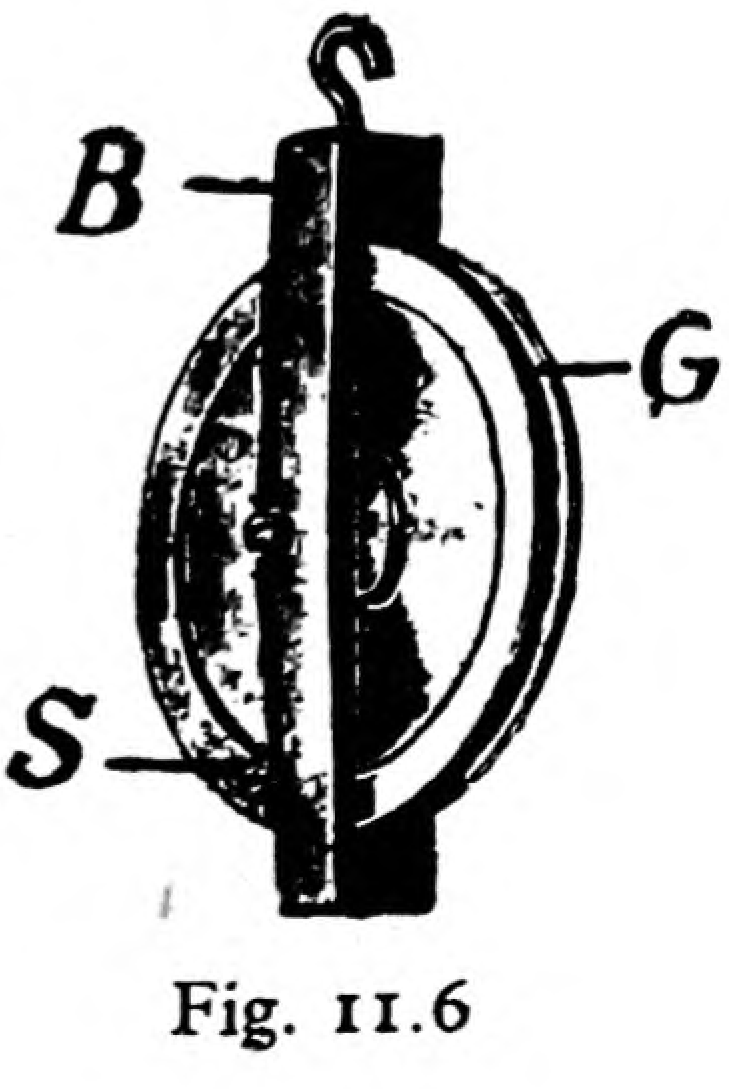
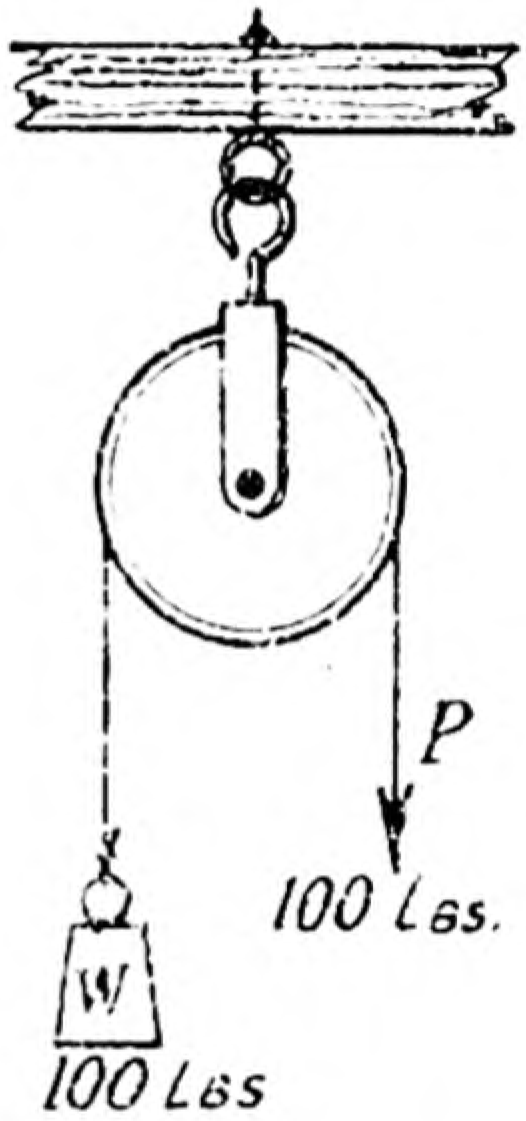


Fig. 11.6

If any body read this
he is fool and also [illegible]
this of this [illegible]

The load or the weight (W) is tied to one end of a rope, which passes round the groove of the pulley, while effort (P) is applied to the other end of the rope. It is to be seen that in order to raise W, the effort has to be applied in the downward direction and a downward pull is easier than an upward lift because one can apply the weight of the body when pulling a rope downwards.



A Single fixed pulley

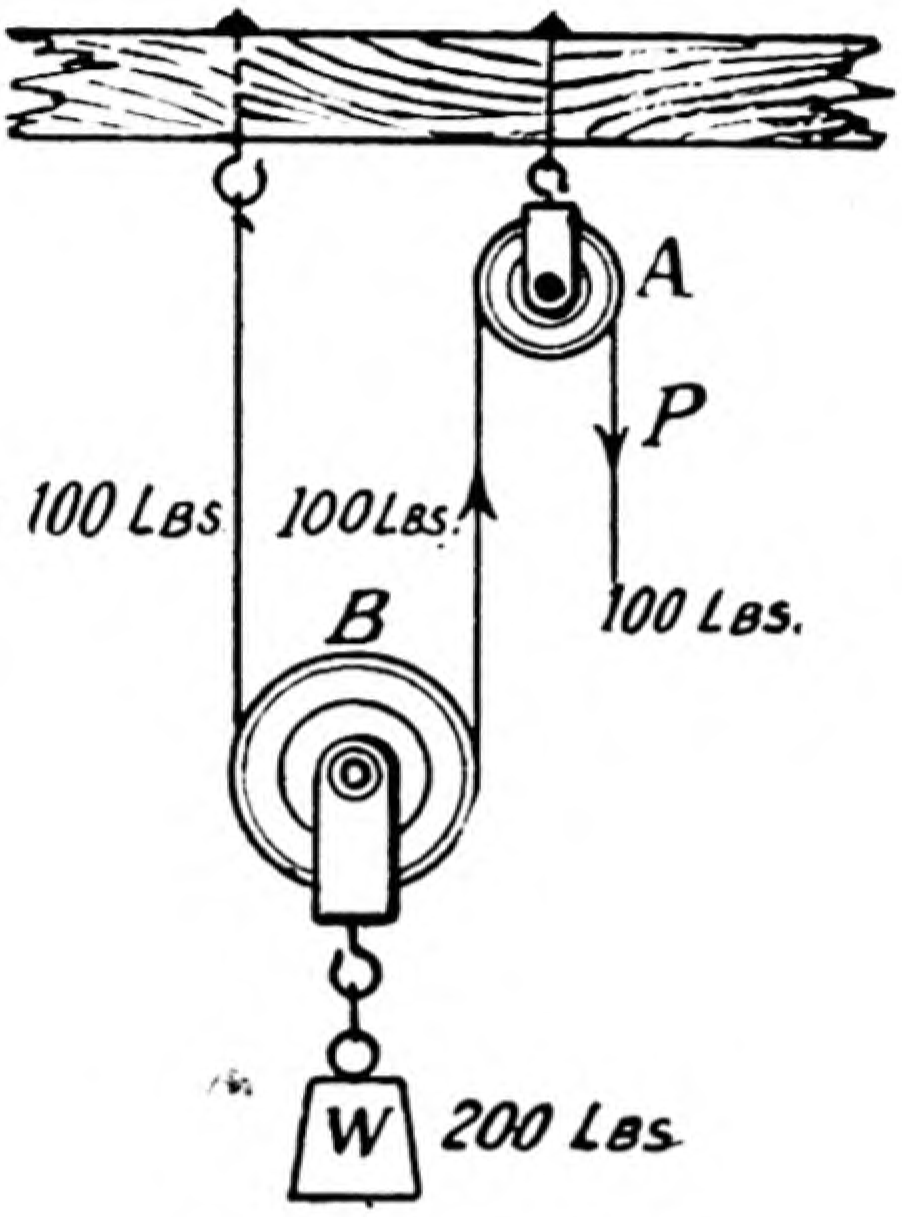
Fig. 11.7

Suppose a point of the rope is pulled down a distance d, by the force P. Then since the rope is inextensible the weight is raised by the same distance d. The work done on the machine is $P \times d$, and by the machine $W \times d$. On equating them

$$W \times d = P \times d \quad \text{or} \quad \frac{W}{P} = 1$$

Therefore a single fixed pulley has no mechanical advantage. It only changes the direction of application of the force needed to lift a body. Suppose the weight of a body is 100 lbs.; then to raise it up, one has to pull down on the rope with a force of 100 lbs. Actually it will be a little more.

Another method of using a pulley is as shown in fig. 11.8. In this case the weight to be raised is attached to the block of the pulley and as the weight is raised, the pulley also rises with it. It is therefore called a movable pulley. The rope used with such a pulley has one end fixed to a firm support, and the rope after passing round the groove of the movable pulley B, is generally again passed round a fixed pulley A, and the effort is applied to the free end of the rope. If P is the pull or effort applied, then neglecting friction, the tension throughout the rope equals P in value. Therefore, as shown in the figure the upward tension in each segment of the rope supporting the weight W equals P. For equilibrium the upward force 2P should be equal to the downward force (W + ω), where ω is the weight of the movable pulley.



A movable Pulley

Fig. 11.8

Therefore $$W + \omega = 2P$$

or $$W = 2P - \omega$$

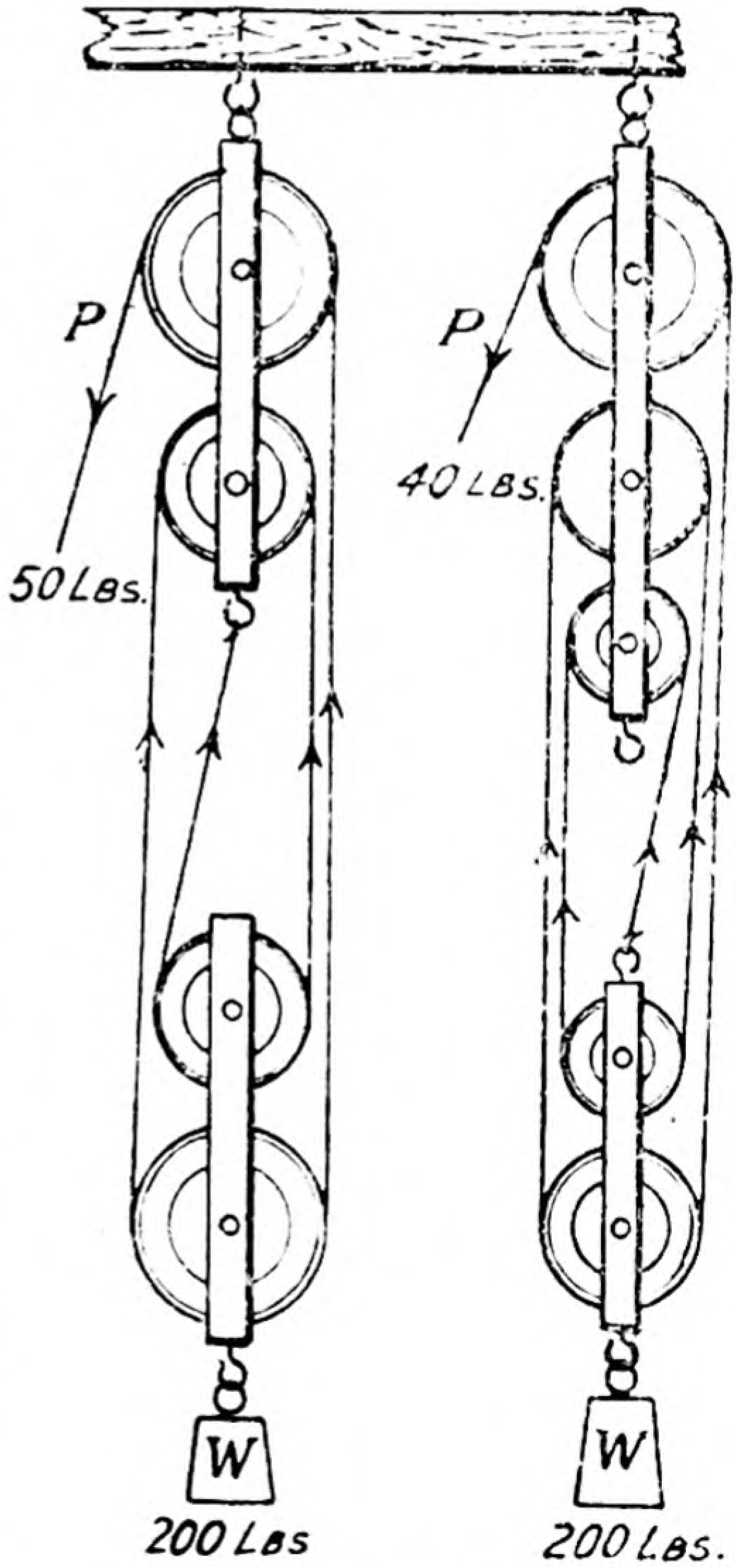
or $$\frac{W}{P} = 2 - \frac{\omega}{P}.$$

If we neglect the weight ω of the movable pulley, the mechanical advantage is equal to two. Thus if a weight of 200 lbs. is being raised with such a pulley the force or effort to be applied is 100 lbs.

It has to be remembered that what is gained in effort is lost in distance. To prove this suppose the weight has been raised through a height equal to *d*. Then each segment of the string or rope has been shortened by a length equal to *d*, or that the force P has pulled off a length 2*d* of the rope. The work done by the effort is P × 2*d*, and the work done by the machine W × *d*. Equating them we find that mechanical advantage equals two.

6. Pulley block or block and tackle.

In order to have a greater mechanical advantage than that obtainable with a single movable pulley, combinations of more than one pulley are frequently used. One such combination is what is called **Block and Tackle**. In this case there is more than one pulley fixed to the same block. One such multiple pulley is used as fixed, while the other is used as movable, and to this movable pulley block the weight to be raised is attached. A single rope has one end fixed to block, and alternately passes around a pulley in the lower or movable and the upper or fixed blocks until it finally comes out from the last pulley in the upper block, and the effort or pull is applied to this portion of the rope. In fig. 11.9 two



Block and Tackle

Fig. 11.9

such pulley systems are shown. In (*a*) the number of pulleys both in the movable and the fixed block is two, which in (*b*) the number of pulleys in the movable block is two and the number in the fixed block is three. It will be noticed that the fixed end of the rope in (*a*) is attached to the block of the upper or fixed pulleys while in (*b*) it is attached to the block in the lower or movable pulleys.

As before if P is the effort applied then neglecting friction P is the tension in each segment of the rope. Therefore in the case shown at (*a*) the suspended weight $(W+\omega)$ is being supported by an upward force of 4P, as there are four segments supporting the weight. For equilibrium

$$W+\omega=4P$$

or

$$\frac{W}{P}=4-\frac{\omega}{P}$$

and the mechanical advantage is nearly four being equal to the number of segments of the rope supporting the weight. Similarly in (*b*) there are five segments of the rope and if P is the tension in each, the upward force is 5P and for equilibrium,

$$W+\omega=5P$$

or

$$\frac{W}{P}=5-\frac{\omega}{P}$$

the mechanical advantage being thus equal to 5.

In general if *n* segments of the rope are supporting the weight, then

$$n\times P=W+\omega$$

or

$$\frac{W}{P}=n-\frac{\omega}{P}$$

Therefore the mechanical advantage nearly equals *n*, that is the number of segments supporting the weight and the movable block.

The velocity ratio is determined by supposing that a length l of the rope has been pulled out by the effort P. Then each of the segments of the rope must have shortened by a length $\frac{l}{n}$ and that is also the height by which the weight has been raised. Therefore the velocity ratio $=$ l $= n$. This means, to raise the weight W through a certain distance d_1, we have to apply a force $P = \frac{W}{n}$, but through a distance d_2 equal to n times d_1, for $W \times d_1$ must be equal to $W \times d_2$ if the machine is perfect. It is therefore obvious that what is gained in power is lost in speed or distance.

7. Differential pulley.

This type of pulley system can give a large mechanical advantage, although the efficiency is not high. Such a pulley system consists of a movable pulley to the block of which the weight is attached, and a fixed pulley block in which there are two pulleys of different ratio held on the same block. The two pulleys of the fixed block instead of being separate, may consist of a single wheel on which two grooves of different radii have been made. Instead of using a rope a chain is used, the parts of the chain interlocking with teeth along the rim or circumference of the pulleys. Use of the chain and teeth prevents slipping of the rope in the groove when very heavy weights are being raised. The chain is of an endless type and it passes around the pulleys as shown in the fig. 11·10.

To calculate the mechanical advantage, suppose the radii of the larger and the smaller pulleys in the fixed block are r_2 and r_1. If the effort P is applied to the part a of the chain, then during one complete rotation of the upper block the length of the chain which has been pulled out at a is $2\pi r_2$. This means that the part b of the chain, which is supporting the weight and the movable pulley, has been shortened by $2\pi r_2$. During the one revolution of the upper or fixed pulley a length of chain equal to $2\pi r_1$ has been fed to the smaller of the two pulleys. This is equivalent to a lengthening of part c of the chain by $2\pi r_1$. Therefore in one revolution of the upper block the chain supporting the weight has been shortened on the side of b by $2\pi r_2$, and lengthened on the side c by $2\pi r_1$. Therefore the net shortening of the whole chain supporting the weight is $(2\pi r_2 - 2\pi r_1) = 2\pi(r_2 - r_1)$. Since there are two segments of the chain holding the weight, the weight has been raised by a height equal to half of the shortening of the length of the chain supporting it. This means that W has been raised by $\pi(r_2 - r_1)$. If we suppose that there is no loss due to friction, and we neglect the weight of the lower or movable pulley itself, it follows from the principle of work that

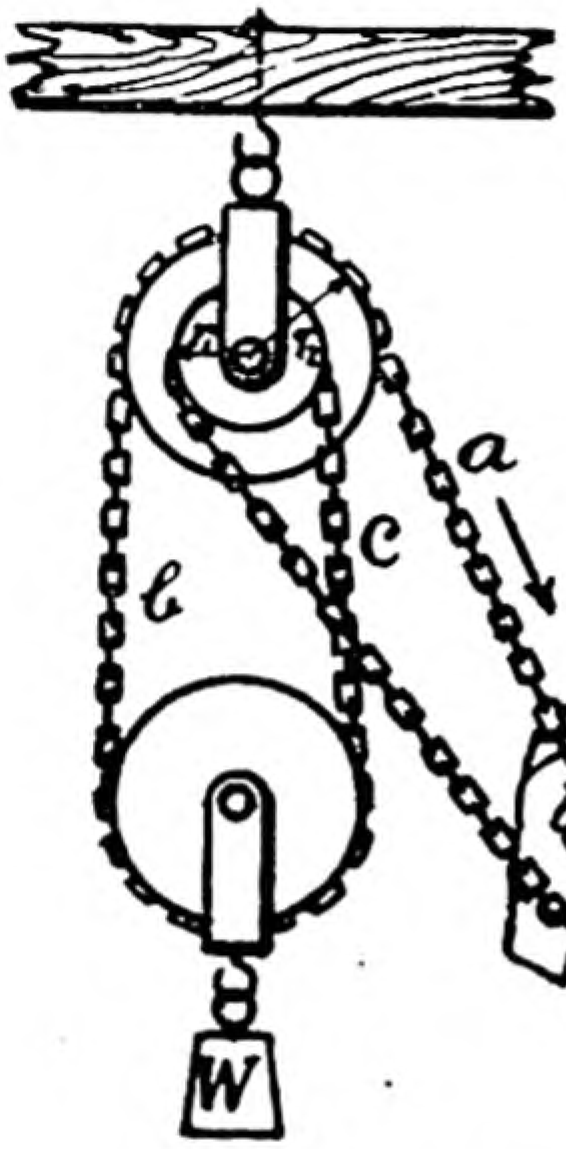


Fig. 11.10

$$W \times \pi (r_2 - r_1) = P \times 2\pi r_2.$$

i.e.,
$$\frac{W}{P} = \frac{2 r_2}{(r_2 - r_1)}.$$

Actually such a pulley possesses an efficiency between 50 and 60% and the mechanical advantage is therefore less than that given by the above equation.

Figure 11.11, shows a heavy weight being lifted by a pulley system, the effort to which is applied by an engine. The machinery shown is a mobile crane, which is of great use in industry.

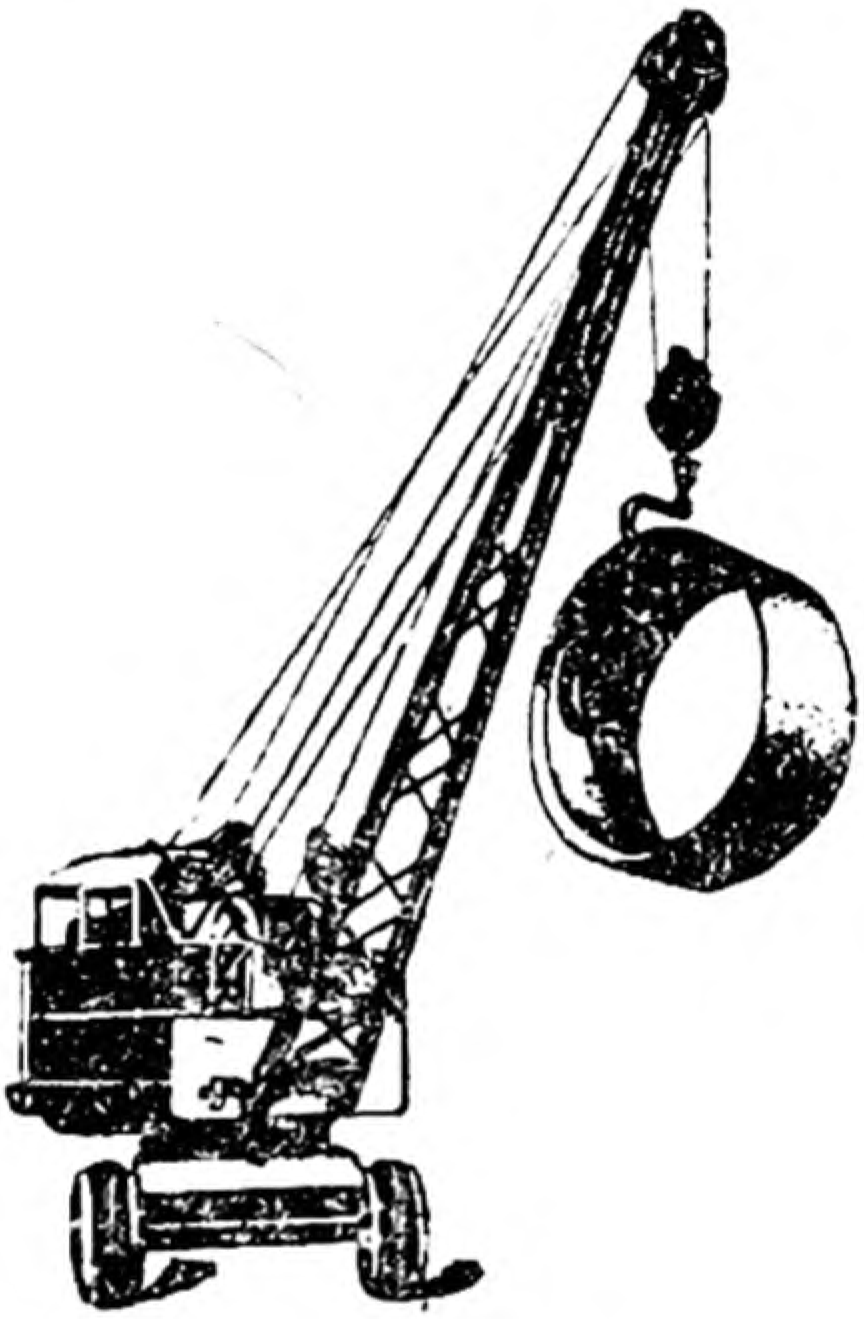


Fig. 11.11

8. The inclined plane.

It is a common experience that when a heavy body has to be raised, we can do it more easily by pushing it or

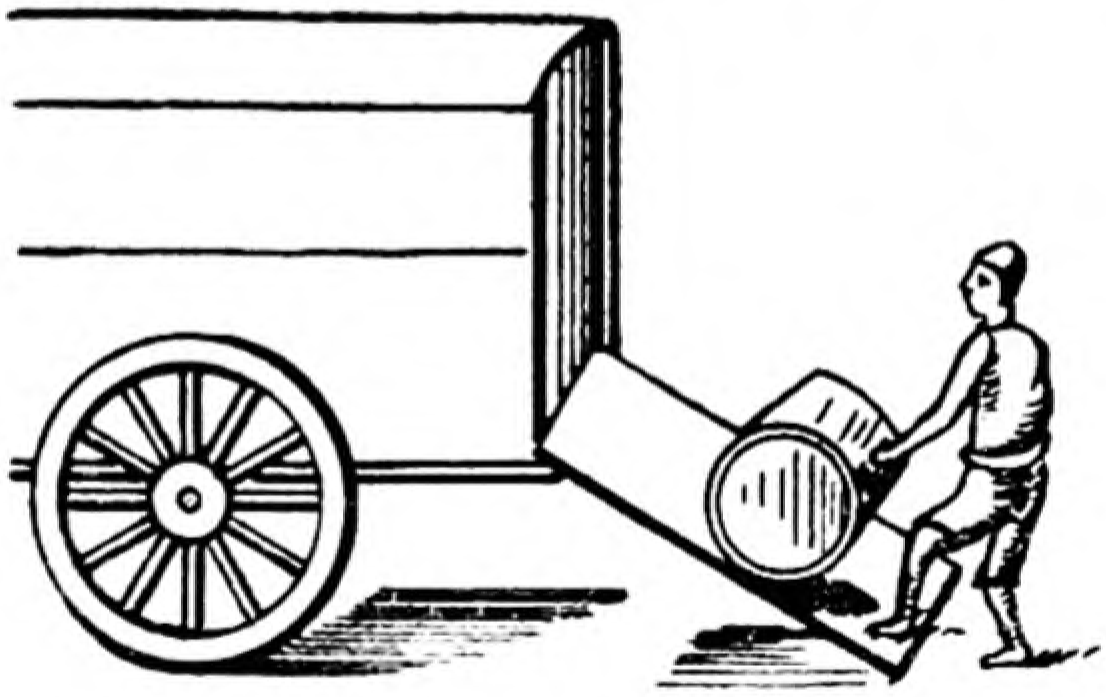
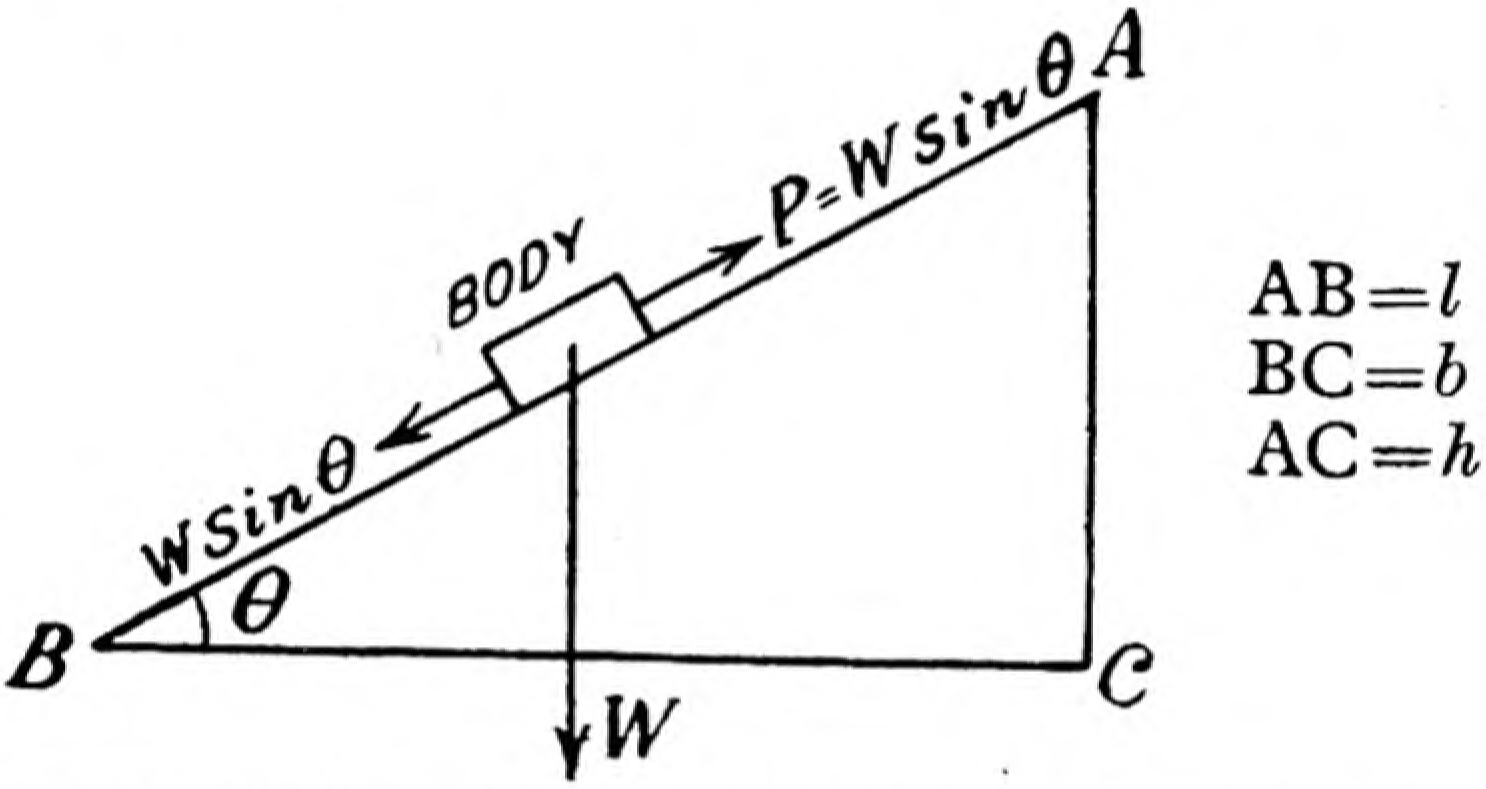


Fig. 11.12

rolling it up on a sloping or inclined surface than to lift it up directly. Work-men loading boxes or barrels to a lorry slide them or role them up a plank, the upper end of which rests on the lorry and the lower end on the ground fig. 11.12.

An inclined plane may be looked upon as a machine as it requires less force to raise a body on the inclined surface than to raise it up vertically. The railway lines in the hills follow a long circuitous route in order to have a less steep incline. It has been shown previously that when a body is lying on an inclined surface with the angle of inclination θ to the horizontal, then if W is the weight of the body its component along the length of the plane is W sin θ fig. 11. 13. Therefore to move the body up the plane the force required is W sin θ if we ignore friction etc. The mechanical advantage of the inclined plane, when the applied force or effort P is parallel to the length of the plane, is therefore

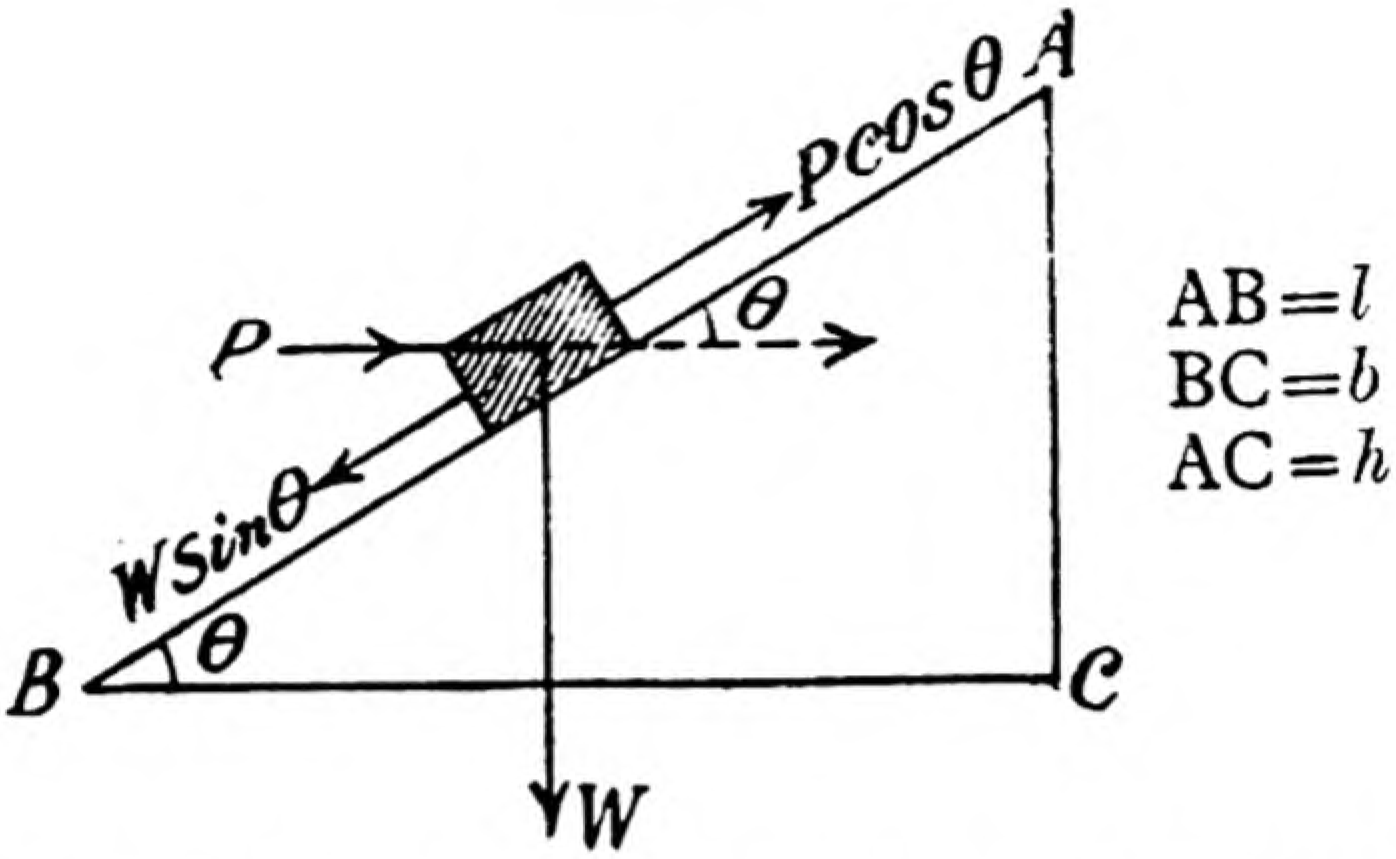


Inclined plan effort acting parallel to the plane

Fig. 11.13

$$\frac{W}{P} = \frac{W}{W \sin \theta} = \frac{1}{\sin \theta} = \frac{l}{h}.$$

If the applied force or effort P is acting not along BA but horizontally, that is in the direction parallel to BC, fig. 11.14, then the component of P upward along the length of the plane, i.e., P cos θ is effective in moving the body or overcoming the force W sin θ. For equilibrium


P cos θ
A
P
θ
W Sin θ
B
θ
C
W
AB = l
BC = b
AC = h

Inclined plane, effort acting in a horizontal direction.

Fig. 11.14

these two forces P cos θ and W sin θ should be equal. Therefore

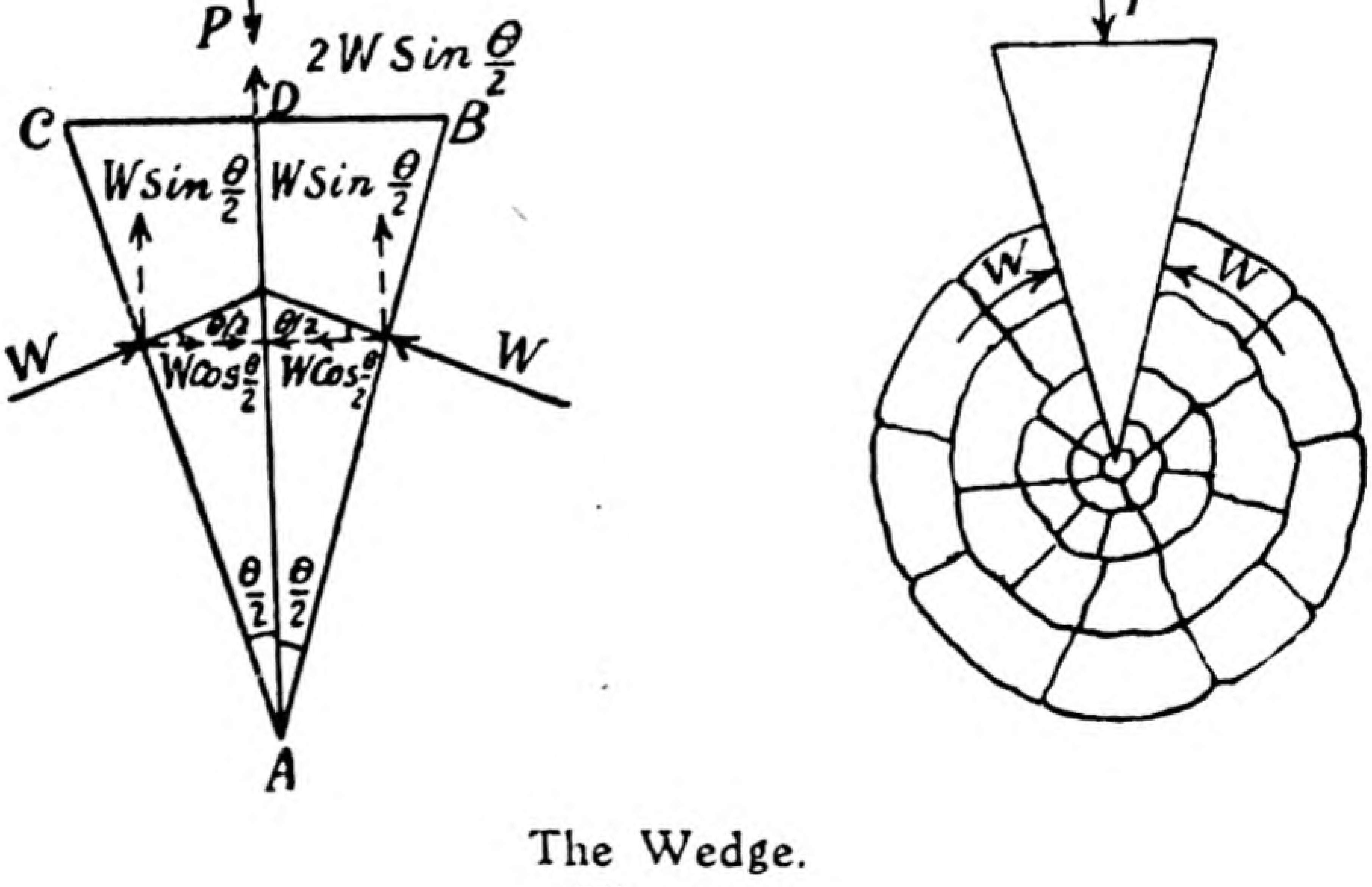
$$W \sin \theta = P \cos \theta$$

or

$$\frac{W}{P} = \frac{\cos \theta}{\sin \theta} = \frac{1}{\tan \theta} = \frac{b}{h}.$$

9. Wedge.

The wedge is used for splitting wood and it may be looked upon as a device which is a double inclined plane.


P
2W Sin θ/2
C
D
B
W Sin θ/2
W Sin θ/2
W
W
θ/2
θ/2
W Cos θ/2
W Cos θ/2
θ/2
θ/2
A
P
W
W

The Wedge.

Fig. 11.15

As shown in fig. 11.15, a force W acts perpendicularly on each side of the wedge, while in order to drive it in, the hammer strikes on the head with a force at least equal to P. If θ is the angle of the sharp edge then the components of W parallel to BC and AD are $W \cos \frac{\theta}{2}$ and $W \sin \frac{\theta}{2}$. The components $W \cos \frac{\theta}{2}$ are acting in opposite directions and therefore cancel each other, while the two components $W \sin \frac{\theta}{2}$ are acting in the same direction and therefore add to each other's effect. Friction between the surfaces of the wedge and the wood play an important part as without friction it would not be possible to leave the wedge in its position. The applied force P must at least be equal to $2 W \sin \frac{\theta}{2}$. Equating these two quantities we get

$$\frac{W}{P}=\frac{1}{2 \sin \frac{\theta}{2}}.$$

This is however a very approximate relation as there is a large amount of friction and the force is applied by striking and not continuously.

The axe, the knife, the razor and a pin are some examples of the wedge.

10. Screw.

The screw is one of the most widely used mechanical devices to overcome strong resistance or lift heavy weights or apply large pressures. Screws are made by cutting a spiral groove along the surface of a cylindrical piece of metal or wood. The spiral ridge so formed is called the thread of the screw. The distance between two consecutive threads measured parallel to the axis of the cylinder, is called the pitch of the screw, fig. 11.16. This is the distance moved by the end or tip of the screw during one complete revolution. Such a screw fits into a similar ridge

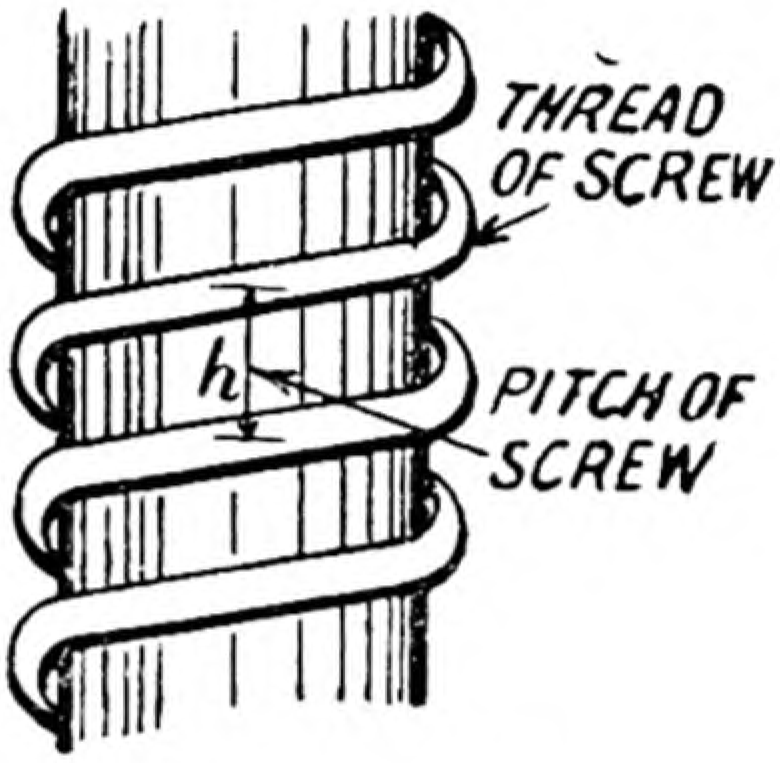


Fig. 11.16

and groove made on the inside of a hollow cylindrical surface. Such a pair of screw surfaces fitting into each other is called a bolt and nut or a screw and nut. Fig. (11.17).

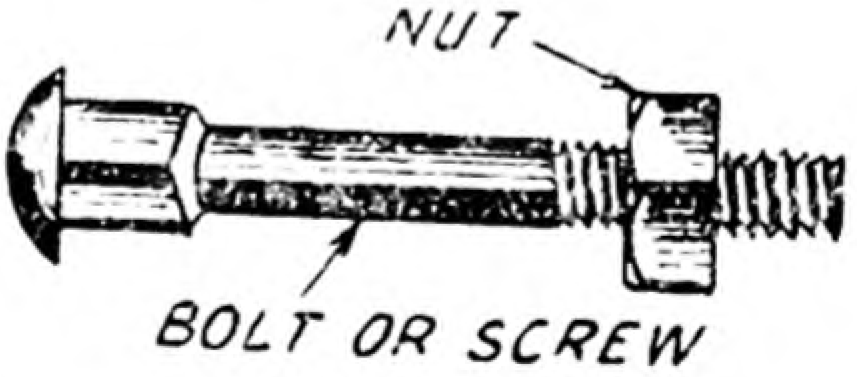
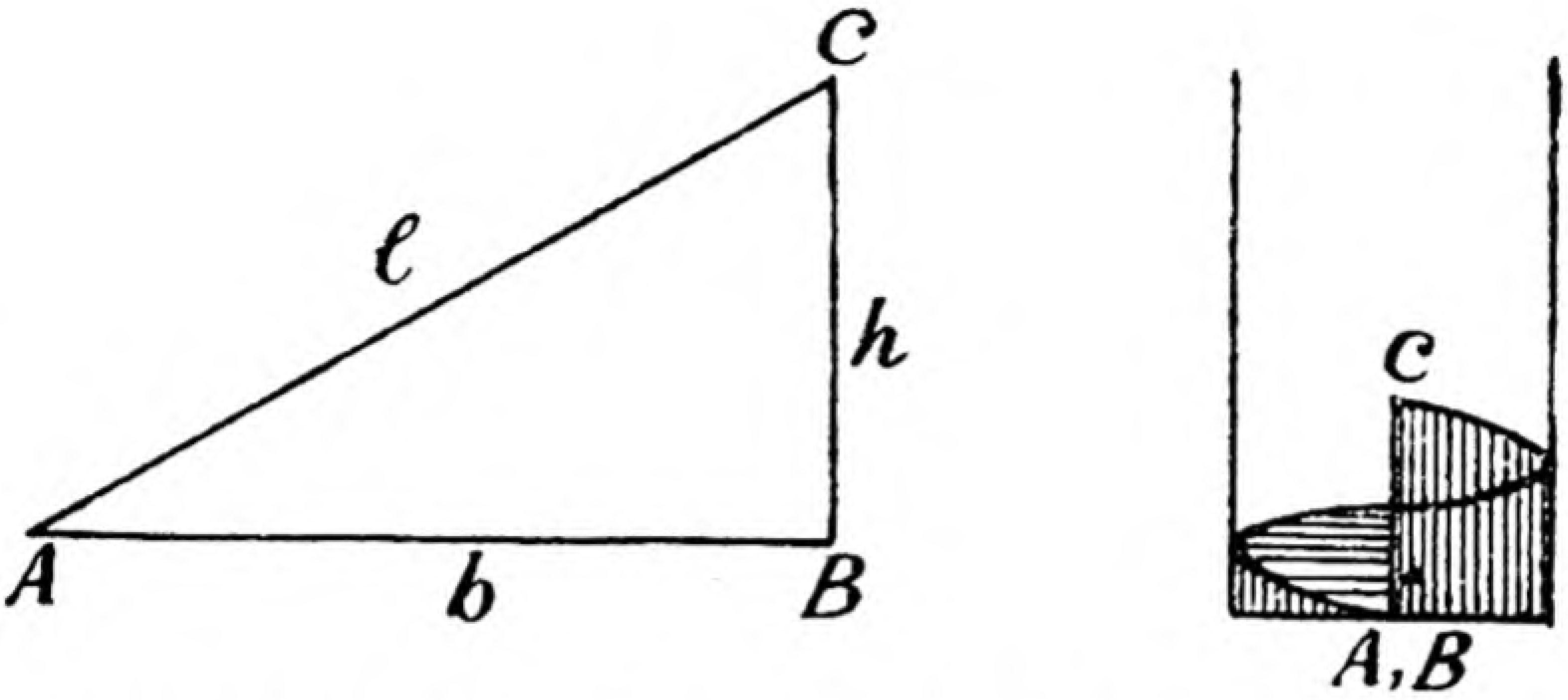


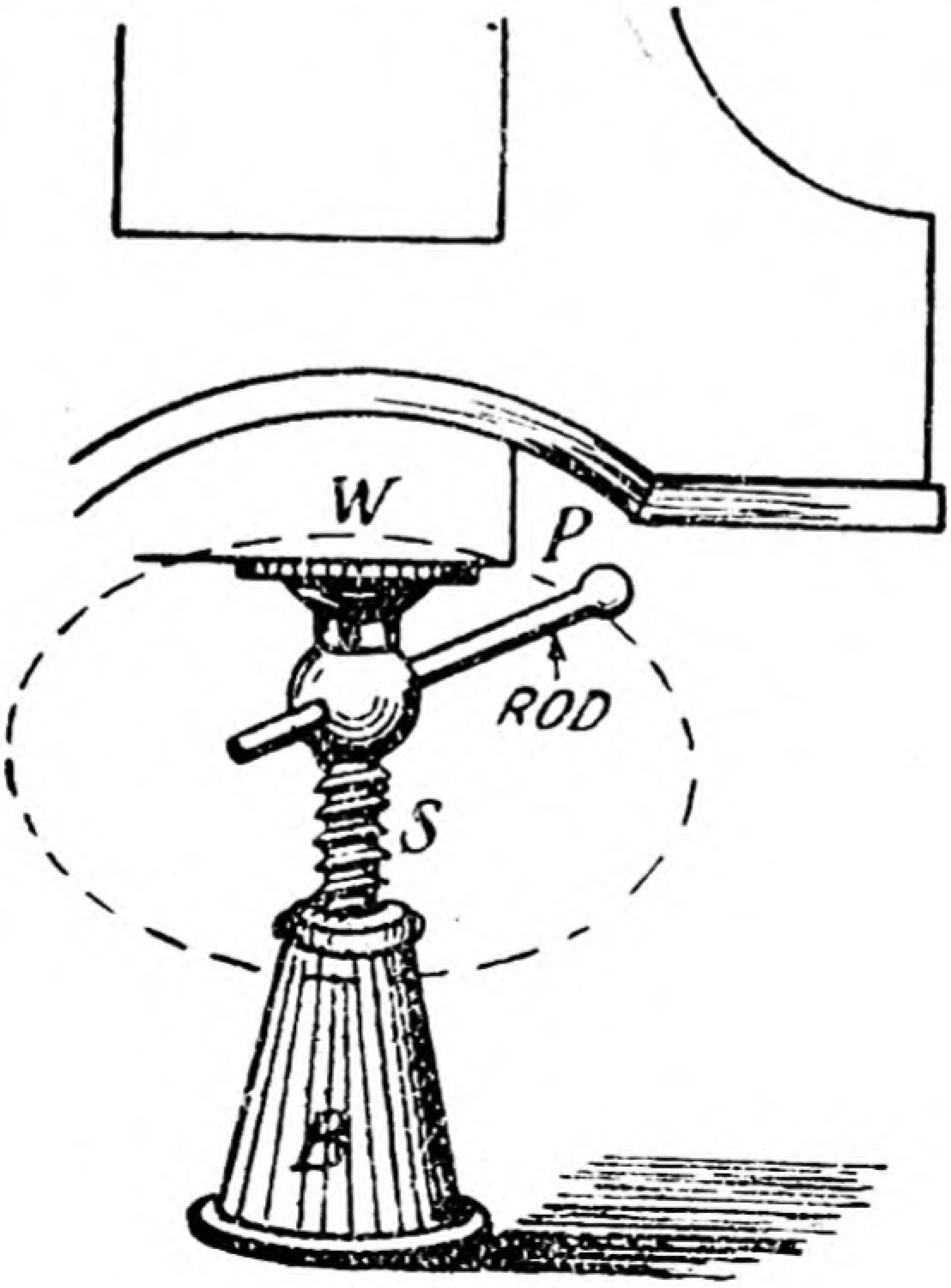
Fig. 11.17

The principle of the working of the screw resembles that of an inclined plane. We have seen that if by moving up a length l of an inclined plane an object rises by a height h, then a mechanical advantage of $\frac{l}{h}$ is obtained. Suppose that the inclined plane, shown in fig. 11.18 was to be cut out and pasted on a cylindrical surface of circumference



length l becomes the thread of the screw and height h becomes the pitch of the screw.

Fig. 11.18



Screwjack.

Fig. 11.19

equal to b, the base of the inclined plane. Then in one revolution the length of the plane will form a part of a spiral and by going once round this spiral, the distance moved parallel to the axis of the cylinder will be h or height of the inclined plane and as described above, this is the pitch of the screw. An example of such a spiral inclined plane occurs in the case of roads and railway lines in the hills when the vertical distance moved is small but a round circuitous path is taken. The spiral staircases are also constructed with the same object in view that is to obtain a mechanical advantage.

The screw jack, which is used to raise a motor car or a truck, when changing the wheels, is another common example of the use of a screw. As shown in fig. 11.19 it consists of a strong screw S working in a nut which has been cut in the upper portion of the base or support B. The object to be raised is placed on a small loose platform at the upper end of the screw, so that when the screw is rotated the loose platform does not rotate with it. A rod inserted through a hole in the screw helps to rotate the screw easily, serving almost as the arm of a lever.

Suppose W is the weight resting upon the screwjack and P is the force or effort applied at the end of the length l from the screw. Then in giving one complete rotation to the rod the work done by the effort is $P \times 2\pi l$. The weight W will have been raised by a height h equal to the pitch of the screw in one rotation of the screw and the work done equals Wh. Neglecting friction, we have from the principle of work

$$W \times h = P \times 2\pi l$$

or

$$\frac{W}{P} = \frac{2\pi l}{h}.$$

Actually there is a large amount of friction in screws and the mechanical advantage is nearly half the above value.

The screw principle is also used in the book-binding press to produce strong pressures.

Example. A screwjack has three threads to an inch and a lever arm 14 inches long. If a force of 20 lbs. can just raise a weight of one ton with the screwjack, find its actual mechanical advantage and efficiency.

$$h=\tfrac{1}{3}'' \qquad \text{and } l=14''$$

$$\therefore \quad \text{Ideal mechanical advantage} = \text{velocity ratio} = \frac{2\pi l}{h}$$

$$= \frac{2\times 22\times 14\times 3}{7\times 1}$$

$$=264.$$

$$\text{Actual mechanical advantage} = \frac{\text{Weight lifted}}{\text{Power applied}} = \frac{2240}{20}$$

$$=112.$$

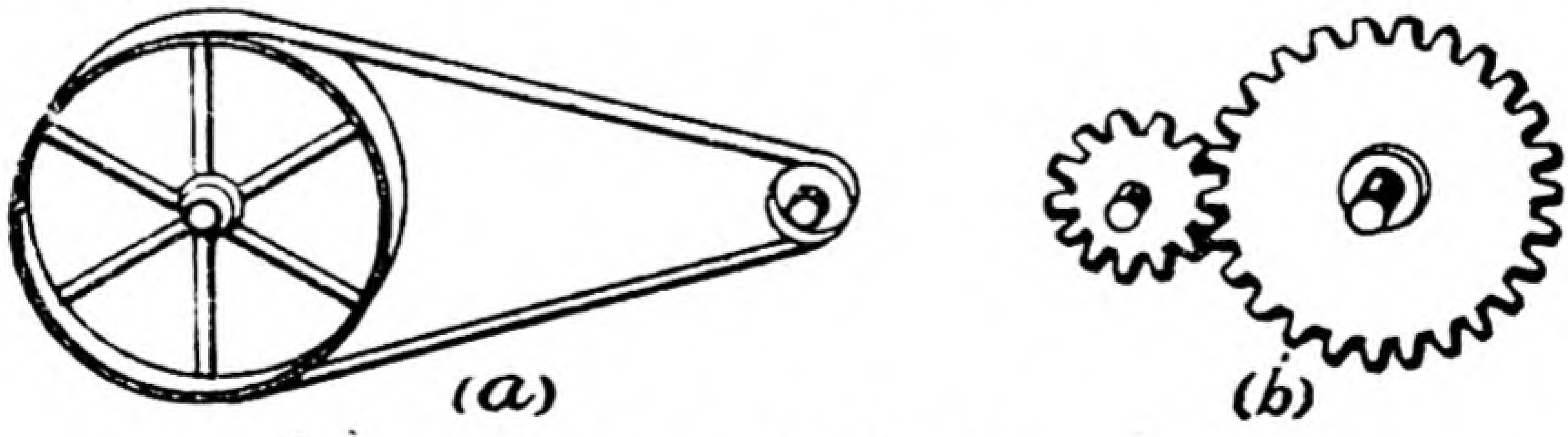
$$\text{Efficiency} = \frac{\text{Actual mechanical advantage}}{\text{Velocity ratio}(=\text{ideal mechanical advantage})}$$

$$= \frac{112}{264} = \frac{14}{33}$$

$$\text{Percentage efficiency} = \frac{14}{33}\times 100 = 42{\cdot}4\%$$

11. Use of wheels for power transmission.

Connected wheels also work as a machine for transmitting power from the effort to the load or the resistance. Such mutually connected wheels have a velocity ratio, mechanical advantage and efficiency as considered previously for other machines, although the movement in this case is one of rotation. Figure 11.20 shows two wheels of



Connected wheels.

Fig. 11.20

different diameters, connected in one case (a) with a belting and in the other case (*b*) by a system of teeth interlocking with each other. When one of the wheels is rotated by an effort, the other wheel rotates and performs work in driving a machine. If the smaller wheel has a radius r, and the bigger wheel radius r_2, then the number of revolutions which the smaller wheel makes for one revolution of the bigger wheel is $\frac{2\pi r_2}{2\pi r_1}=\frac{r_2}{r_1}=r$. This therefore is the ratio of their angular speeds that is speed reduction or speed increase, and assuming the system to be frictionless it is also their mechanical advantage. The toothed wheels are used in the gear-box of a motor car, in which one of the wheels is driven by the engine sheft and the others to which it can be geared, drive the back wheels of the car. By changing the gear the speed and the mechanical advantage can be changed. On a level track the speed is greater but the mechanical advantage is less while on going up hill the speed is less and mechanical advantage greater. This again shows that in machines what is gained in speed is lost in power and also what is gained in power is lost in speed.

In the case of a bicycle the toothed pedal wheel or spoked wheel is rotated by the force applied on the pedals. The sprocket wheel is connected to a smaller toothed wheel called the hub wheel mounted on the same axle as the back wheel of the bicycle. The two toothed wheels are connected by a chain and tend to give a bigger velocity ratio to the machine.

12. Beam balance and measurement of mass.

We have seen previously that a spring balance measures the weight of a body at a place, since the extension in its spring is proportional to the downward pull of the earth on the body that is suspended from it. It does not measure the mass of body, because the same mass can have different weights at different places on the earth. We can however, compare the masses of two bodies with a spring balance by weighing them at the same place. Suppose w_1 and w_2

are the weights of two masses m_1 and m_2 as recorded by a spring balance. Then.

$$w_1=m_1g \text{ and } w_2=m_2g$$

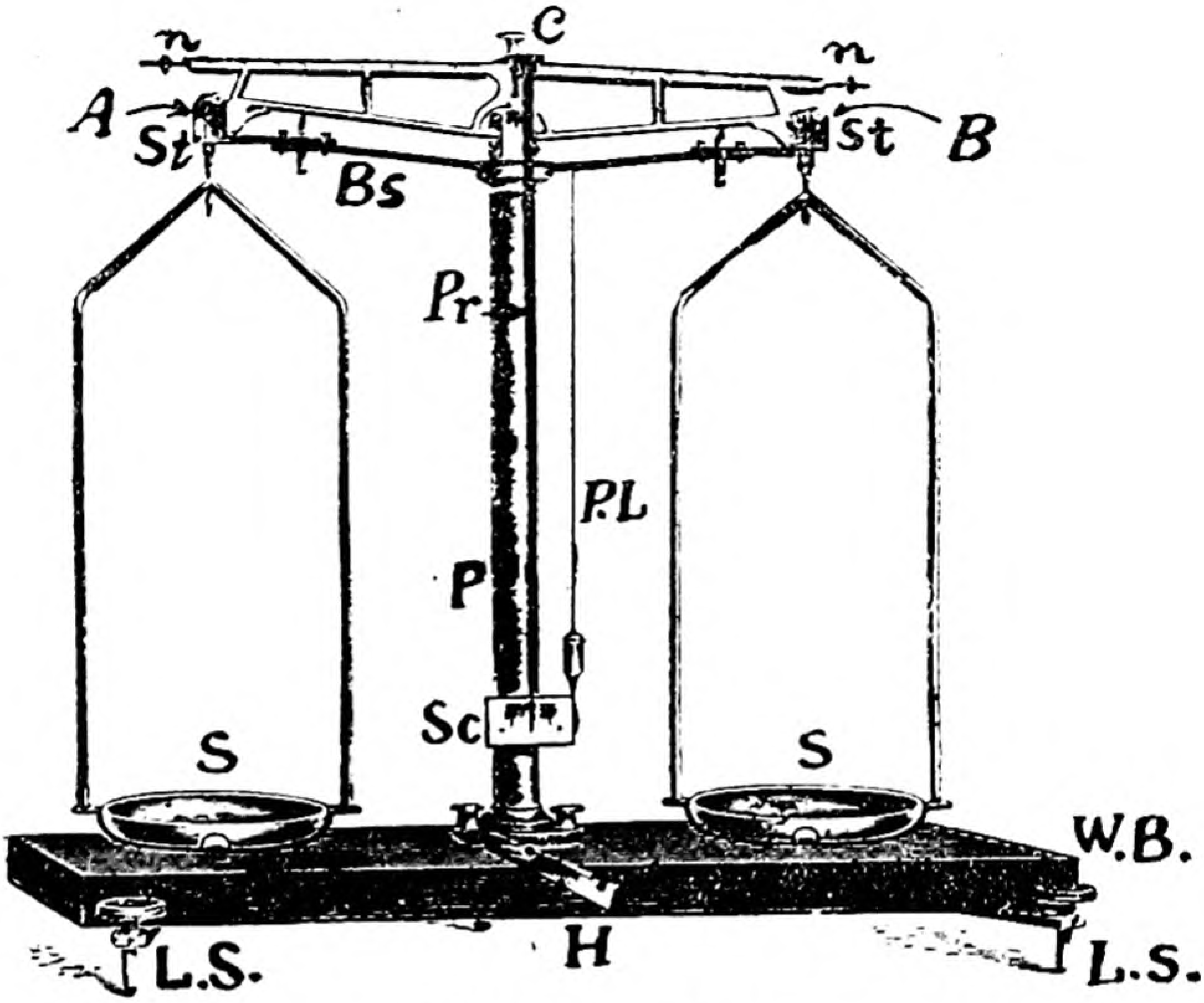
or $$\frac{w_1}{w_2}=\frac{m_1g}{m_2g}=\frac{m_1}{m_2}$$

This means that at any one place, masses are proportional to their weights and therefore bodies having equal weights have equal masses

An ordinary beam balance which is a lever of the first order with equal arms, measures masses on this principle. When a body in one pan balances a standard weight in the other pan it means that the earth is pulling them with an equal force and therefore the mass of the body must be equal to the mass of the standard weight. This balance will give the same value of the mass of a body at all places of the earth whereas a spring balance will give different readings of the weight of the same body because the force of gravity is not constant at all places.

13. The Laboratory balance.

A sensitive laboratory balance (fig. 11.21) consists of a rigid metallic beam ACB having equal arms AC and BC. The fulcrum C consists of an agate knife-edge, resting on a hard smooth agate plate on the top of the vertical rod inside the hollow pillar P which is fixed to the wooden base (W.B.) of the balance. The rod carrying the agate plate can be raised or lowered by means of the lever handle H. The arms also have knife edges at the points A and B near the ends from which scale pans of equal weights (S,S) are suspended from the stirrups (St, St), which rest on the knife-edges. The beam carries small adjustment nuts (*n*, *n*) for fine adjustment in order to make the beam horizontal. The end of a long pointer, Pr, fixed perpendicularly to the beam from its middle moves over a scale, Sc, near the base of the hollow pillar P. A plumb limb (P. L.) hangs from the beam support (B.S.), along the side of the vertical pillar. The wooden base (W. B.) of the



Laboratory balance.

Fig. 11.21

balance, supporting the pillar, rests on three legs, two of which are levelling screws (L. S.).

Whenever it is required to use the balance, it is first of all levelled with the help of the levelling screws, so that the tip of the plumb line comes exactly over a sharp point attached to the side of the pillar P. When the balance is not in use, the beam rests on the beam support (B. S.) and not on the agate plate to avoid unnecessary rubbing of the knife edges against the agate surfaces. When the lever handle H is, turned the beam is raised and, supported on the agate plate, swings about the knife edge C as shown by the movement of the pointer on the scales. If the balance has been properly adjusted the pointer oscillates equally about the zero of the scale, otherwise it will swing more towards one side. The adjustment nut on the lighter arm is moved outwards or that on the heavier side is moved inwards until the pointer has equal amptitudes about the zero of the scale. After such a setting the balance is ready for weighing.

The balance is usually housed in a glass case to protect it from draughts.

14. Requisites of a Balance.

A good balance used to compare the masses of the two bodies should fulfil the following conditions.

(*i*) It should be true.
(*ii*) It should be sensitive.
(*iii*) It should be stable.

Truth : A balance is said to be true if the beam becomes horizontal when equal masses are added to or removed from the pans. To satisfy this condition the moments in clockwise and counterclockwise directions should be the same.

Let M_1 and M_2 be the masses of right and left pans respectively and let l_1 and l_2 be the corresponding arms of the beam. The beam remains in the horizontal position. Therefore,

$$M_1 \, g \, l_1 = M_2 \, g \, l_2$$

$$\text{or,} \quad M_1 \, l_1 = M_2 \, l_2 \qquad \dots \quad (i)$$

Now equal masses m are placed in each pan and again the beam remains horizontal.

$$(M_1+m) \, g \, l_1 = (M_2+m) \, g \, l_2$$

$$\text{or} \quad (M_1+m) \, l_1 = (M_2+m) \, l_2 \qquad \dots \quad (ii)$$

Subtracting (*i*) from (*ii*), we get

$$m \, l_1 = m \, l_2$$

$$\text{or} \quad l_1 = l_2$$

Substituting this in the (*i*), we get

$$M_1 = M_2$$

Thus we see that for a balance to be true, its arms should be of equal lengths and its pans should have equal masses.

Sensitiveness : A balance is said to be sensitive when a small difference between the masses of the loaded pans is shown clearly by the considerable deviation of the beam from its horizontal position. As a rule the beam balance has the same sensitiveness over a considerable range of loads ; the sensitiveness for any particular load being defined as the deflection, in scale divisions, produced when an excess of 1 milligram is added to the scale pan. The following consideration gives the factors which influence the sensitiveness of a balance.

Suppose a vertical section is taken through the centre of the beam, cutting the knife edges at A, B, C. (Fig. 11.22). l is the length of each arm and h is the distance

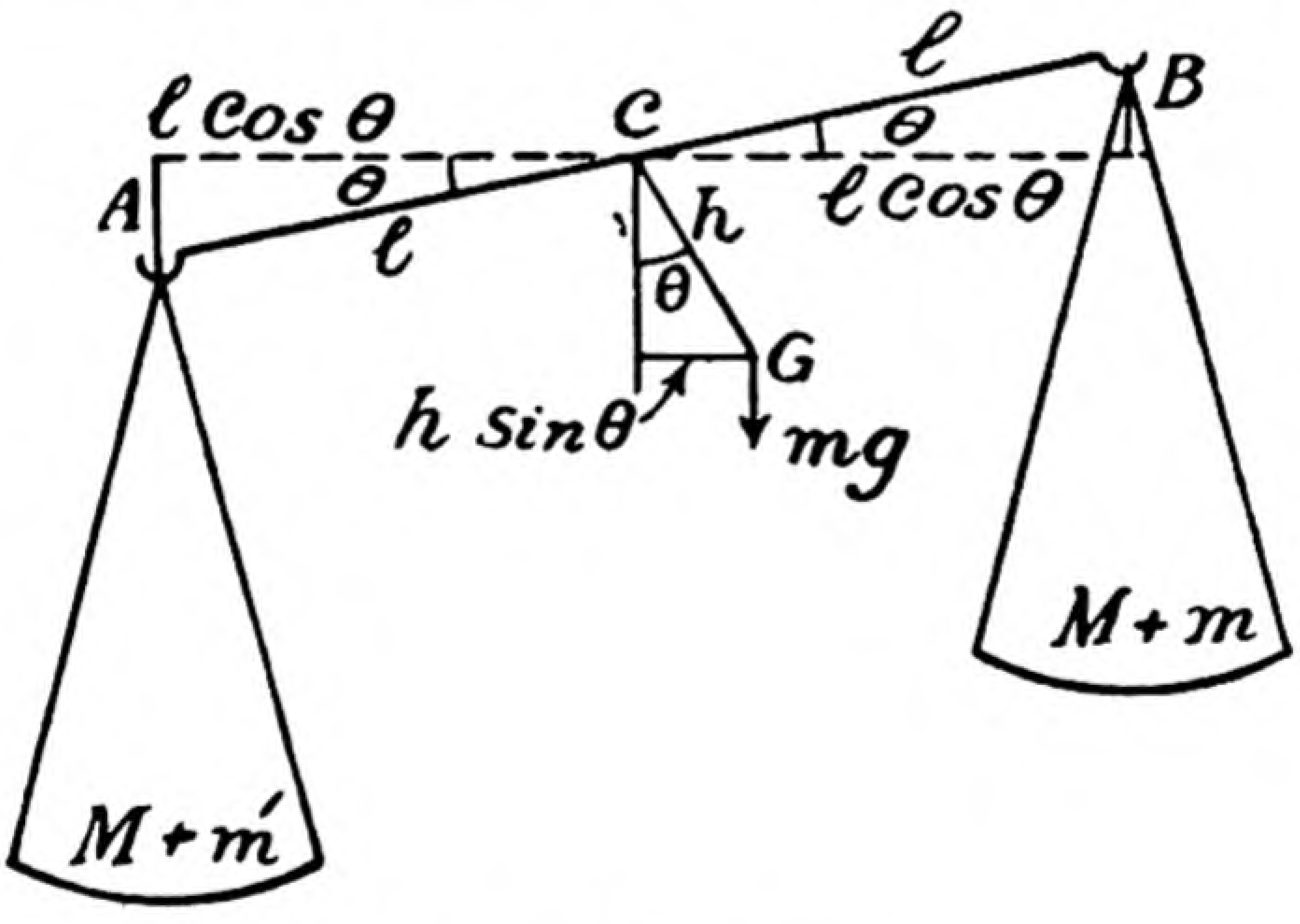


Fig. 11.22

of the centre of gravity G below C. Let the mass of the beam be μ, the masses of the scale pans M ; and m, m' are the masses placed in the right and left hand pans respectively. Let the beam come to rest inclined at an angle θ to the horizontal.

In the equilibrium position the moments of the system in the clockwise direction must be equal to the moments in the counterclockwise direction. Taking moments about C, we have

Clockwise moments $=(M+m)\,gl\cos\theta+\mu.gh\sin\theta.$

Counter ,, ,, $=(M+m')\,gl\cos\theta.$

For equilibrium

$$(M+m)\,gl\cos\theta+\mu.\,gh\sin\theta=(M+m')\,gl\cos\theta.$$

or $$(M+m)\,l\cos\theta+\mu h\sin\theta=(M+m')\,l\cos\theta.$$

or $$\mu\,h\sin\theta=(m'-m)\,l\cos\theta.$$

or $$\tan\theta=\frac{(m'-m)\,l}{\mu h.}$$

To make θ appreciable, i.e. to make $\tan\theta$ have large value, for a small difference $(m'-m)$ between the masses of the loaded scale pans, l should be made large, μ should have small value and also h, the distance between C and G should be small. In actual practice l cannot be made large for the balance would become unmanageable. Nor the beam can be made lighter otherwise it would bend by heavy weights. So the only alternative left to increase the sensitiveness of the balance is to make h, the distance of the centre of gravity G below C, as small as possible.

Stability : A balance is said to be stable when it comes to equilibrium quicker after it is disturbed.

When the pans are equally loaded and the beam is oscillating, the moments of the right and left pans remain equal at all stages. The only moment which tries to restore it to equilibrium position is that of the weight of the beam, which is equal to $\mu gh\sin\theta$, or $\mu gh\,\theta$ for θ is small. For a certain fixed value of θ, the expression will have large value when μ, the mass of the beam is large and h, the distance of G below C is also large. Though increasing μ, the mass of the beam helps in increasing the restoring moment, yet at the same time the mass to be moved also becomes greater. Thus increasing the mass μ of the beam does not help in increasing stability.

The only way left is to increase the value of h, the distance between C and G. This would decrease the sensitiveness of the balance. So a balance cannot be stable and sensitive at the same time.

15. To find the correct weight of a body with a false balance.

(1) A balance may have equal arms (l, l) but pans may be of unequal weights (M_1, M_2). The beam of such a balance will not come to rest in the horizontal position when the pans are empty. To find the true weight of a body with such a balance, we place the body in the left hand pan and find the weight x which, when placed in the right hand pan, makes the beam horizontal. If W be the true weight of the body then taking moments about the fulcrum of the beam,

$$(M_1+W)\,l=(M_2+x)l$$

or $$M_1+W=M_2+x\ldots\ldots(i)$$

Now we place the body in the right hand pan and find the weight y to be placed in the left hand pan to make the beam horizontal. Taking moments about the fulcrum again,

$$(M_2+W)l=(M_1+y)l$$

or $$M_2+W=M_1+y\ldots\ldots(ii)$$

Adding equations (i) and (ii) we have

$$M_1+M_2+2W=M_1+M_2+x+y$$

or $$W=\frac{x+y}{2}.$$

∴ True weight = Arithmetic mean of the weights of the body when weighed in the left and the right pans.

(2) A balance may have unequal arms l_1 and l_2 and yet the weights of the pans be such that the beam is horizontal

when the pans are empty. In this case

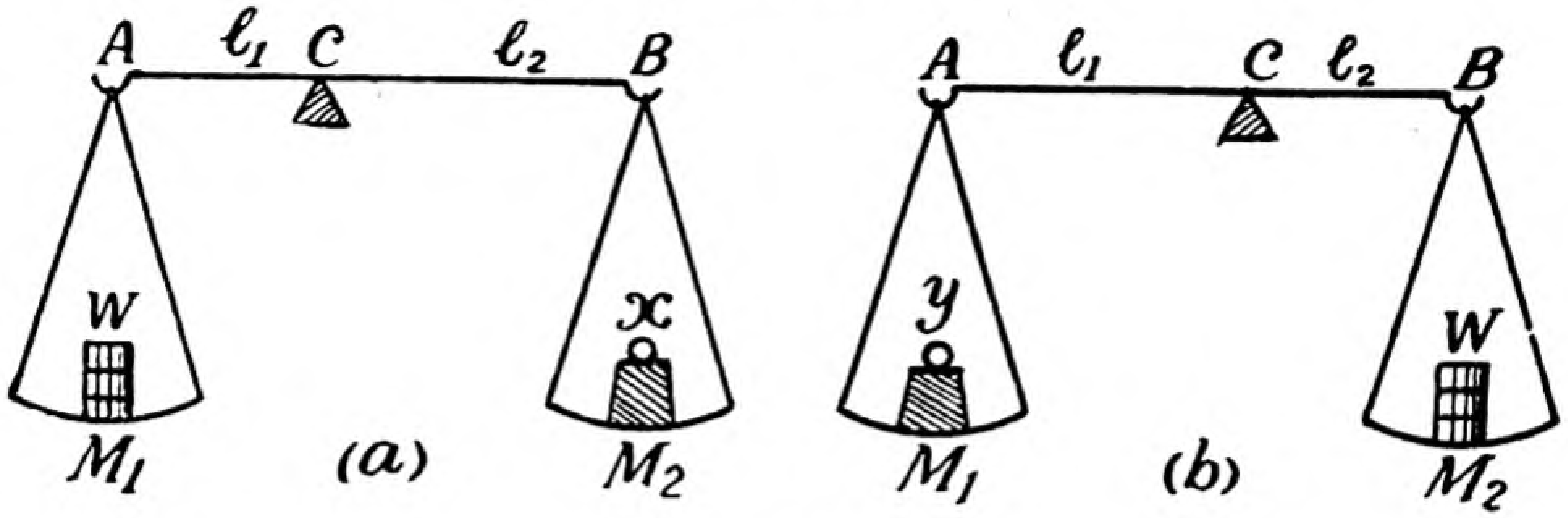


Fig. 11.23

$$M_1 l_1 = M_2 l_2 \qquad \dots \quad (iii)$$

suppose the weight of a body as shown by this balance is x when it is placed in the left hand pan as in fig. 11.23*a*. If W be the true weight then by taking moments about the fulcrum *c*,

$$(W + M_1)\, l_1 = (x + M_2)\, l_2$$

or $$Wl_1 + M_1 l_1 = xl_2 + M_2 l_2$$

but $$M_1 l_1 = M_2 l_2 \qquad \dots \quad (iii)$$

∴ By subtraction

$$Wl_1 = xl_2 \qquad \dots \quad (iv)$$

If the weight of the body, as given by the balance be y, when it is placed in the right hand pan as in fig. 11.23*b*, then by taking moments about C again

$$(y + M_1)\, l_1 = (W + M_2)\, l_2$$

or $$yl_1 + M_1 l_1 = Wl_2 + M_2 l_2$$

but $$M_1 l_1 = M_2 l_2 \qquad \dots \quad (iii)$$

∴ by subtraction

$$yl_1 = Wl_2 \qquad \dots \quad (v)$$

Multiplying equations (*iv*) and (*v*)

$$W^2 l_1 l_2 = xy l_1 l_2$$

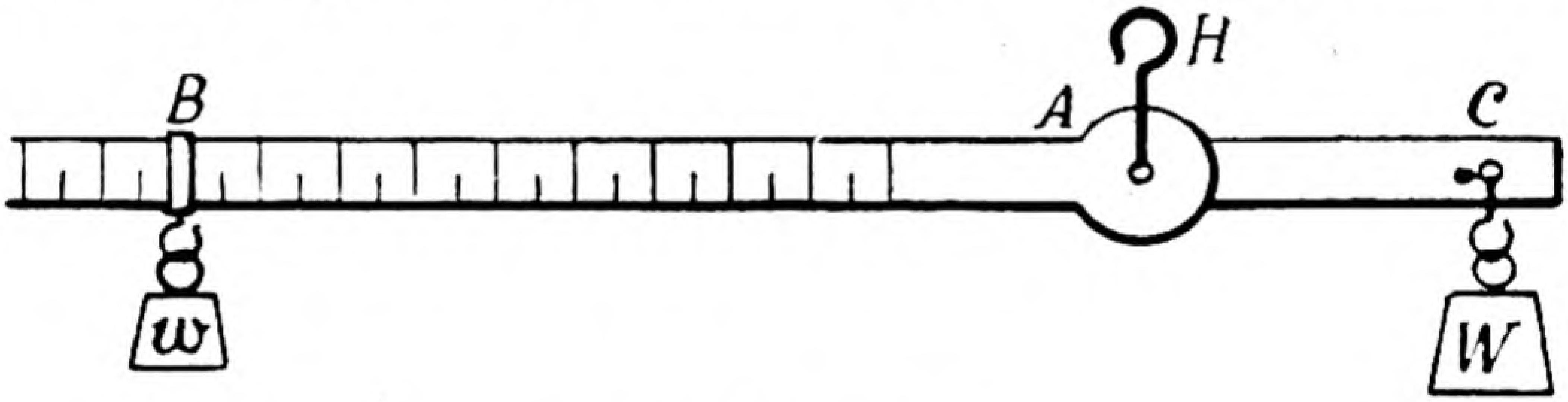
or $$W^2 = xy$$

or $$W = \sqrt{xy}$$

Therefore true weight = Geometric mean of the weights shown by the balance when the body is placed in the left and the right pans.

(3) This is called the substitution method and can be used to find the true weight of a body when the balance has any one or both of the above defects. The object to be weighed is placed in the left hand pan and sand or any other fine material is placed in the right hand pan until the beam is horizontal. Now the body is removed from the left hand pan and weights are added in the pan so that the beam again becomes horizontal. The weights added in the left hand pan give the true weight of the body.

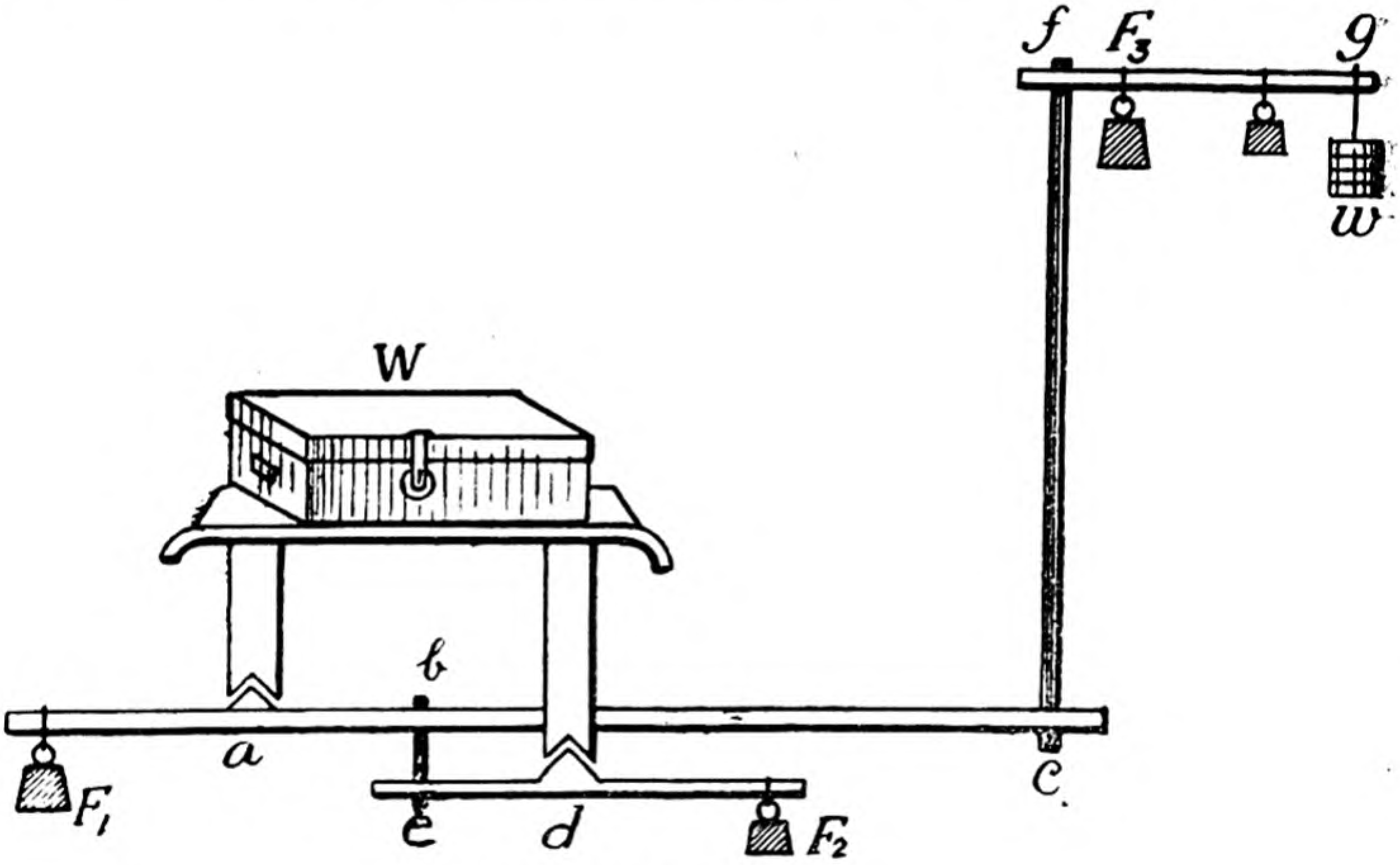
The steel-yard and the weighing machine. The common steel-yard utilises the principle of the lever of the first order, and it enables heavier articles to be weighed with a comparatively much small weight. It consists of a uniform steel rod, fig. 11.24, which is pivoted at A, from which



The Steel-yard
Fig. 11.24

place it is suspended by a hook H. One of the arms AC is shorter and of a fixed length, the object to be weighed being suspended from a point C at the end of this arm. The other arm is much longer and a weight ω can be moved along it, being brought to such a position that the steelyard takes up a horizontal direction. This longer arm AB has been previously calibrated or marked with graduations by suspending known weights from C and noting the position of the smaller movable weight ω on the longer arm. When the rod is horizontal the moments of W and ω around the fulcrum A are equal. It is a handy machine enabling heavier objects to be easily weighed.

The weighing machine, commonly seen on the platforms of railway stations for quickly weighing luggage and heavy parcels, is also made on the principle of the lever. Instead of a single lever, three levers suitably connected are used in such machines in order to get equilibrium between a small effort and a large weight, also keeping in mind that the machine is made as compact as possible. The weight W is placed on the platform of the weighing machine and a downward force is communicated by two legs from the platform, resting on wedges at the points *a* and *d*, fig. 11.25., of the levers *abc* and *de* with their fulcrums at F_1 and F_2. The force at *a* pushes the lever *abc* downwards. Therefore exerting a downward force at the point *c*. Similarly the force at *d* pushes the point *e* of the lever *de* downwards. Since *e* is connected to *b*, the downward movement of *e* causes a downward force on *b*, which is communicated as a downward force at *c*. Thus the total effect of the weight W is to produce a downward force acting at *c*. *c* is connected with the rod *cf* to the lever *fg* with its fulcrum at F_3. Thus a



A weighing machine

Fig. 11.25

downward force at c gives a downward force at f and it is counterbalanced by placing suitable weight ω suspended from g. A fine adjustment is made by moving a small weight between g and the fulcrum of the lever F_3.

The weighing bridge, by which motor trucks etc. can be weighed straigtaway, consists of a large platform level with the ground, and a system of levers on which it is supported. It works on the same principle as the weighing machine above described.

Summary.

A machine is a device by which force acting at one point may be made available at another point, or weaker forces acting over a longer distance may be converted into stronger forces acting over a shorter distance or *vice-versa*.

In any machine

$$\frac{\text{Resistance overcome or weight lifted}}{\text{Power or effort applied}}$$

$$= \frac{W}{P} = \text{Mechanical advantage.}$$

$$\frac{\text{Distance through which power moves}}{\text{Distance through which weight moves}}$$

$$= \frac{a}{b} = \text{Velocity Ratio}$$

$$\text{Efficiency of the machine} = \frac{\text{out put}}{\text{Input}} = \frac{W \times b}{P \times a} = \frac{W/P}{a/b}$$

$$= \frac{\text{Mechanical advantage}}{\text{velocity ratio}}$$

Percentage efficiency $=$ efficiency $\times 100$

In an ideal or perfect machine

Output $=$ Input or $W \times b = P \times a$

and $\frac{W}{P} = \frac{a}{b}$ or Mechanical advantage $=$ Velocity ratio.

and efficiency $= 1$.

But in an actual machine.

Output is always less than input.

Mechanical advantage is always less than velocity ratio.

Efficiency is always less than one.

Principle of a lever is

Power × Power arm = Weight × weight arm

$$\left.\begin{array}{l}\text{Mechanical advant-}\\\text{age of a lever}\end{array}\right]=\frac{\text{Weight}}{\text{Power}}=\frac{\text{Power arm}}{\text{weight arm}}$$

$$\left.\begin{array}{l}\text{Mechanical advantage}\\\text{of a wheel and } a\times b\end{array}\right]=\frac{\text{W}}{\text{P}}=\frac{\text{Radius of wheel}}{\text{Radius of } a\times b}=\frac{\text{R}}{r}.$$

A fixed pulley has no mechanical advantage, $\left(\frac{\text{W}}{\text{P}}=1\right)$.

In the case of a movable pulley

$\frac{\text{W}}{\text{P}}=2$, when friction and weight of the pulley are neglected.

For pulley block or block and tackle.

$\frac{\text{W}}{\text{P}}=n=$ number of segments of rope or string

Supporting the lower movable block.

The number of segments supporting the lower movable block is usually equal to the total number of pulleys in the pulley block.

Mechanical advantage of a Differential pulley $=\frac{2r_2}{(r_2-r_1)}$,

where r_2 is the radius of the bigger fixed pulley and r_1 is the radius of the smaller fixed pulley.

Inclined plane of inclination θ.

(*a*) When power acts parallel to the inclined plane

$$\frac{W}{P}=\frac{1}{\sin\theta}=\frac{\text{Length of inclined plane}}{\text{height of inclined plane}}=\frac{l}{h}.$$

(*b*) When power acts in the horizontal direction

$$\frac{W}{P}=\frac{1}{\tan\theta}=\frac{\text{Base of inclined plane}}{\text{height of inclined plane}}=\frac{b}{h}.$$

If θ be angle of the sharp edge of a wedge, then

$$\text{Mechanical advantage of the wedge}=\frac{1}{2\sin\frac{\theta}{2}}.$$

The distance between two consecutive threads of a screw, measured parallel to the axis of the cylinder on which the screw is cut, is called the pitch of the screw,

Mechanical advantage of a screw jack $=\frac{2\pi l}{h}$, where l is the length of the lever arm and h is the pitch of the screw.

Gear wheels are used to transmit power from one part of the machinery to another part. The velocities of two geared or connected wheels are in the inverse ratio of their radii or the number of teeth on them.

If A and B be two geared wheels, then

$$\frac{\text{velocity of wheel A}}{\text{velocity of wheel B}}=\frac{\text{Radius of wheel B}}{\text{Radius of wheel A}}$$
$$=\frac{\text{Number of teeth on wheel B}}{\text{Number of teeth on wheel A}}$$

The three requisites of a good balance are

(1) truth, (2) sensitiveness and (3) stability.

Conditions for truth

(*a*) Arms should be of equal length.

(*b*) Pans should be of equal weights.

(*c*) Centre of gravity of the beam should be vertically below its point of support.

Conditions for sensitiveness

(*a*) A light and long beam.

(*b*) Centre of gravity of the beam to be very near to the point of support.

Conditions for stability.

(*a*) Beam to be heavy.

(*b*) Centre of gravity of the beam should be at a distance from its point of support.

Weighing with a false balance :—

(*a*) When arms are of equal lengths but pans are of unequal weights.

True weight = Arithmetic mean of the weights of the body when weighed in the left and the right pans.

(*b*) When arms are unequal but beam becomes horizontal when the pans are empty.

True weight = Geometric mean of the weights shown by the balance when the body is placed in the left and the right pans.

(*c*) Substitution method. Place the body in one pan and make the beam horizontal by putting sand or any fine material in the other pan. Remove the body and put weights in its stead to make the beam horizontal again. Weights substituted for the body give the true weight of the body.

Exercises

1. (*a*) Define mechanical advantage, velocity ratio and efficiency of a machine. How are the three quantities related to one another ?

(*b*) Explain what is meant by the statement, "With a machine, what we gain in power we lose in distance."

2. Find mechanical advantage in the case of
(*a*) Differential pulley
(*b*) Wedge.

3. Find the mechanical advantage of an inclined plane—
(*i*) When the effort acts parallel to the plane.
(*ii*) When the effort acts in a horizontal plane.

(*P. U.*)

$$(i)\ \frac{1}{\sin\theta},\ (ii)\ \frac{1}{\tan\theta}.$$

4. Sketch the forces acting on a body of weight W supported on a smooth inclined plane by a force P acting up the plane, and find the relation between P and W in terms of the height and length of the plane.

Why is the above arrangement known as a "machine"?

(*P. U.*)

$$\frac{W}{P}=\frac{\text{Length}}{\text{height}}.$$

5. Find the mechanical advantage in the case of a screw. The lever of a screw is 21 inches long. If the screw has four threads to the inch, find the mechanical advantage of the machine. Neglecting friction, what effort is needed to lift 40,000 lb? (*P. U.*)

$$\text{Mechanical advantage}=\frac{2\pi l}{h}=\frac{2\times22\times21\times4}{7\times1}=528.$$

$$\text{Effort}=40000/528=75.8 \text{ lb}.$$

6. State the principle of work as applied to machines. Give examples. Sketch the system of pulleys in which the same string passes round all the pulleys and use the above principle to obtain a value for the mechanical advantage of the system.

7. A nut is placed one inch from the fulcrum of a nut cracker. If a force of 8 lbs. is applied at the end of the handle that is 7″ long, calculate the force with which the nut will be pressed.

Nut cracker is a lever of the second order

$$P=8 \text{ lb.} \qquad a=7''$$
$$W=? \qquad b=1''$$

$$P\times a=W\times b$$
$$8\times 7=W\times 1 \qquad \therefore \quad W=56 \text{ lb. wt.}$$

8. A mass of 150 lbs. resting on a smooth inclined plane is supported by a force of 15 lbs acting parallel to the plane. Find the inclination of the plane.

$$\frac{W}{P}=\frac{1}{\sin\theta}$$

or $$\sin\theta=\frac{P}{W}=\frac{15}{150}=\frac{1}{10}.$$

So inclination is one in ten.

9. In a pulley block consisting of 6 pulleys a force of 20 lbs. can balance a weight of 100 lbs. Find the velocity ratio, the mechanical advantage and efficiency of the machine. What is the weight of the lower block ?

Velocity ratio = no. of pulleys = 6

mechanical advantage $=\frac{W}{P}=\frac{100}{20}=5.$

$\therefore$ $$\text{Efficiency}=\frac{5}{6}\times 100=83{\cdot}3\%$$

If wt. of lower block $=w$

then $$100+w=6\times 20$$

$\therefore$ $$w=20 \text{ lb.}$$

10. The beam of a balance is horizontal when the pans are empty, but an object weighs 8 lbs. in one pan

and 9 lbs. in the other. What is the defect in the balance ? What is the true weight of the object ? unequal arms

$\sqrt{8 \times 9} = 8{\cdot}48$.

11. A plank weighing 100 lbs is suspended from a pulley block of 5 pulleys. A man weighing 140 lbs stands on the plank and pulls at the free end of the string of the pulley block. What force must he apply to support himself on the plank ?

Total weight to be lifted $= 100 + 140 = 240$ lbs.

If the man pulls the free end of pulley system with a force of P lbs wt. the total force supporting the plank and the man is 6P, since in this case 6 segments of the string are supporting the weight.

$$\therefore \quad 6\,P = 240$$

or $\quad P = 40$ lb wt.

12. A box weighing 400 lbs is to be loaded on a truck that is 4 ft. high by sliding it on a plank that is 20 ft long. If the force of friction between the plank and the box be 20 lbs wt. find the least force that will move the box on the plank.

100 lbs wt.

13. A wheel and axle is used to raise a bucket of coal weighing 240 lbs. The radius of the wheel is 24″ and of the axle is 4″. A force of 60 lbs applied at the wheel raises the bucket with uniform velocity. (*a*) Find the mechanical advantage of the wheel and axle. (*b*) What is the efficiency of the machine ?

(*a*) 4, (*b*) 66·7%

(14) A load of one ton is to be hauled up with a differential pulley whose radii are 12″ and 10″. If the weight of the lower movable pulley and frictional forces be neglected, find the least force that will raise the load.

$$\text{mechanical advantage} = \frac{2r_2}{(r_2 - r_1)} = \frac{2 \times 12}{(12 - 10)} = 12.$$

$$\therefore \quad \frac{W}{P} = 12$$

or $$P=\frac{W}{12}=\frac{2240}{12}=186.7 \text{ lbs wt.}$$

15. In a machine weight is lifted through 4″ when the point of application of power moves through 4 ft. Calculate the force that will be able to lift a load of 1800 lbs with this machine. 150 lbs.